Before this ---

LP ~ 

$$\max \ \vec{c}\cdot\vec{x} \quad \text{s.t.} \quad \underline{A}\vec{x} \le \vec{b}, \quad \vec{x} \ge \vec{0} \qquad \vec{x} = \vec{x}_{dec}$$

Roughly dictionary:

$$\underbrace{\left[\begin{array}{c:c} A & \underline{I} \end{array}\right]}_{A_{big}} \underbrace{\begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{slack} \end{bmatrix}}_{\vec{x}} = \vec{b}$$

$$z = \vec{c}\cdot\vec{x}_{dec} = \begin{bmatrix} \vec{c} \\ \vec{0} \end{bmatrix} \cdot \begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{slack} \end{bmatrix}$$

Math 340, Nov. 20

Sensitivity Analysis --

$$\max \ 4x_1 + 5x_2 \quad \text{s.t.}$$
$$x_1 + 2x_2 \le 8$$
$$x_1 + x_2 \le 5$$
$$2x_1 + x_2 \le 8$$
$$x_1, x_2 \ge 0$$

solve

What if 5 in $\max 4x_1 + 5x_2$ increases?

What if $\vec{b}$ change a little?

$$\boxed{z = \vec{c}_N^{\,T}\vec{x}_N + \vec{c}_B^{\,T}\vec{x}_B}$$
$$= \vec{c}_N^{\,T}\vec{x}_N + \vec{c}_B^{\,T}\left(A_B^{-1}\left(\vec{b} - A_N x_N\right)\right)$$
$$= \vec{c}_B^{\,T}A_B^{-1}\vec{b} + \left(\vec{c}_N^{\,T} - \vec{c}_B^{\,T}A_B^{-1}A_N\right)\vec{x}_N$$

Revised Simplex:

$$\vec{x}_B = \text{- -}$$
$$z = \text{- - -} \left(\left(\vec{c}_B^{\,T}A_B^{-1}\right)A_N\right)$$

① ② → difficulty

if you can't beat simplex method here

$$\vec{x}_{slack} = \vec{b} - A_{orig}\vec{x}_{dec}$$
$$= A_{big}\cdot\vec{x}_{all} = \vec{b}$$

$\vec{x}_N$ = non-basic

$\vec{x}_B$ = basic

$$A_B\vec{x}_B + A_N\vec{x}_N = \vec{b}$$

$$\boxed{\vec{x}_B = A_B^{-1}\left(\vec{b} - A_N x_N\right)}$$

$$z = \vec{c}_{big}\cdot\vec{x}_{all}$$

① Perturbation, ② Revised Simplex:

$$\vec{X}_B = A_B^{-1}(\vec{b} - A_N X_N)$$

$$z = \vec{C}_N^T \vec{X}_B + \vec{C}_B^T (\quad)$$

$$= \underbrace{C_B^T A_B^{-1} \vec{b}}_{\text{not so important}} + (\vec{C}_N - \vec{C}_B A_B^{-1} A_N) X_N =$$

Parenthesis

Max $4x_1 + 5x_2$

s.t. $x_1 + 2x_2 \le 8$

$x_1 + x_2 \le 5$

$2x_1 + x_2 \le 8$

$x_1 = 2 - x_4 + x_3$

$x_2 = 3 - x_4 + x_3$

$x_5 = 1 + x_4 + 3x_3$

$z = 23 - x_4 - 3x_3$

$x_1 + 2x_2 \le 8$ (tons espresso)

$x_1 + x_2 \le 5$ (volume to ship)

$2x_1 + x_2 \le 8$ (milk)

Cap.

$x_1 = 2$

$x_2 = 3$ (hopefully not needed)

$x_5 = 1$

$z = 23 - x_4 - 3x_3$

Before:

$$\vec{X}_{slack} = \vec{b} - A\vec{X}_{dec}$$

$$\vec{b} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \\ \vdots \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \vdots \end{bmatrix}$$

Why does perturbation work

$$\vec{X}_B = A_B^{-1}\left(\vec{b} + \begin{bmatrix} \quad \end{bmatrix}\right) - A_B^{-1} A_N \vec{X}_N$$

z = less important

Perturb

$4x_1 + 5x_2$

=

Make up units:

x_1 = turkey spice cappuccino

x_2 = voodoo drink cans

Max $4x_1 + 5x_2 = \$$

$\$$/Capp. Cap

5 x_2 voodoo cans

$\$$/Voodoo

$\$$/Voodoo price 5 → $5.01/voodoo

→ $4.99

important

X_B [blah blah blah] $\vec{b}$ | unimportant — — | $\vec{X}_N$

$z = (\text{unimportant}) + (\vec{C}_N^T - \vec{C}_B^T A_B^{-1} A_N)\vec{X}_N$

$= X_B = \text{blah}\ \vec{b}$ | unimportant

$z = (\text{unimport}) + (\text{blah}\ \vec{C}\ \text{blah blah})\vec{X}_N$

① change $\vec{C}$, then z-row coefs change

② change $\vec{b}$, constants for dictionary change

$X_B = A_B^{-1}\vec{b} - A_B^{-1} A_N \vec{X}_N$

$\begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix}$

$z = (C_B^T A_B^{-1}\vec{b}) + (C_N^T - C_B^T A_B^{-1} A_N) \cdot X_N$

say $4x_1 + 5x_2 \rightsquigarrow 4x_1 + 4.9 x_2$

$= C \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $C = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

$\hat{z} = 4(2 - x_4 - x_3) + (5.01)(3 - x_4 + x_3)$

$= 4(2 - x_4 - x_3) + 5(3 - x_4 + x_3)$ } old z

$+ (.01)(3 - x_4 + x_3)$

$= 23 - x_4 - 3x_3 + .03 - .01 x_4 + .01 x_3$

$= (23 + (.01)3) + (-1 - .01)x_4 + (-3 + .01)x_3$

$23 + (.01)\cdot\beta_1$

max $4x_1 + 5x_2$ ⇝ $5.01 x_2$ ⇝ $4.99 x_2$

s.t.

$x_1 + 2x_2 \le 8$

$x_1 + x_2 \le 5$

$2x_1 + x_2 \le 8$

$x_1 = 2 - x_4 - x_3$

$x_2 = 3 - x_4 + x_3$

$x_5 = 1 - x_4 + 3x_3$

$z = 23 - x_4 - 3x_3$

$\hat{z} = 4x_1 + (5.01)x_2$