Math 340, Nov 23

- Sensitivity Analysis
  1. Generalities
  2. $\vec{b}$ perturbations
  3. $\vec{c}$ perturbations

$\max \vec{c}\cdot\vec{x}$ s.t.

$A\vec{x} \le \vec{b}$

$\vec{x} \ge 0$

- Applications: - Game Theory
  - Matching Theory

Review sample exam problems

$c = \begin{bmatrix} 4 \\ \cancel{8} \end{bmatrix}$

$\alpha = 10$, $\alpha = 8$, $\alpha = 5$, $\alpha = 4$, $\alpha = 3$

$\max\ 4x_1 + \alpha x_2$

$x_1 + 2x_2 \le 8$

$x_1 + x_2 \le 5$

$2x_1 + x_2 \le 8$

$x_1, x_2 \ge 0$

$c = \begin{bmatrix} 4 \\ \cancel{8} \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$

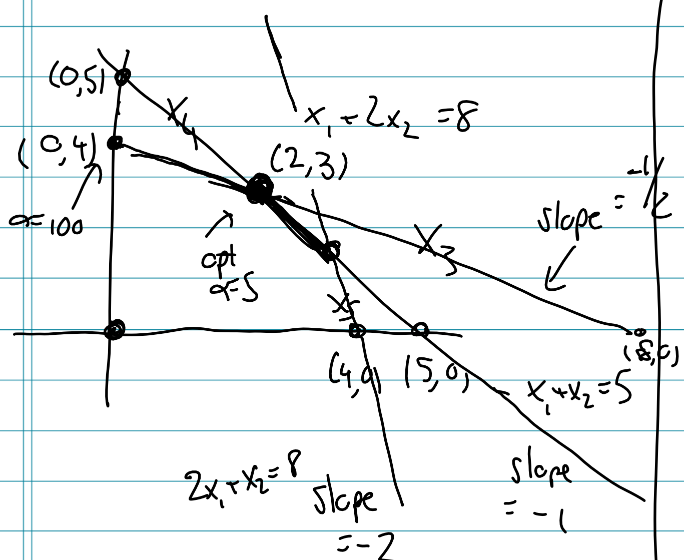
$\max\ 4x_1 + 5x_2$ ← profit price changes $5 \to 5.01$, $5 \to 4.99$

Generally!

$$\boxed{x_B = A_B^{-1}(\vec{b} - A_N \vec{x}_N)}$$

$$z = c_B^T x_B + c_N^T x_N$$

$$\boxed{z = c_B^T A_B^{-1}\vec{b} + (c_N^T - c_B^T A_B^{-1} A_N)\, x_N}$$



For $z = 4x_1 + 5x_2$ — op (2,3)

$z = 4x_1 + 100x_2$ — (0,4)

$z = 4x_1 + 8x_2$ — either (0,4) or (2,3)

Start

$x_3 = 8 - x_1 - 2x_2$

$x_4 = 5 - x_1 - x_2$

$x_5 = 8 - 2x_1 - x_2$

$z = 4x_1 + 5x_2$

→

$x_1 = 2 + x_3 - 2x_4$

$x_2 = 3 - x_3 + x_4$

$x_5 = 1 - x_3 + 3x_4$

$z = 23 - 3x_3 - x_4$

$z = 4x_1 + \alpha x_2$

α varies...

$\vec{c} = \begin{bmatrix} 4 \\ \alpha \\ 0 \\ 0 \\ 0 \end{bmatrix}$ ← $\vec{b}, A_N, A_B$ is the same

$\vec{c}$ varies, only need look at $z = \ldots$

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$x_1 = 2$
$x_2 = 3$ ~~+ax_3 + blah~~ blah
$x_5 =$

$z = 4x_1 + (4.01)x_2$
$= \underbrace{\text{old } z}_{4x_1+5x_2} + (-0.99)x_2$
$\boxed{= 23 - 3x_3 - x_4}$
$+ (-.99)(3 - x_3 + x_4)$
$= 23 + (-.99)3$
$+ (-3 + (-.99)(-1))x_3$
$+ (-1 + (-.99) \cdot 1)x_4$

for $z = 4x_1 + 3x_2$ | optimal $(3,2)$
$z = 4x_1 + (.001)x_2$ | $(4,0)$

at $z = 4x_1 + 4x_2$ $(2,3)$ or $(3,2)$ are optimal
$4x_1 + 3.99x_2$ $(3,2)$ only

e.g.
max $z = 4x_1 + (4.01)x_2$
favor x_2

$x_2 = \begin{cases} -3 + x_4 \\ -8 + x_1 + 2x_2 \quad 5 - x_1 - x_2 \end{cases}$
$x_2 - 3$

$z = 8 + 12.03$
$+ (4 - 4.01)x_3$
$+ (-8 + 4.01)x_4$
$= 20.03 - (.01)x_3$
$- (3.99)x_4$

$z = 4x_1 + (4.01)x_2$
$\boxed{\begin{aligned} &= 4(2 + x_3 - 2x_4) \\ &+ 4.01(3 - x_3 + x_4) \end{aligned}}$

$x_3 = 8 - x_1 - 2x_2$
$x_4 = 5 - x_1 - x_2$
$x_5 = 8 - 2x_1 - x_2$

$x_1 = 2 + x_3 - 2x_4$
$-2x_4 = -10 + 2x_1 + 2x_2$
$x_3 = 8 - x_1 - 2x_2$

$z = 4x_1 + 3.99x_2$

$x_1 = 2 + x_3 - 2x_4$

$x_2 = 3 - x_3 + x_4$

$x_5 = 1 - x_3 + 3x_4$

$\cancel{z = 23}$

$$\boxed{z = \_ + (.01)x_3 + (-5.01)x_4}$$

Now not optimal

at $z = 4x_1 + 4.01x_2$

$z = \_ - (\ )x_3 - (\ )x_4$

max $4x_1 + 3.99x_2$

s.t. $\equiv$

opt $(2,3) \to (3,2)$

$z = 4(2 + x_3 - 2x_4)$

$3.99(3 - x_3 + x_4)$

$= 8 + 11.97 + (4 - 3.99)x_3$

$(-8 + 3.99)x_4$

Pivot x_3 in?

x_5 leaves

$x_5 = 1 - x_3 + 3x_4$

$x_3 = 1 - x_5 + 3x_4$

$z = \text{const} + (.01)x_3$

$+ (-5.01)x_4$

$= \text{const} + (.01)(1 - x_5 + 3x_4)$

$+ (-5.01)x_4$

$= \text{const} + (-.01)x_5 + (-5.01 + \text{little})x_4$