Math 340 Nov 18

Revised Simplex:

blah blah blah blah blah blah

" " " $\underbrace{\vec{c}_B^{\,T} A_B^{-1} A_N}_{\text{main obstacle}}$ blah...

$\Big(\underbrace{(\vec{c}_B^{\,T} A_B^{-1})}_{\text{first}} \overset{\text{second}}{\cdot} A_N \Big) \vec{x}_N$ ...

$n = \#$ dec
$m = \#$ slack

just this, for an A_N $m \times n$ can cost $O(mn)$ operations w/o some assumption on A_N

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Last time: If A_N is sparse ☺

If we don't need all of A_N, $(\vec{c}_B^{\,T} A_B^{-1} \cdot A_N) \vec{x}_N$

i.e. don't need all the non-basic coefficients in z-row ... ☺

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First: $\Big(\underbrace{\vec{c}_B^{\,T}}_{1\times m \text{ row vector}} \cdot \bar{A}_B^{-1} \Big)$ ...

subtle issue as to how you can speed this up

A_B: $m \times m$
naive inverting: m^3 operations

Fast ways to invert A_B?

- Not generally
- In some applications yes...

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Eta matrices: Described Chvatal.

$$\underset{m \times m}{A_B} = \begin{bmatrix} 1 & 4 & -2 \\ 2 & 5 & 1 \\ 3 & 7 & 5 \end{bmatrix} \underbrace{\begin{bmatrix} x_4 \\ x_5 \\ x_2 \end{bmatrix}}_{\vec{x}_B}$$

$$- \Big(\underset{1\times m}{c_B^T} \underset{m\times m}{A_B^{-1}} \Big) - \text{---}$$

$x_4 \quad = \quad x_3$
$x_5 \quad\quad\quad x_3$
$x_2 \quad\quad\quad x_3$

x_3 in $\quad x_5$ out

Next step

$$\begin{array}{c} \begin{bmatrix} 1 & \alpha & -2 \\ 2 & \beta & 1 \\ 3 & \gamma & 5 \end{bmatrix} \begin{bmatrix} x_4 \\ x_3 \\ x_2 \end{bmatrix} \\ x_4 \quad\quad x_2 \end{array}$$

$$\vec{x}_B \begin{cases} x_4 = \text{- - -} \\ x_5 = \text{- -} \\ x_2 = \text{- -} \end{cases}$$

$z =$

$$A_B \vec{x}_B = \vec{b} - A_N \vec{x}_N$$

$$\begin{bmatrix} 1 & \alpha & -2 \\ 2 & \beta & 1 \\ 3 & \gamma & 5 \end{bmatrix} = \begin{bmatrix} 1 & ? & -2 \\ 2 & ? & 1 \\ 3 & ? & 5 \end{bmatrix} \begin{bmatrix} 1 & \cancel{\ } & 0 \\ 0 & \cancel{\ } & 0 \\ 0 & \cancel{\ } & 1 \end{bmatrix}$$

good start

$$=$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & \cancel{\ } \\ 2 & \cancel{\ } \\ 3 & \cancel{\ } \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix} = \begin{bmatrix} \cancel{\ } & -2 \\ \cancel{\ } & 1 \\ \cancel{\ } & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & u & -2 \\ 2 & v & 1 \\ 3 & w & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{matrix} \cancel{1} & \cancel{\alpha} \\ \cancel{2} & \cancel{8} \\ 3 & 7 \end{matrix}$$

vs.

$$\begin{bmatrix} 1 & u & -2 \\ 2 & v & 1 \\ 3 & w & 5 \end{bmatrix} \qquad \begin{bmatrix} 1 & \alpha & -2 \\ 2 & \beta & 1 \\ 3 & \gamma & 5 \end{bmatrix}$$

$$A_{B_{now}} \qquad A_{B_{next}}$$

$$\underbrace{\begin{bmatrix} 1 & \alpha & -2 \\ 2 & \beta & 1 \\ 3 & \gamma & 5 \end{bmatrix}}_{A_{B_{next}}} = \underbrace{\begin{bmatrix} 1 & u & -2 \\ 2 & v & 1 \\ 3 & w & 5 \end{bmatrix}}_{A_{B_{now}}} \underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}_{E}$$

eta matrix

$$\begin{bmatrix} s \\ t \\ r \end{bmatrix} = A_{B_{now}}^{-1} \cdot \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

quick

assuming A_B^{-1} can be applied to vectors

x_3 part of A

$$\underbrace{\begin{bmatrix} 1 & \alpha & -2 \\ 2 & \beta & 1 \\ 3 & \gamma & 5 \end{bmatrix}}_{A_{B_{next}}}^{\text{know}} = \underbrace{\begin{bmatrix} 1 & u & -2 \\ 2 & v & 1 \\ 3 & w & 5 \end{bmatrix}}_{A_{B_{now}}}^{\text{know}} \underbrace{\begin{bmatrix} 1 & s & 0 \\ 0 & t & 0 \\ 0 & r & 1 \end{bmatrix}}_{E}$$

s.t. $A_{B_{now}}$

$$\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 & u & -2 \\ 2 & v & 1 \\ 3 & w & 5 \end{bmatrix} \begin{bmatrix} s \\ t \\ r \end{bmatrix}$$

$$c_{B_{next}}^T A_{B_{next}}^{-1}$$

$$c_{B_{next}}^T (A_{B_{now}} E)^{-1}$$

$$\cdots c_{B_{now}}^T A_{B_{now}}^{-1} \cdots \Big\} \; c_{B_{next}}^T A_{B_{next}}^{-1}$$

$$C^T_{B_{2nd\,next}} \left(A_{B_{now}} E E_2\right)^{-1}$$

$$= C^T_B E_2^{-1} E_1^{-1} A_{B_{now}}^{-1}$$

$=$

Eventually

$$C^T_B \underbrace{E_k^{-1} E_{k-1}^{-1} \cdots E_2^{-1} E_1^{-1}}_{\substack{\text{too expensive} \\ \text{too much error}}} A_{B_{now}}^{-1}$$

$$\left(A_{B_{now}} E\right)^{-1} = E^{-1} A_{B_{now}}^{-1}$$

$$\left(C^T_{B_{next}} E^{-1}\right) \cdot A_{B_{now}}^{-1}$$

quick

$$\left(\begin{bmatrix}4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 7 & 0 \\ 0 & 9 & 0 \\ 0 & 2 & 1\end{bmatrix}^{-1}\right)$$

quick

$$A_{B_{2nd\,next}} = A_{B_{next}} E_2$$

$$= A_{B_{now}} E E_2$$

$$\begin{bmatrix}4 & 5 & 6\end{bmatrix} = \begin{bmatrix}x & y & z\end{bmatrix} \begin{bmatrix}1 & 7 & 0 \\ 0 & 9 & 0 \\ 0 & 2 & 1\end{bmatrix} \cancel{\begin{bmatrix}x & y & z\end{bmatrix}}$$

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$$\begin{bmatrix}4 & 5 & 6\end{bmatrix} = \begin{bmatrix}x & 7x + 9y + 2z & z\end{bmatrix}$$

$4 = x$:)

$6 = z$:)

$$5 = 7x + 9y + 2z$$

$$5 = 28 + 9y + 12$$

$$y = \frac{5 - 28 - 12}{9}$$

Say

$$\begin{bmatrix}4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 7 & 0 \\ 0 & 9 & 0 \\ 0 & 2 & 1\end{bmatrix}^{-1} = ?$$

Do we invert? ... no...

$$\begin{bmatrix}4 & 5 & 6\end{bmatrix}\begin{bmatrix} & & \end{bmatrix}^{-1} = \begin{bmatrix}x & y & z\end{bmatrix}$$