Math 340 Oct 30

Ch. 5: Duality

e.g.

$$\max\ z = 4x_1 + 5x_2 \text{ s.t.}$$
$$x_1 + 2x_2 \le 8$$
$$x_1 + x_2 \le 5$$
$$2x_1 + x_2 \le 8$$
$$x_1, x_2 \ge 0$$

There is dual LP.

" " a simplex method, dual simplex method

" " a pivot, dual pivot

... blah, dual blah

...

2 problems: max v ... 1 var to 2

maybe $v < 0$ ... $v \sim v_1 - v_2$

if we have $x_1 + 3x_2 = 5$ one equ.

$$\left.\begin{array}{l} x_1 + 3x_2 \le 5 \\ x_1 + 3x_2 \ge 5 \end{array}\right\} \text{two}$$

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$$z = 4x_1 + 5x_2$$
$$x_3 = 8 - x_1 - 2x_2$$
$$x_4 = 5 - x_1 - x_2$$
$$x_5 = 8 - 2x_1 - x_2$$

This is the dual !!

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Dual: $\max\ \vec{c}\cdot\vec{x}$ s.t. $A\vec{x} \le \vec{b}$, $\vec{x} \ge 0$ $\quad (\vec{c}, A, \vec{b}) \leadsto (-\vec{b}, -A^T, -\vec{c})$

$$z = 4x_1 + 5x_2$$

x_3 $\quad (x_1 + 2x_2 \le 8) \cdot y_1 \quad (y_1 \ge 0)$

x_4 $\quad (x_1 + x_2 \le 5) \cdot y_2 \quad y_2\,!$

x_5 $\quad (2x_1 + x_2 \le 8) \cdot y_3 \quad y_3\,!$

$$(y_1 + y_2 + 2y_3)x_1 + (2y_1 + y_2 + y_3)x_2 \le 8y_1 + 5y_2 + 8y_3$$

minimize $8y_1 + 5y_2 + 8y_3$

s.t.

x_1: $4 \le y_1 + y_2 + 2y_3 \quad (y_4)$

x_2: $5 \le 2y_1 + y_2 + y_3 \quad (y_5)$

$(y_1, y_2, y_3 \ge 0)$

Side remark:

$x_3 = 8 - x_1 - 2x_2$	$y_4 = -4 + y_1 + y_2 + 2y_3$
$x_4 = 5 - x_1 - x_2$	$y_5 = -5 + 2y_1 + y_2 + y_3$
$x_5 = 8 - 2x_1 - x_2$	
$z = 4x_1 + 5x_2$ (max)	$w = -8y_1 - 5y_2 - 8y_3$ (max)

$x_1 = 2 - x_4 - x_3$	$y_1 = 1$? ?
$x_2 = 3 - x_4 + x_3$	$y_2 = 3$ ...
$x_5 = 1 - x_4 + 3x_3$	
$z = 23 - x_4 - 3x_3$	$w = -23 - 2y_4 - 3y_5 - y_3$?

$z = 4x_1 + 5x_2$

$z = 23 - x_4 - 3x_3$

$= 23 - (\cancel{8} - x_1 - \cancel{x_2})$

$-3(8 - x_1 - 2x_2)$

we had better get $4x_1 + 5x_2$

$y_1 = 1,\ y_2 = 3,\ y_3 = 0$

We claim: this is dual

Suggest: optimal is $z = 23$

$x_1 = 2$

$x_2 = 3$ $\quad x_3, x_4 = 0$

$x_5 = 1$

Dual optimal: $\quad w = -23$

$y_1 = 1$

$y_2 = 3$ $\quad y_4, y_5, y_3 = 0$

Maybe: $(x_1 + 2x_2 \le 8) \cdot 1$

x_3

$(x_1 + x_2 \le 5) \cdot 3$

x_4

$(2x_1 + x_2 \le 8) \cdot 0$

$\boxed{4x_1 + 5x_2 \le 8 + 15 = 23}$