Math 340, Nov 25

Today: Finish Sensitivity Analysis

Back to Poker / Games . .

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Last time:

max $4x_1+5x_2$ s.t.

$x_1+2x_2\le 8$

$x_1+x_2\le 5$

$2x_1+x_2\le 8$

$x_1,x_2\ge 0$

$x_3=8-x_1-2x_2$

$x_4=5-x_1-x_2$

$x_5\ 8-2x_1-x_2$

Anticipate $(x_1,x_2)=(2,3)$ optimal

Computation is direct

$z=4x_1+5x_2 \to 4x_1+\alpha x_2$

$z=4(2+x_3-2x_4)+\alpha(3-x_3+x_4)$

$=(8+3\alpha)+(4-\alpha)x_3$

$(-8+\alpha)x_4$

expect $(2,3)$ optimal,

$4x_1+\alpha x_2=4\cdot 2+3\cdot\alpha$

$$\begin{aligned}
x_1&=2+x_3-2x_4\\
x_2&=3-x_3+x_4\\
x_5&=1-x_3+3x_4\\
z&=23-x_3-3x_4
\end{aligned}$$

$4x_1+5x_2 \to 4x_1+\alpha x_2$

Near $\alpha=5$ nothing much changes, same final dictionary basic & non-basic . . .

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$\begin{matrix}8\\5\\8\end{matrix} \leftrightarrow \begin{matrix}8\\\beta\\8\end{matrix} \to ?$

$\alpha=5$

$z=23-x_3-3x_4$

as long as $(4-\alpha)$, $(-8+\alpha)$ ≤ 0

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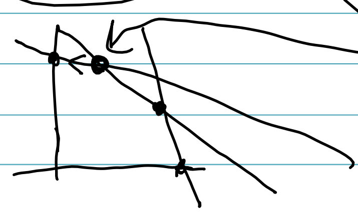
So $4\le\alpha\le 8$

Dictionary basic vars, $\vec{x}^*=(2,3)$

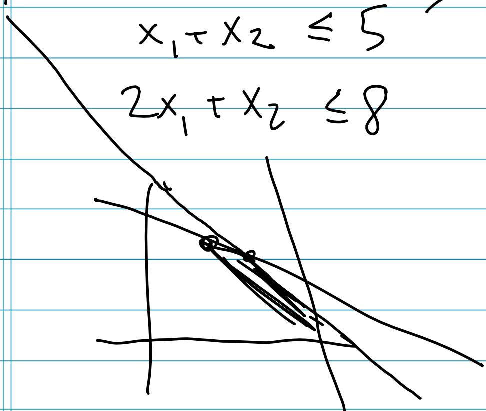
$\alpha<4$ or $\alpha>8$



We expect one pivot,
$\alpha < 4$ but near 4

Try

$$\max\ 4x_1+5x_2$$
$$x_1+2x_2 \le 8$$
$$x_1+x_2 \le 5 \to \le \beta$$
$$2x_1+x_2 \le 8$$



$\alpha = 4.01$:

$$z = 8+3\alpha + \underbrace{(4-4.01)}_{<0} x_3$$
$$+(\quad) x_4$$

$\alpha = 3.99$

$$z = \_\_ + (4-3.99)\, x_3$$
$$(\quad)\, x_4$$

$$z = \_\_ + (0.01)\, x_3 + (\quad)\, x_4$$

$$x_1 = 2 + \_\_$$
$$x_2 = 3 + \_\_$$
$$x_5 = 1 + \_\_$$
$$z = 23$$

Solution at $x_B = \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix}$ is changing...

... :(

$$z = 4x_1+5x_2$$
$$x_3 = 8 - x_1 - 2x_2$$
$$x_4 = 5 - x_1 - x_2 \quad (5 \to \beta)$$
$$x_5 = 8 - 2x_1 - x_2$$

$$x_1 = 2 + x_3 - 2x_4$$
$$x_2 = 3 - x_3 + x_4$$
$$x_5 = 1 - x_3 + 3x_4$$
$$z = 23 - x_3 - 3x_4$$

$$x_B = A_B^{-1}(b - A_N x_N)$$
$$z = (c\ b) + (\quad)\, x_N$$

C's

$$b = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix} \to \begin{bmatrix} 8 \\ \beta \\ 8 \end{bmatrix}$$

if x_1, x_2, x_5 basic:

$$\begin{matrix} x_1 \\ x_2 \\ x_5 \end{matrix} = (A_B^{-1}) \begin{bmatrix} 8 \\ \beta \\ 8 \end{bmatrix} + \sim$$

$x_2 = (\text{funct of } \beta)$

$x_5 = (\text{funct of } \beta)$

$z = \underline{\qquad} - x_3 - 3x_4$

$\text{funct}(\beta) \qquad - 3\tilde{x}_4$

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$x_1^* = 2\beta - 8$ as long as $2\beta - 8 \geq 0$

$x_2^* =$

$x_3^* =$

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What happens when turns slightly negative?

Tricky method:

$$x_4 = 5 - x_1 - x_2$$

$$\tilde{x}_4 = \beta - x_1 - x_2$$

$$= 5 - x_1 - x_2 + (\beta - 5)$$

$$\tilde{x}_4 - (\beta - 5) = x_4 = 5 - x_1 - x_2$$

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$$x_1 = 2 + x_3 - 2x_4$$

$$= 2 + x_3 - 2(\tilde{x}_4 - (\beta - 5))$$

$$= 2 + 2(\beta - 5) + x_3 - 2\tilde{x}_4$$

$$= 2\beta - 8$$

$\max \vec{c} \cdot \vec{x}$

s.t. $A\vec{x} \leq \vec{b}$

$\max -\vec{b} \cdot \vec{y}$

$-A^T \vec{y} \leq -\vec{c}$

modifying ☹

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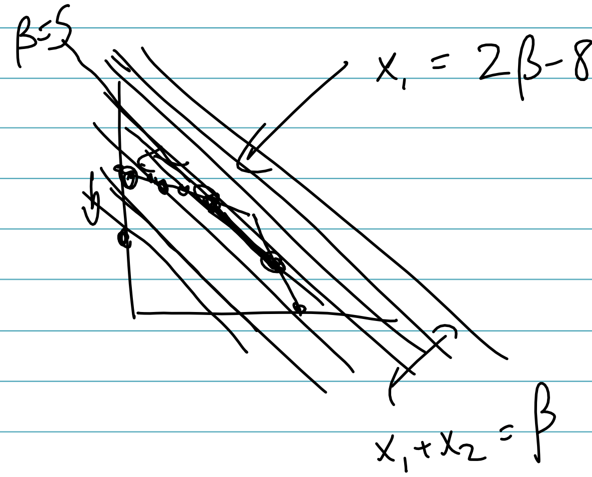

2-for-1

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Dual from original

$x_3 = 8 - x_1 - 2x_2$

$x_4 = \beta - x_1 - x_2$

$x_5 = 8 - 2x_1 - x_2$

$z = 4x_1 + 5x_2$

$\max -8y_1 \overset{-\beta}{-5y_2} - 8y_3$

$4 \leq y_1 + y_2 + 2y_3$

$5 \leq 2y_1 + y_2 + y_3$

Dual:

$A\vec{x} \leq \vec{b}$ ← modifying $\vec{b}$ ☹

$\max \vec{c} \cdot \vec{x}$ ← modifying $\vec{c}$ direct

$x_3 = 8 - x_1 - 2x_2$
$x_4 = 5 - x_1 - x_2$
$x_5 = 8 - 2x_1 - x_2$
$z = 4x_1 + 5x_2$

$y_4 = -4 - y_1 - y_2 - y_3$
$y_5 = -5 - y_1 - y_2 - y_3$
$w = -8y_1 - 5y_2 - 8y_3$

$x_1 = 2 + x_3 - 2x_4$
$x_2 = 3 - x_3 + x_4$
$x_5 = 1 - x_3 + 3x_4$
$z = 23 - x_3 - 3x_4$

$y_1 = 1 - y_4 + y_5 + y_3$
$y_2 = 3 + 2y_4 - y_5 - 3y_3$
$w = -23 - 2y_4 - 3y_5 - 1y_3$
$=$
$w = -8y_1 - 5y_2 - 8y_3$
$-8y_1 - \beta y_2 - 8y_3$

$x_1 \longleftrightarrow y_4$
$x_2 \longleftrightarrow y_5$
$x_3 \longleftrightarrow y_1$
$x_4 \longleftrightarrow y_2$
$x_5 \longleftrightarrow y_3$

$w = -8(\qquad\qquad)$
$-\beta(\qquad\qquad)$
$-8y_3$
$= (\ )y_4\ (\ )y_5\ (\ )y_3$

"Dual Pivot"

$y_1 = +$
$y_2 = +$
$w \quad -\_\_ - \_\_ - \_\_$