

Fact: If "primal" $\max \vec{c} \cdot \vec{x}$ s.t. $A\vec{x} \leq \vec{b}$
 $\vec{x} \geq 0$

is feasible and bounded, then dual is " " "

dual objective in optimality $\min \vec{b} \cdot \vec{y} = \max \vec{c} \cdot \vec{x}$
 of the primal objective in optimality.

Dictionary in primal \rightsquigarrow Dictionary in dual

Ch 5: Duality:

LP: $\max \vec{c} \cdot \vec{x}$ s.t. $A\vec{x} \leq \vec{b}$
 $\vec{x} \geq 0$ \rightsquigarrow dual LP

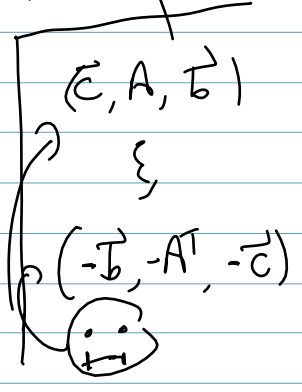
Derivation:

$\vec{y} \cdot (A\vec{x} \leq \vec{b})$ s.t. $\vec{y} \geq 0$

$\vec{c} \cdot \vec{x} \leq (\vec{y}^T A) \vec{x} \leq \vec{y} \cdot \vec{b}$

$\rightsquigarrow \max -\vec{b} \cdot \vec{y}$

s.t. $-A^T \vec{y} \leq -\vec{c}$



$x_1, x_2 \geq 0$
 $(x_1 \leq 5) \quad y_1$
 $y_1 x_1 \leq 5 y_1$
 $Z = x_1 + x_2$ is never $\leq y_1 x_1$
 for any $y_1 \dots$

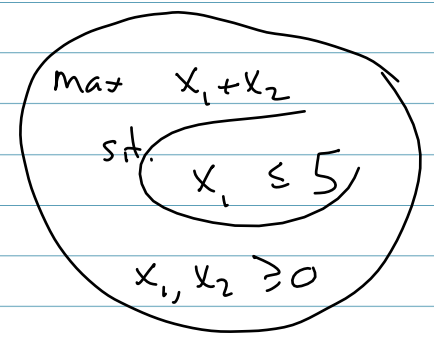
We want $\min 5y_1$ s.t.

$1 \leq y_1$

$1 \leq 0 \cdot y_1 \leftarrow$ impossible

$y_1 \geq 0$

Primal is unbounded \rightsquigarrow Dual is infeasible



for $x_1 = 0, x_2 = t \geq 0$, objective = t

$t = 10^{1010}, x_2 = 10^{1010}, x_1 = 0$

$\vec{x} = \begin{pmatrix} 0 \\ 10^{1010} \end{pmatrix}$ is feasible $Z = 10^{1010}$

We can't bound from above

$$\max z = 10x_2 \quad \text{s.t.}$$

$$x_1 + x_2 \leq 7$$

$$x_1, x_2 \geq 0$$

Looks like $x_1 = 0, x_2 = 7 \dots$

$$z = x_1 + 10x_2 \quad \text{bound via } x_1 + x_2 \leq 7$$

$$x_1 + x_2 \leq 7$$

$$z = x_1 + 10x_2 \leq 10x_1 + 10x_2 \leq 70$$

Since $x_1 = 0$ we don't mind

best upper bound

Last class

$$\max 4x_1 + 5x_2 \quad \text{s.t.}$$

$$x_3 \rightarrow x_1 + 2x_2 \leq 8$$

$$x_4 \rightarrow x_1 + x_2 \leq 5$$

$$x_5 \rightarrow 2x_1 + x_2 \leq 8$$

Final dictionary:

$$z = 23 - x_3 - 3x_4$$

$$(x_1 + 2x_2 \leq 8) \cdot 1$$

$$(x_1 + x_2 \leq 5) \cdot 3$$

$$(2x_1 + x_2 \leq 8) \cdot 0$$

optimal dual solution

$$z = 4x_1 + 5x_2 \leq 23$$

$$\max 4x_1 + 5x_2 \quad x_1, x_2 \text{ decision}$$

s.t. blah blah blah

x_3, x_4, x_5 slack

$$z = 23 - x_3 - 3x_4$$

$(x_1 + 2x_2 \leq 8) \cdot 1$

$(x_1 + x_2 \leq 5) \cdot 3$

$$z = 4x_1 + 5x_2 \leq 23$$

all slack

$$z = 70 - 9x_1 - 10x_3$$

x_1, x_2 decision

decision

slack

$$z = x_1 + 10x_2$$

$$x_3 = 7 - x_1 - x_2$$

x_1 enters, or x_2 .

$$x_2 = 7 - x_1 - x_3$$

$$z = x_1 + 10(7 - x_1 - x_3)$$

$$= 70 - 9x_1 - 10x_3$$

$$z = x_1 + 10x_2 = 70 - 10(7 - x_1 - x_2)$$

i.e.

$$z + 9x_1 = 10x_1 + 10x_2 \leq 70$$

Z + non-neg comb of decision
 = non-neg comb of slack

(e.g. $Z = 70 - 9x_1 + 10x_3$)
 (dec) (slack)

$x_3 \leftrightarrow x_1 + x_2 \leq 7$
 $x_3 = 7 - x_1 - x_2$

So 1st ineq: some decisions $\leq b_1$

2nd some other comb dec $\leq b_2$

Z + non-neg comb of decision =

= optimal value

Proof that the dual LP has its min $\bar{y} \cdot \bar{b}$
 = primal max $\bar{c} \cdot x$ assume both are primal is feasible and bounded...

Since primal is feasible and bounded, we arrive at a final dictionary:

$Z = \bar{c} - a_1x_1 - a_2x_2 - \dots - a_nx_n$
 $- a_{n+1}x_{n+1} - \dots - a_{n+m}x_{n+m}$
 (basic) (slack variables)

$Z + a_1x_1 + \dots + a_nx_n = -a_{n+1}x_{n+1} - \dots - a_{n+m}x_{n+m}$
 Z + non-neg combo of $x_1, \dots, x_n =$

2 for 1: ② \Rightarrow ③

Simplex method \rightarrow dual simplex method

=

Complementary slackness:

Max $4x_1 + 5x_2$

$x^* = \begin{pmatrix} x_1=2 \\ x_2=3 \end{pmatrix}$

1. $x_1 + 2x_2 \leq 8$ (8 \leq 8)

3. $x_1 + x_2 \leq 5$ (5 \leq 5)

0. $2x_1 + x_2 \leq 8$ (7 \leq 8)

th
 n
 be

IF

① an LP is feasible & bounded \Rightarrow dual LP is " " "

② an LP is feasible & unbounded \Rightarrow dual LP is infeasible

③ If dual is feasible & unbounded \Rightarrow original LP is infeasible

② \Rightarrow if dual LP is blah blah blah then dual(dual LP) is blahhhhhh
 LP