Math 340, Nov 27

- Sensitivity Analysis
  2 for 1 deal

- Today & Monday:
  Poker game 2^{52} strategies A
  $\qquad\qquad\qquad 2 \qquad$ Betty

  (Convexity)

Take solved LP, solved by simplex method

$x_B = A_B^{-1}(\vec{b} - A_N \vec{x}_N)$

$z = \;\ldots\; + (c_N^T - \underbrace{c_B^T A_B^{-1} A_N}) \vec{x}_N$

Modify $\vec{c}$ slightly, expect same final dictionary, ...
... same basic x_B
non-basic x_N

If $\vec{c}$ changes slightly
$\vec{b}$ — — —
A_B, A_N " "

In practice: change $\vec{c}$, just need to recompute z row

if $\vec{c}$ changes slightly beyond the point where x_B, x_N are still a final dictionary

$x_B =$

$z = 36 - 2x_2 - 5x_3 + 0.01x$
— — —

expect only one pivot away from optimal ...

If $\vec{b}$ changes ... ☹

max $4x_1 + 5x_2$

s.t.

$x_1 + 2x_2 \le 8$

$x_1 + x_2 \le \beta$ ← vary from 5 to ...

$2x_1 + x_2 \le 8$

$x_1, x_2 \ge 0$

$x_1 = 2 + \ldots$
$x_2 = 3 + \ldots$
$x_5 = 1 + \ldots$
$z = 23 + \ldots$

$\beta = 5$

$\beta \ne 5$?

Dual dictionary

$x_1 = 2$
$x_2 = 3$
$x_5 = 1$
$z = 23$ — —

$y_1 = 1 +$
$y_2 = 3 +$ etc.

$w = -23 - y_3 - 2y_4 - 3y_5$

Dual LP:

$(y_4)\quad 4 \le y_1 + y_2 + 2y_3$
$(y_5)\quad 5 \le 2y_1 + y_2 + y_3$
$w = -8y_1 - 5y_2 - 8y_3$ β

Say we move slightly beyond feasibility

funct(β)

$x_1 =$ (+) — —
$x_2 =$ slightly — — — $-(.01)$
$x_5 =$ (+) — —
$z =$ — —

leave the simplex method ☹

= ... but 2-for-1 deal ...

Dual LP:

$4 \le y_1 + y_2 + 2y_3$

— — — —

$w = -8y_1 - 5y_2 - 8y_3$

$(— \le 8, y_1$
$(— \le 5, y_2$
$(— \le 8, y_3$

Dual pivot = simplex method pivot in the dual

Simplex pivot $\xrightarrow{\text{Duality}}$ Dual pivot

$x_1 = +3$
$\quad = +2$
$\quad = -1$
$\quad = +5$
$z =$ — $-3x_1 - 5x_9 + 2x_7$

Dual

$= 3$
$= 5$
$= 2$

$-3y - 2y + 1y$
$-5y$

Similarly:

optimal dictionary

- can usually add a new decision variable easily in simplex method (add a drink

max $4x_1 + 5x_2$

$\hookrightarrow$ max $4x_1 + 5x_2 + 2x_3$)

=

- usually adding a inequality can be more difficult

(dual) adding a variable

$y_4 = -4 + y_1 + y_2 + 2y_3 \geq 0$

$y_5 = -5 + 2y_1 + y_2 + y_3 \geq 0$

max $w = \underbrace{-8y_1 - 5y_2 - 8y_3}_{\text{dual fens}}$

not fasible

$y_2 = 10$

$y_1 = y_3 = 0$

$\Rightarrow w = -50$

(claim $w^* = -23$)

Poker Game: 2^{52} stretegies..

Alice & Betty put 1 penny each into the middle

Alice draws a card

(Old: Black or Red
50% 50%)

New:
High Card #52, Next highest #51,
A Spades, A Hearts ...,
2nd to Lowest, Lowest

$(x_1 + 2x_2 \leq 8)$ y_1

$(x_1 + x_2 \leq 5)$ y_2

$(2x_1 + x_2 \leq 8)$ y_3

$(3x_1 + 4x_2 \leq 12)$ (new y_4)

=

Theme 2-for-1 in simplex method via duality.

=

Game theory..

=

Alice draws card # C

$C = 52, 51, \dots, 1$

Betty wins $\frac{51-(C-1)}{51}$

Alice wins: $C-1, C-2, \dots, 1$

$$\frac{C-1}{51}$$

=

Betty: Calls or Folds

Alize: Ace Spades: Fold or Bet
Ace Hearts: Fold or Bet
⋮
Lowest, 2 : Fold or Bet

2 choices for 52 cards

Alize looks at card:
folds or bets (1 penny)

Betty (no info) (draws a card) Betty can't see it
folds or calls

Alice sees Ace Spades

51 cards left:

Alice wins $\frac{51}{51}$ times

Alice sees Ace Hearts

wins $\frac{50}{51}$ times

Large # rows

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 5 & 6 \\ 7 & 10 \\ 10 & 7 \end{bmatrix}$$

2 cds