

Before this ---

LP ~ (in)

$$\begin{aligned} \max \quad & \vec{c} \cdot \vec{x} \\ \text{s.t.} \quad & A\vec{x} \leq \vec{b} \\ & \vec{x} \geq \vec{0} \end{aligned} \quad \vec{x} = \vec{x}_{dec}$$

Roughly dictionary:

$$\begin{bmatrix} A & I \\ \hline A_{big} & \end{bmatrix} \begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{stack} \end{bmatrix} = \vec{b}$$

$$z = \vec{c} \cdot \vec{x}_{dec} = \begin{bmatrix} \vec{c} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{stack} \end{bmatrix}$$

Math 340, Nov. 20

Sensitivity Analysis ---

$$\begin{aligned} \max \quad & 4x_1 + 5x_2 \quad \text{s.t.} \\ & x_1 + 2x_2 \leq 8 \\ & x_1 + x_2 \leq 5 \\ & 2x_1 + x_2 \leq 8 \\ & x_1, x_2 \geq 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{solve}$$

What if 5 in  $\max 4x_1 + 5x_2$  increases?

What if change a little?

$$\begin{aligned} [z] &= \vec{c}_N \cdot \vec{x}_N + \vec{c}_B \cdot \vec{x}_B \\ &= \vec{c}_N^T \vec{x}_N + \vec{c}_B^T (A_B^{-1} (\vec{b} - A_N \vec{x}_N)) \end{aligned}$$

$$= \vec{c}_B^T A_B^{-1} \vec{b} + \left( \vec{c}_N^T - \vec{c}_B^T A_B^{-1} A_N \right) \vec{x}_N$$

Revised Simplex:

$$\begin{aligned} \vec{x}_B &= \dots \\ \vec{z} &= \dots \left( \begin{bmatrix} \vec{c}_N^T & A_B^{-1} \\ \vec{c}_B & A_B \end{bmatrix} A_N \right) \end{aligned}$$

difficulty  
if you can't beat simplex method here

$$\vec{x}_{stack} = \vec{b} - A_{orig} \vec{x}_{dec}$$

$$A_{big} \cdot \vec{x}_{all} = \vec{b}$$

$\vec{x}_N$  = non-basic

$\vec{x}_B$  = basic

$$A_B \vec{x}_B + A_N \vec{x}_N = \vec{b}$$

$$\begin{aligned} \vec{x}_B &= A_B^{-1} (\vec{b} - A_N \vec{x}_N) \\ z &= \vec{c}_{big} \cdot \vec{x}_{all} \end{aligned}$$

① Perturbation, ② Revised Simplex:

$$\vec{x}_B = A_B^{-1} (\vec{b} - A_N x_N)$$

$$z = \vec{c}_N^T \vec{x}_B + \vec{c}_B^T$$

$$= \underbrace{\vec{c}_B^T A_B^{-1}}_{\text{not so important}} \vec{b} + (\vec{c}_N - \vec{c}_B A_B^{-1} A_N) x_N$$

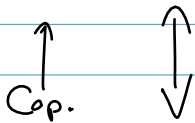
Max  $4x_1 + 5x_2$   
 s.t.  $x_1 + 2x_2 \leq 8$   
 $x_1 + x_2 \leq 5$   
 $2x_1 + x_2 \leq 8$

$x_1 = 2 - x_4 + x_3$   
 $x_2 = 3 - x_4 + x_3$   
 $x_5 = 1 + x_4 + 3x_3$   
 $z = 23 - x_4 - 3x_3$

$x_1 + 2x_2 \leq 8$  (tens espresso)

$x_1 + x_2 \leq 5$  (volume to ship)

$2x_1 + x_2 \leq 8$  (milk)



$x_1 = 2$   
 $x_2 = 3$   
 $x_5 = 1$

(hopefully not needed)

$z = (23) - x_4 - 3x_3$

Before:

$$\vec{x}_{\text{std}} = \vec{b} - A \vec{x}_{\text{dec}}$$

$$\vec{b} + \begin{bmatrix} \epsilon \\ \epsilon_2 \\ \epsilon_3 \\ \vdots \end{bmatrix} \text{ or } \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \end{bmatrix}$$

Why does perturbation work

$$\vec{x}_B = A_B^{-1} \left( \vec{b} + \begin{bmatrix} \epsilon \\ \epsilon_2 \\ \vdots \end{bmatrix} \right) - A_B^{-1} A_N \vec{x}_N$$

$z =$  less important

Perturb

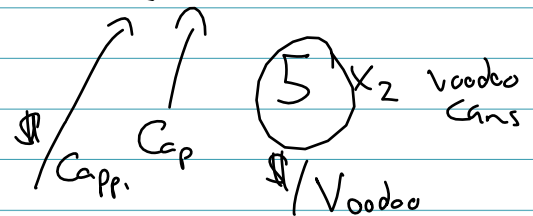
$$4x_1 + 5x_2$$

Make up units:

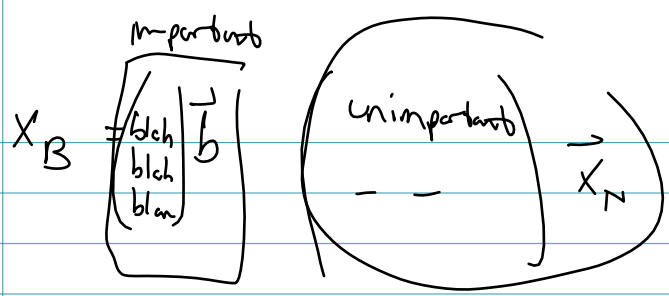
$x_1 =$  turkey spice cappuccino

$x_2 =$  voodoo drink cans

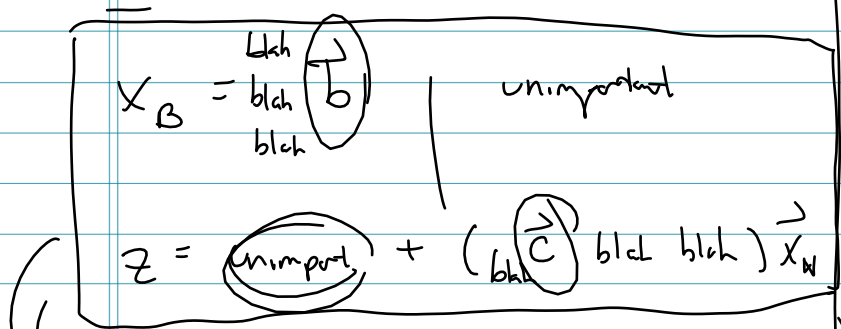
Max  $4x_1 + 5x_2 = \$$



$\$/\text{Voodoo price } 5 \rightarrow \$5.01/\text{vood}$   
 $\rightarrow \$4.99$



$$Z = (\text{unimportant}) + \underbrace{\left( \begin{matrix} \rightarrow^T & \rightarrow^T \\ C_N & -C_B A_B^{-1} A_N \end{matrix} \right)}_{\text{unimportant}} X_N$$



- 1) change  $\vec{c}$ , then z-row coeffs change
- 2) change  $\vec{b}$ , constants for dictionary change

$$X_B = A_B^{-1} \vec{b} - \underbrace{\left( A_B^{-1} A_N \right)}_{\text{unimportant}} X_N$$

$$Z = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_5 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_5 \end{pmatrix} = \left( C_B^T A_B^{-1} \vec{b} \right) + \left( C_N - C_B^T A_B^{-1} A_N \right) \cdot X_N$$

say  $4x_1 + 5x_2 \rightsquigarrow 4x_1 + 4.9x_2$

$$= C \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad C = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\begin{aligned}
 \hat{z} &= 4(2 - x_4 - x_3) + (5.01)(3 - x_4 + x_3) \\
 &= 4(2 - x_4 - x_3) + 5(3 - x_4 + x_3) + (.01)(3 - x_4 + x_3) \\
 &= 23 - x_4 - 3x_3 + .03 - .01x_4 + .01x_3 \\
 &= \underbrace{(23 + (.01)3)}_{23 + (.01)3} + \underbrace{(-1 - .01)}_{(-1 - .01)} x_4 + \underbrace{(-3 + .01)}_{(-3 + .01)} x_3
 \end{aligned}$$

$$\begin{aligned}
 \max \quad & 4x_1 + 5x_2 \rightsquigarrow 5.01x_2 \\
 \text{s.t.} \quad & x_1 + 2x_2 \leq 8 \\
 & x_1 + x_2 \leq 5 \\
 & 2x_1 + x_2 \leq 8
 \end{aligned}$$

$$\begin{array}{l}
 x_1 = 2 - x_4 - x_3 \\
 x_2 = 3 - x_4 + x_3 \\
 x_5 = 1 - x_4 + 3x_3 \\
 z = 23 - x_4 - 3x_3
 \end{array}$$

$$\hat{z} = 4x_1 + (5.01)x_2$$