Math 340, Nov 16

What is the revised simplex method?

(1) Do this, do that - -

it does well...

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Truth: Revised simplex method only works well in certain cases...

$$X_B = \text{blah blah blah}$$

$$z = \text{blah blah}\Big(\text{blah} - \underbrace{\vec{c}_B^{\,T} A_B^{-1} A_N}_{\text{key to the whole method..}}\Big) X_N$$

Idea: don't have to write the whole dictionary..

don't write all this

$$\begin{cases} X_B = \left(A_B^{-1}\vec{b}\right) - A_B^{-1} A_N \vec{X}_N \end{cases}$$

$$z = - - \left(\vec{c}_N - \vec{c}_B A_B^{-1} A_N\right) \vec{x}_N$$

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Start max $\vec{c}\cdot\vec{x}$ s.t.

$$A\vec{x} \le \vec{b}$$
$$\vec{x} \ge 0$$

$\vec{x}$ n-dimensional, $A = A_{orig}$ is an $m \times n$ matrix, $\vec{b}$ m-dimensional

Rough idea:

(1) Focus on $z = - \left(\vec{c}_B^{\,T} A_B^{-1} A_N\right) \vec{x}_N$

$$\ldots\Big(\ldots \quad \vec{c}_B^{\,T} \cdot A_B^{-1} \cdot A_N \Big) \vec{x}_N$$

$\vec{c}_B^{\,T}$: $1 \times m$ row vector

A_B^{-1}: $m \times m$ — serious problem

A_N: serious problem

(I) If A_B^{-1} is ~~hard~~ costly to compute - - -

- m small, say $m = n^{1/4}$ - -

Naive invert $m^3 = n^{3/4}$

$$\left[A \mid I\right]\begin{pmatrix} \vec{x}_{dec} \\ \hline \vec{x}_{slack} \end{pmatrix} = \vec{b}$$

$$z = \begin{bmatrix} \vec{c} \\ \hline 0 \end{bmatrix} \cdot \begin{bmatrix} \vec{x}_{dec} \\ \hline \vec{x}_{slack} \end{bmatrix}$$

n dec vars, m slack vars

$$\vec{x}_{slack} = \vec{b} - A_{orig}\vec{x}_{dec}$$

basic vars always m

non-basic always n

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Naive simplex method takes about $\underline{\underline{m \cdot n}}$ $\begin{cases}\text{mults} \\ \text{additions}\end{cases}$ per pivot

If

$$\left(\vec{c}_B^{\,T} A_B^{-1} \underbrace{[\cdots\cdots]}_{m} \right) \underbrace{\left[\vdots \right]}_{n \text{ cols}} A_N$$

then this multiplication takes mn mults, mn adds

For (II) often A_N is sparse ✓

$$A_N \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & & & \\ 0 & \vdots & & & \\ \vdots & 0 & & & \\ 0 & 0 & & & \end{bmatrix}$$ mostly zeros

(I) A_B^{-1} ?

- If A_B has a special structure s.t. A_B^{-1} ~~easy~~ fast to invert,
- From step to step, A_B and A_B^{-1} doesn't change much

(II) Look at

$$\left(\vec{c}_B^{\,T} A_B^{-1} \right) \cdot A_N$$

$1 \times m$ row vector

$m \times n$

If A_N is not sparse - -

For (II)

$$z = - - . \left(c_N^T - c_B^T A_B^{-1} A_N \right) \vec{x}_N$$

Do we need all of z row?

- Yes, at the final dictionary:

$$z = \_\_ - 3x_1 - 5x_7 - 20x_2 \cdots$$

all negatives

But in the middle

$$z = 20 + 5x_4 + 30x_5 + 20x_9 - 3x_{44} - 17x_{27} \cdots$$

Remark:

$$\vec{x}_B = A_B^{-1}\vec{b} - A_B^{-1} A_N \vec{x}_N$$

99% of each row & col is 0's

$m = n^{1/4}$ so

A_B^{-1} no problem..

$$[\vdots\vdots\vdots] \begin{bmatrix} 0\,0\,0 & \cdots & 0\,0\,0\,0\,0\,0\,0 \\ 0\text{'s} & \cdots \; 0\text{'s} & \cdot \; 0\text{'s} \end{bmatrix}$$

Have to compare with <u>naive</u> simplex method, mn FLOPs

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Rather than evaluating all of

$$c_B^T A_B^{-1} \cdot A_N X_N$$

do it one coefficient at a time.

What if

$$z = 30 + 2x_1 + 5x_3 - 4x_7 - 9x_6 + 12231\, x_8 + 21974\, x_{11}$$

- take x_8, x_{11} into basic
- guess that both into basis does well

$$\left(\cdot\ c_B^T A_B^{-1} \qquad A_N\right) \vec{x}_N$$

$$\underbrace{[\cdot - - - - -]}_{1\times m}\ [\ \vdots\ ]$$

x_5

z = blah blah blah $+30x_5$ blah blah blah

x_5 can enter...

(II) $\underbrace{\left(c_B^T A_B^{-1}\right)}_{\text{If you know vector}} A_N \vec{x}_N$

to find a z-row coef takes just m mult/add for each coeff.

Other possibility:

evaluate all of z row,

this give you guess as to which to try next few times

then

- one pivot: find all z row
- next few pivots find $c_B^T A_B^{-1} A_N x_N$ at a few like candidates to enter basis

Next time .. A_B^{-1} ..