Math 340, Nov 13

Revised Simplex Method ...

-- with honesty

$$\begin{bmatrix} A & \vdots & I \end{bmatrix} \begin{bmatrix} x_{dec} \\ \hline x_{slack} \end{bmatrix} = \vec{b}$$

big form of A

$$\begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \hline x_3 \\ x_4 \\ x_5 \end{bmatrix} = \vec{b} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

"x_1" "x_2"

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_5 = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

$\vec{x}_B$ = basic, $\vec{x}_N$ = non-basic variables

cols of $[A \mid I]$ corresp to x_B

$$A_B x_B + A_N x_N = \vec{b}$$

$$\vec{x}_B = A_B^{-1}\left(\vec{b} - A_N x_N\right)$$

not issue

$$z = \underbrace{\vec{c}_B^{\,T} \vec{x}_B}_{\text{big problem}} + \underbrace{\vec{c}_N^{\,T} \vec{x}_N}_{\text{no problem}}$$

$$\underbrace{\vec{c}_B^{\,T} A_B^{-1} (\vec{b}}_{\text{constant}} - A_N x_N)$$

no problem

$$\boxed{\underbrace{\vec{c}_B^{\,T} A_B^{-1} A_N}_{\text{the whole problem}}}$$

Pivot:

$$x_3 = 7 - 2x_1 - 9x_4 - \dots$$

x_4 enters $\quad x_3$ leaves

Computation

$$9x_4 = 7 - 2x_1 - x_3 - \dots$$

$$\boxed{x_4 = \frac{7}{9} - \frac{2}{9}x_1 - \frac{1}{9}x_3 \dots}$$

one division for each non-basic, n divisions

m slack, basic; n decision, non-basic

formulas

$$\vec{x}_B = A_B^{-1}\vec{b} - A_B^{-1} A_N \vec{x}_N$$

$$\vec{z} = \vec{c}_B^{\,T} A_B^{-1} \vec{b} + \left(\vec{c}_N^{\,T} - \vec{c}_B^{\,T} A_B^{-1} A_N\right) \vec{x}_N$$

The real issue!

1. $c_B^T A_B^{-1} A_N$ — the main problem

2. $A\vec{x} \le \vec{b}$ — $\boxed{A: m \times n \text{ matrix}}$

each pivot order mn operations (FLOPs)

Revised simplex is faster than simplex, i.e. fewer than Order $(m \cdot n)$ FLOPs per pivot, only in certain situations ...

$=$

Which situations?

Problem in revised simplex:

$$\left(\vec{C}_N - \vec{C}_B^{T} A_B^{-1} A_N\right) x_N$$

for the z-row

$$\begin{aligned} x_2 &= \quad +2x_4 \\ x_{20} &= \quad -3x_4 \\ x_{17} &= \quad +9x_4 \\ &\vdots \end{aligned} \Bigg\} \; m \text{ equations}$$

$x_4 =$ new thing

roughly mn operations, i.e. FLOPs (addition, subtraction, division, ...)

(2) Any fancier simplex has to beat mn FLOPs per pivot.

To invert A_B $m \times m$ naively, m^3 FLOPs

$$\left[A_B \,\middle|\, I_{m \times m}\right] \rightsquigarrow \left[I \,\middle|\, A_B^{-1}\right]$$

Simplex (ordinary) takes mn operations

if $n = m^4$,

$mn = m^5$

$$\Big(\big((-C_B^T)(A_B^{-1})\big) \cdot A_N\Big) \vec{x}_N$$

$(1 \times m)$ $\quad$ $(m \times n)$

(1) If A_N is sparse then revised simplex can work well...

$$\ldots\left(-C_B^T \quad A_B^{-1} \quad A_N\right) \vec{x}_N$$

$1 \times m \qquad m \times m$

$=$

Toy situation: Say that n much bigger...

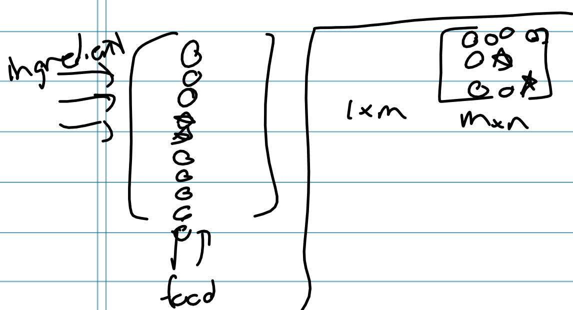
$n = m^4, m^5, \ldots$

takes ~~then~~ roughly

$O(mn)$ FLOPs

$$\left[A \mid I\right] \begin{bmatrix} x_{dec} \\ \hline x_{slack} \end{bmatrix} = \vec{b}$$

what if they have mostly 0's?



Now: naively multiply

$(1 \times m) \quad (m \times n)$