Ch 5 – "Economic interpretation of dual variables"

Idea max $4x_1 + 5x_2$ s.t.

$x_1 + 2x_2 \le 8$

Say $z = 4x_1 + 5x_2$ is \$

Say x_1 = # tables produce

x_2 = # chairs "

$4 = ?$ $\underbrace{4}_{\text{table}} x_1 = \$$ units

$4 = 4$ \$/table

$x_1 + 2x_2 \le 8$ hours of work

x_1 = tables

$1 \cdot x_1 + 2x_2 \le 8$ hours work

1 hour work / table . . .   could be 7.9, 8.1 . . .

hours work $(1 \cdot x_1 + 2x_2 \le 8) \cdot y_1$

≤ bound on \$

⟶ y_1 \$ / hours of work

Sensitivity analysis ⟷ Perturbation Analysis

Ch 7: Revised Simplex Method (without Revised Simplex Method...)

– Good ideas:

1. $x_{basic} = \underbrace{\text{— — —}\ x_{non\text{-}basic}}_{\text{General Formula}}$

2. 2a You don't need the entire dictionary for the simplex method

2b Change in dictionary per iteration does not change much

3. You can speed up the simplex with various techniques

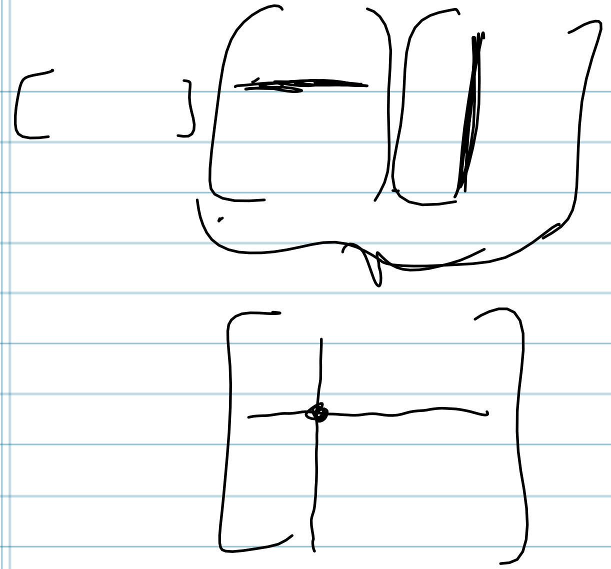
– General ideas

– Applied: pretty technical

e.g.

$\underbrace{[\ \ \ ]}_{\text{100 entries}}$ (1×100) $\begin{bmatrix} 100 \times 100 \end{bmatrix} \begin{bmatrix} 100 \times 100 \end{bmatrix}$

$ABC = (AB)C$
$\quad\ \ = A(BC)$

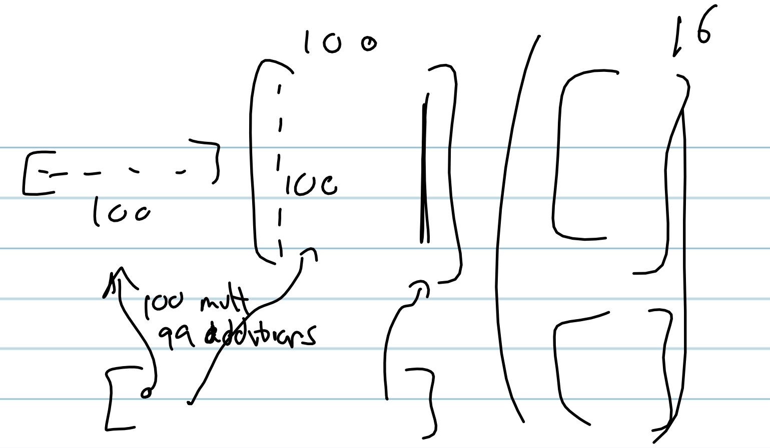


Have

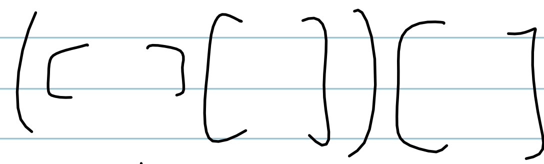
(100^2 entries, each times)

(100 mult, 99 adds)

order 100^3 FLOPS



roughly 100 (100 mult, 99 adds)



roughly

2 times 100 (100 mults, 100$\pm$1 adds)

order 100^2 FLOPS

Idea!

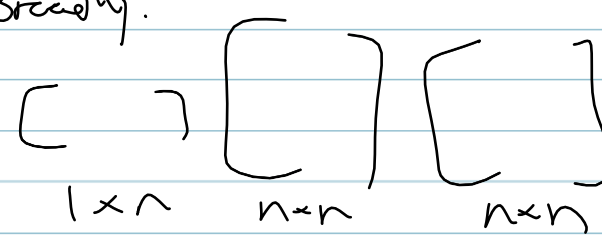
max $4x_1 + 5x_2$ s.t.

$$x_1 + 2x_2 \le 8$$
$$\rightarrow \quad x_1 + x_2 \le 5$$
$$2x_1 + x_2 \le 8$$

$$x_3 = 8 - x_1 - 2x_2$$

$$\begin{aligned} x_1 + 2x_2 + x_3 \quad\quad &= 8 \\ x_1 + x_2 \quad + x_4 \quad &= 5 \\ 2x_1 + x_2 \quad\quad + x_5 &= 8 \end{aligned}$$

Broadly:



one way ~ order(n^2) ops

another " ~ order(n^3) ops

=

Revised Simplex long story

but has <u>many nice ideas</u>

~~Guess... maybe optimal dictionary~~

Say we want to take a look dictionary with x_1, x_2, x_5 basic

$$\begin{bmatrix} 1 & 2 & | & 1 & 0 & 0 \\ 1 & 1 & | & 0 & 1 & 0 \\ 2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} x_5$$

$$\begin{bmatrix} \ \end{bmatrix} x_1 + \begin{bmatrix} \ \end{bmatrix} x_2 + \begin{bmatrix} \ \end{bmatrix} x_5 = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix} + \begin{bmatrix} \ \end{bmatrix} x_3 + \begin{bmatrix} \ \end{bmatrix} x_4$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} x_3 - \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} x_4$$

$$\begin{bmatrix} 1 & 2 & | & 1 & 0 & 0 \\ 1 & 1 & | & 0 & 1 & 0 \\ 2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} A & | & I \end{bmatrix} \begin{bmatrix} x_{dec} \\ \hline x_{slack} \end{bmatrix} = \vec{b}$$

max $\vec{c} \cdot \vec{x}$ s.t.

$$A\vec{x} \le \vec{b} \qquad A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}, \vec{b} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix}$$

$x_3 = \; - \; x_1 \quad x_2 \quad -$

$x_4 = \; - \; x_1 \quad x_2 \quad -$

$x_5 = \; - \; x_1 \quad x_2 \quad -$

$z = \; - \; (\ ) x_1 \quad (\ ) x_2$

look for positive coefs

$$\vec{x}_B = A_B^{-1} \vec{b} - A_B^{-1} A_N x_N$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

Objective:

$$4x_1 + 5x_2 = \vec{c} \cdot \vec{x}_{dec}$$

$$= \begin{bmatrix} 4 \\ 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \vec{C}_{Big} \begin{bmatrix} x_{dec} \\ \hline x_{slack} \end{bmatrix}$$

$$\begin{bmatrix} \text{part of} \\ [A \mid I] \\ \text{corresponding} \end{bmatrix} X_B = \begin{bmatrix} 8 \\ 5 \\ 8 \end{bmatrix} - \begin{bmatrix} \text{part of} \\ [A \mid I] \end{bmatrix} X_N$$

to Basic x's — corresponding to Non-basic

$$z = C_B X_B + C_N X_N$$

$$= \begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

$$A_B X_B = \vec{b} - \bar{A}_N \vec{X}_N$$

$$X_B = A_B^{-1} (\vec{b} - A_N \vec{x}_N)$$

Literature!

$\boxed{A_B \text{ written as } B}$

$$\vec{x}_B = A_B^{-1}\vec{b} - A_B^{-1} A_N x_N$$

$$z = (\quad) + \left(c_N^T - c_B^T A_B^{-1} A_N\right) x_N$$

$\vec{c}_B^{\,T} A_B^{-1} \vec{b}$

Rem

$$c_B^T \quad A_B^{-1} \quad A_N$$

want..

$$\left(c_B^T A_B^{-1}\right) A_N$$

not

$$\text{''} \left(\text{''} \quad \text{''}\right)$$

Abstractly!

$$z = \vec{c}_B^{\,T} \vec{x}_B + \vec{c}_N^{\,T} \vec{x}_N$$

dictionary want

$$z = \text{const} \pm \text{coef}\ x_{\text{non-basic 1}} \pm \ldots$$

$$z = \vec{c}_B^{\,T}\left(A_B^{-1}\vec{b} - A_B^{-1} A_N x_N\right) + c_N^T x_N$$

$$= \text{const} + \underbrace{[\text{vect}]^T}_{\left(c_N^T - c_B^T A_B^{-1} A_N\right)} x_N$$

$$\vec{c}_B^{\,T} A_B^{-1} \vec{b} + \left(c_N^T - c_B^T A_B^{-1} A_N\right) \vec{x}_N$$