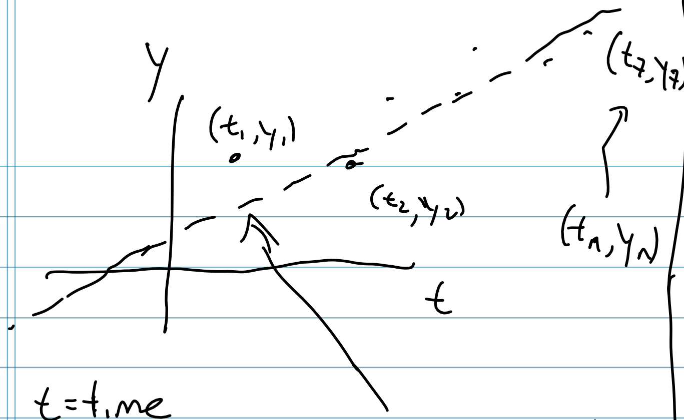
Math 340, Oct 26

- Slides will probably be posted...
- L^∞-fit, L^1-fit (Vanderbei, maybe Chvatal?)
- $\boxed{\text{Ch. 5}}$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ \hline 9 & 10 & 11 & 12 \\ 10 & 9 & 11 & 12 \end{bmatrix} \quad \left(\begin{array}{cc|cc} 9 & 10 & 11 & 12 \\ 10 & 9 & 11 & 12 \end{array}\right)$$

Ch 5 = Duality Theory

Application: Curve fitting:



t = time

$y = y(t)$ model... $y(t) = a + bt$

error $_{i\text{-th}}$ data point:

Text

$\text{error}_i = y_i - a - bt_i$

If you could find a, b s.t.

$\text{error}_1 = 0$, $\text{error}_2 = 0$, ..., $\text{error}_n = 0$

then exact linear fit..

Given n data points

$(t_1, y_1), \ldots, (t_n, y_n)$

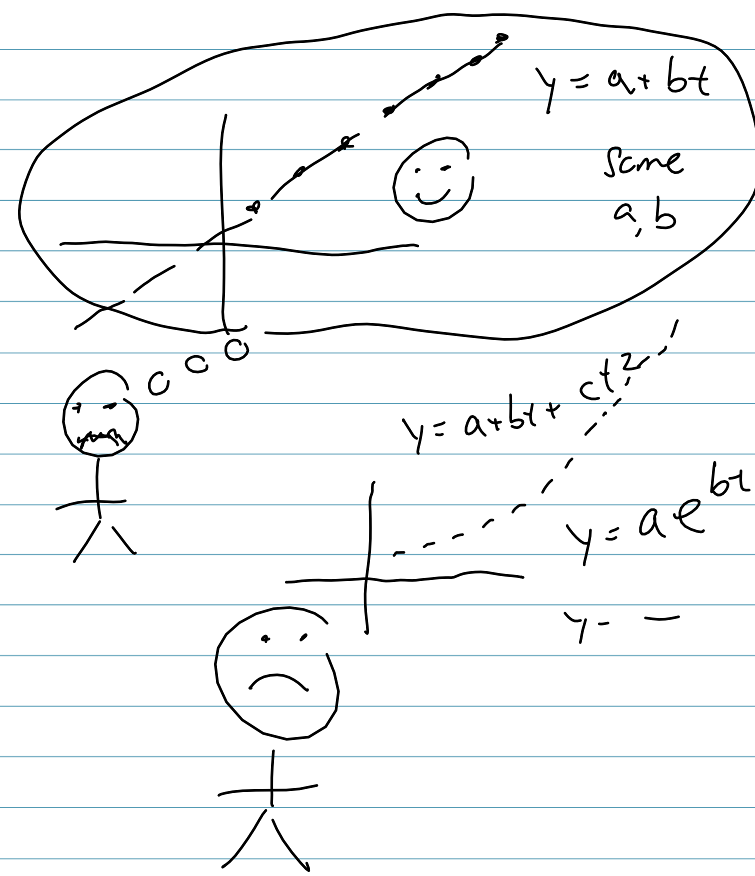
For any a, b,

$\text{error}_i = y_i - a - bt_i$

So

$$\text{OverError} = \max_{i=1,\ldots,n} |\text{error}_i|$$

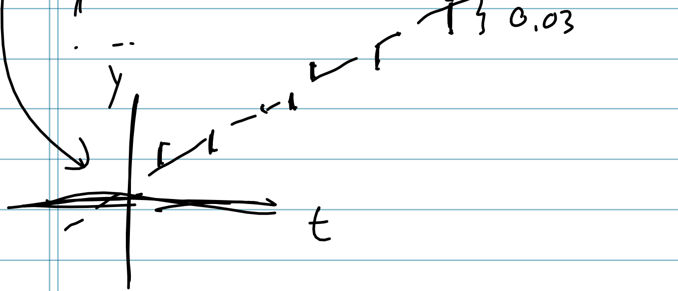
Best L^∞ fit, Best max fit, Chebytchev, Tchebyshev, -- fit



(t_s, y_s)

$y = a + bt$

$y = a + bt + ct^2$



Claim 1: Given $(t_1, y_1), \ldots, (t_n, y_n)$ there is an a, b such that OverallMaxError(a,b)

$$= \max_{i=1,\ldots,n} |y_i - a - bt_i|$$

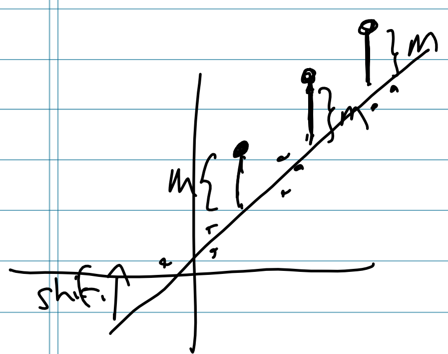
is attained

Overall Error(a,b)

$$= \begin{cases} \max_i |\text{error}_i| & \text{max fit} \\ \sum_{i=1}^{n} |\text{error}_i| & L^1 \text{ fit} \\ \sum_{i=1}^{n} (\text{error}_i)^2 & \text{Least squares fit} \end{cases}$$

Min, over all a, b, of Overall Error(a,b)

Overall Error$_{\boxed{\max}}(a,b)$



So: MaxError(a,b) must be one i s.t.

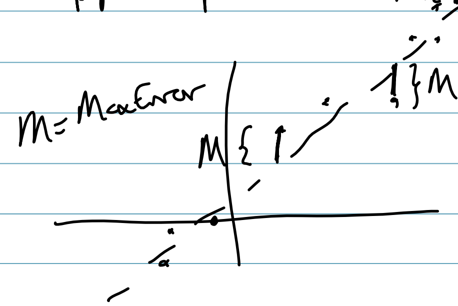
$|\text{error}_i| =$ MaxError(a,b)

for such i, (t_i, y_i) "support point"

There must be some support points error$_i$ = Max, some error$_i$ = −Max

Claim 2: $n \geq 2$, OverallMaxErr$(a,b) > 0$

Then there are at least three "support points"



there are at least three values of i s.t.

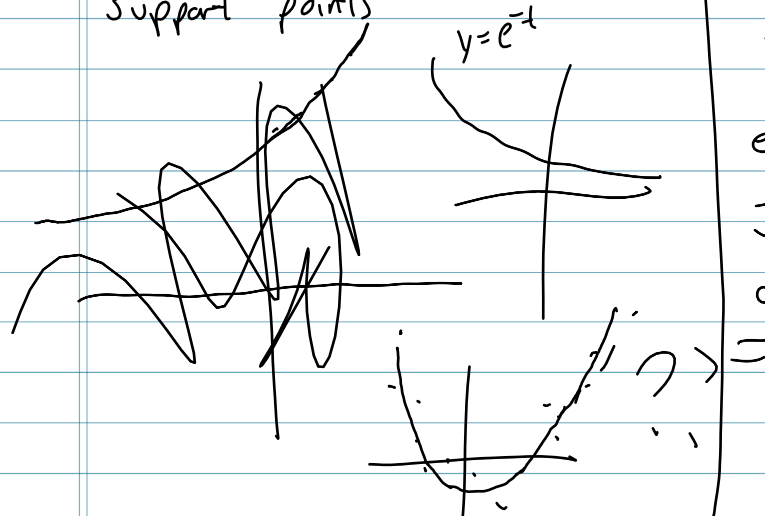
$$\text{MaxErr}(a,b) = |y_i - a - bt_i| = |\text{error}_i|$$

Imagine:

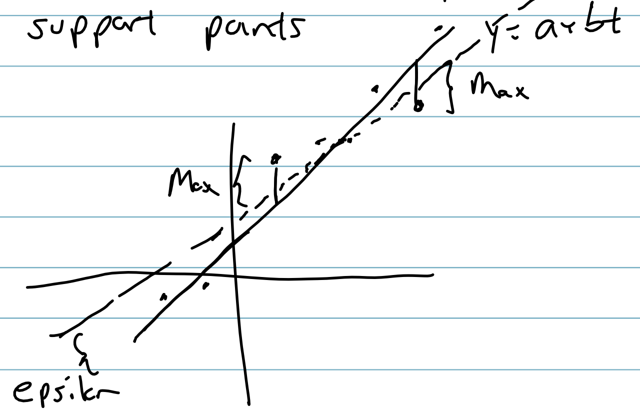
$$y = a + bt + ct^2 + de^{-t}$$

four parameters.

Claim: You have 5 support points



Could there be only 2? support points



If only 2, we can perturb and do better.

minimize

$$M = \operatorname{Max}\left(|y_1 - a - bt_1|, |y_2 - a - bt_2|, \ldots |y_n - a - bt_n| \right)$$

minimize M, max $-M$

subject to

$-M \le y_1 - a - bt_1$

$y_1 - a - bt_1 \le M$

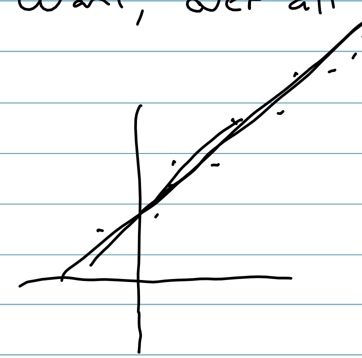
$-M \le y_2 - a - bt_2$

$y_2 - a - bt_2 \le M$

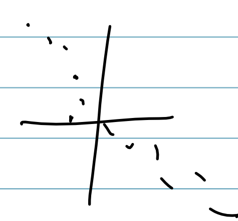
$\vdots$

$(t_1, y_1), \ldots (t_n, y_n)$

Want, over all a, b



(say we knew $a, b > 0$ in the best max fit)



We know:

First dictionary

$$x_{(slack,1,-)} = y_1 - a - bt_1 + M \geq 0$$

$$x_{(slack,1,+)} = M - (y_1 - a - bt_1) \geq 0$$

⋮

End: Final Dictionary

a, b, M

Basic Vars

(twice # points we don't really care)

Non-basic Vars 3

3 slack variables

(we know $M \geq 0$, assume we know $a, b \geq 0$)

maximize $-M$ s.t.

slack → $-M \leq y_1 - a - bt_1$

slack → $y_1 - a - bt_1 \leq M$

⋮

$$-M \leq y_n - a - bt_n \leq M$$

given $t_1, \dots, t_n, y_1, \dots, y_n$ variables are a, b, M

also $M, a, b \geq 0$

What about

$$y(t) = a + bt + c\left(e^{e^{-t}+5}\right) + d\sin(\sin(t))$$

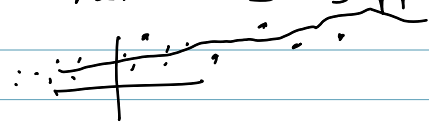
four parameters

MaxError:

$$-M \leq a + bt_1 + c\left(e^{e^{-t_1}+5}\right) + d\sin(\sin(t_1)) \leq M$$

5 decision variables

⟶ hence 5 support points



($M > 0$, $a > 0$, $b > 0$)

Could we have $x_{slack,7,-}$ and $x_{slack,7,+}$ both non-basic in final dictionary?

In the optimal solution $x_{slack,7,-} = 0$ and $x_{slack,7,+} = 0$?!

No: must have 3 slack variables with different (t_i, y_i).