Last time:

Complementary slackness:

1. $X_{dec} \overset{1-1}{\longleftrightarrow} Y_{slack}$ (constraint in dual)

   $X_{slack} \overset{1-1}{\longleftrightarrow} Y_{dec}$

   orig — dual

2. In optimality if $X_i \leftrightarrow Y_j$ then optimal solutions $\vec{x}^*, \vec{y}^*$ have either one x_i, y_j is zero, or both, but at least zero. I.e. $x_i y_j = 0$

Math 340, Nov 6

Complementary Slackness:

I:

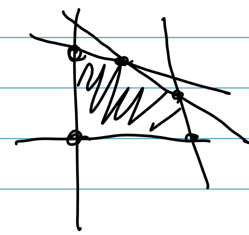
$\max 4x_1 + 5x_2$

$x_1 + 2x_2 \le 8 \quad [y_1]$

$x_1 + x_2 \le 5 \quad [y_2]$

$2x_1 + x_2 \le 8 \quad [y_3]$

"typical with no degeneracies"



II:

$\max 10x_1$

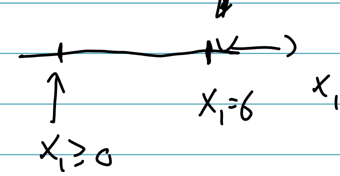
$\cancel{x_1} \le 6$

$x_1 \le 6,$

$x_1 \le 6$

$2x_1 \le 12$

extremely degenerate (a lot of redundancy)



$$\max 4x_1 + 5x_2$$

s.t.

$(x_1 + 2x_2 \le 8) \quad y_1 \ge 0$

$(x_1 + x_2 \le 5) \quad y_2 \ge 0$

$(2x_1 + x_2 \le 8) \quad y_3 \ge 0$

dual: minimize $8y_1 + 5y_2 + 8y_3$

s.t.

$Y_4 \rightarrow$ $\boxed{\begin{array}{l} 4 \le y_1 + y_2 + 2y_3 \\ 5 \le 2y_1 + y_2 + y_3 \end{array}}$ Dual Feasibility

Proposing:

$x_1 > 0 \longrightarrow y_4 = 0$

$x_2 = 0$

$x_3 > 0 \longrightarrow y_1 = 0$

$x_4 > 0 \longrightarrow y_2 = 0$

$x_5 > 0 \longrightarrow y_3 = 0$

$\cancel{y_4} = \cancel{y_1} + \cancel{y_2} + \cancel{2y_3} - 4$

$0 \quad 0 \quad 0 \quad 0$

$0 = -4$ ☹

2'. If we have feasible $\vec{x}, \vec{y}$, and for each $x_i \leftrightarrow y_j$ we have $x_i y_j = 0$ then both $\vec{x}, \vec{y}$ are optimal

e.g. In I we guess $(x_1, x_2) = (3, 0)$

$x_1 = 3 \longleftrightarrow y_4$

$x_2 = 0 \longleftrightarrow y_5$

$x_3 = 8 - x_1 - 2x_2 = 5 > 0 \longleftrightarrow y_1$

$x_4 = 5 - x_1 - x_2 > 0 \longleftrightarrow y_2$

$x_5 = 8 - 2x_1 - x_2 = 2 > 0 \longleftrightarrow y_3$

$x_1 = 4$ OK $\rightarrow$ $y_4 = 0$ OK

$x_2 = 0$ " $y_5 = ?$

$x_3 = 4$ " $y_1 = 0$ OK

$x_4 = 1$ " $y_2 = 0$ OK

$x_5 = 0$! $y_3 = 2$ OK

must have $y_5 \geq 0$

$$y_5 = 2\cancel{y_1} + \cancel{y_2} + y_3 - 5 = \text{anything (non-neg)}$$

$$2 - 5$$

$$= -3$$

Have to check all $\vec{y}$'s

$x_1 = 4, x_2 = 0$: ? is optimal?

$x_1 = 4 > 0 \longrightarrow y_4 = 0$ OK

$x_2 = 0 \longrightarrow y_5 = ?$

$x_3 = 4 > 0 \longrightarrow y_1 = 0$ OK

$x_4 = 1 > 0 \longrightarrow y_2 = 0$ OK

$x_5 = 0 \longrightarrow y_3 = ?$ 2 OK

$y_4 = 0 = y_1 + y_2 + 2y_3 - 4$

$y_5 = ?$ $2y_1 + y_2 + y_3 - 5 = ?$

$y_4 = 0 = \cancel{y_1} + \cancel{y_2} + 2y_3 - 4$

$y_3 = 2$.

A-I $x_1 = 6$

Insight

$x_1 = 6 \rightarrow 0 = y_5$ ≥ 0

$x_2 = 0 \rightarrow$ $y_1 = ?$ ≥ 0

$x_3 = 0$ $y_2 = ?$ ≥ 0

$x_4 = 0$ $y_3 = ?$ ≥ 0

$x_5 = 0$ $y_4 = ?$ ≥ 0

So!

$$0 = y_1 + y_2 + y_3 + 2y_4 - 10$$

no unique solution ... but ...

$y_2 = y_3 = y_4 = 0$ safe

$y_1 = 10$ OK

max $10x_1$ | dual

$x_1 \leq 6 \xleftarrow{x_2} y_1$

$x_1 \leq 6 \xleftarrow{x_3} y_2$

$x_1 \leq 6 \xleftarrow{x_4} y_3$

$2x_1 \leq 12 \xleftarrow{x_5} y_4$

$10 \leq y_1 + y_2 + y_3 + 2y_4$

$x_1 = 5 \Rightarrow 0 = y_5 = y_1 + y_2 + y_3 + 2y_4 - 10$

$x_2 = 1 \Rightarrow 0 = y_1$

$x_3 = 1 \Rightarrow 0 = y_2$

$x_4 = 1$ $0 = y_3$

$x_5 = 2$ $0 = y_4$

$y_1 = \ldots = y_5 = 0$ then $y_5 = y_1 + y_2 + y_3 + 2y_4 - 10$

Dictionary

$x =$
$x =$
$x =$
"

$z = 103 - 2x_1 - 5x_2 - 20x_9 \ldots$

- Make sure you
  → 2 Phase Method
  → Solving 2×2 games

Betty 2 pure strategies

Alice
a lot

$$\begin{bmatrix} , & , \\ , & , \\ , & \\ , & , \end{bmatrix}$$

L^1 fit, L^∞ fit

simple method ⇒ optimal solution has # zero ≥ # ~~non~~ basic vars = # dec vars

~~1 2 3 4~~
~~8 7 6 5~~
100 101 102 103
1004 1001 . .

Scaling out of 36

$0 \to 0$

$12 \to \geq 50$

$27 \to \geq 80$

$36 \to \geq 100$

100