Unbounded: LP is unbounded

$$\max\ 4x_1 + 9x_2$$
s.t. $x_1 \le 10$ (x_3)
$x_1 \le 3$ (x_4)
$x_1, x_2 \ge 0$

$z = \underline{4x_1} + \underline{9x_2}$

$x_3 = 10 - x_1$

$x_4 = 3 - x_1$

Here x_1 or x_2 enters

Math 340, Oct. 21

Exam on Friday, in BIOL 2000,

Solutions to HW 3 posted "immediately"

Office hours: (Math Building, room 210)

Today (Wednesday): 4:30 - 5:30 pm

Thursday: 4:45 - 5:45 pm

No notes, calculators, etc.

Covering up to part of Ch. 3 which is 2-phase method

BUT NO - Unbounded

- Degenerate Pivot, Cycling, Perturbation

So z's value $\left(\begin{matrix} x_1 = 0 \\ x_2 = \text{large} \end{matrix}\right)$ is arbitrarily large

The linear program is "unbounded."

$\Leftrightarrow$ for some dictionary,

a variable enters,

no variable leaves

Now: the simplex method always has a version that works (!)

Say x_2 enter:

$x_3 = 10 - x_1 \ge 0$ no constraint on x_2

$x_4 = 3 - x_1 \ge 0$ " "

x_2 enters, (z increases)

BFS $x_1 = 0$, x_2 increases

nothing leaves

Hence if we hold $x_1 = 0$

x_2 can be arbitrarily large

$z = 4x_1 + 9x_2$ (hold $x_1 = 0$)

$= 9x_2$ arbitrarily large

So ① We can never cycle, i.e. we see each dictionary at most once.

② In each dictionary:

- some variable enters
  - ~~no~~ variable leaves (unbounded)
  - some variable leaves (z gets larger)
- no variable enter (i.e. all z row coefficients are ≤ 0)
  $\Rightarrow$ have final dictionary solved LP

Why?

If $\max \vec{c}^T \cdot \vec{x}$ s.t.

$$A\vec{x} \leq \vec{b}$$

$$\vec{x} \geq 0$$

and all $\vec{b}$'s components are ≥ 0 ...

$$\vec{b} \to \vec{b} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \\ \vdots \end{bmatrix}$$

1^{st} dictionary $z = \vec{c} \cdot \vec{x}$ is 0

2^{nd} dictionary z larger

3^{rd} " " " "

Run phase 1:

$$\vec{x}_{slack} = \vec{b} - A\vec{x}_{dec} + \begin{bmatrix} x_0 \\ x_0 \\ \vdots \\ x_0 \end{bmatrix} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \\ \vdots \end{bmatrix}$$

~~New~~ obj

$w = -x_0$ maximize

pivot x_0 in ...

if x_0 cannot attain $x_0 = 0$

LP infeasible

Otherwise $\vec{x}_B = \ldots \vec{x}_N$ $\cancel{3x_0}$ $\cancel{7x_0}$ $\cancel{5x_0}$

feasible dictionary

$z = \vec{c} \cdot \vec{x}$ (Phase 2)

So after finitely many steps we are done

Best choice of entering variables?

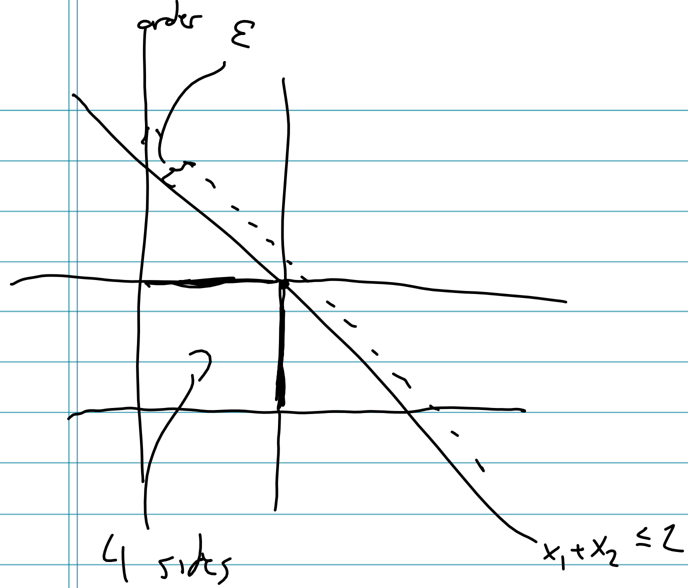
Ans: BIG BIG NEWS

If $\max \vec{c} \cdot \vec{x}$ s.t.

$$A\vec{x} \leq \vec{b}$$

$$\vec{x} \geq 0$$

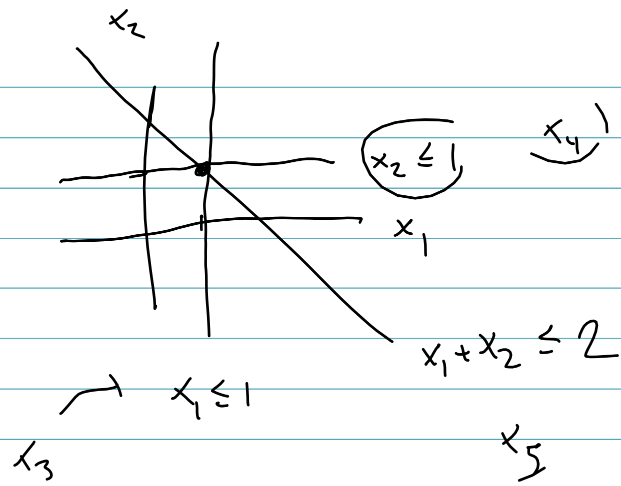
but $\vec{b}$ has some negative entries ...



$x_1+x_2 \le 2+\varepsilon$

$x_1 \le 1+\varepsilon^2$

$x_2 \le 1+\varepsilon^3$



$x_1 \le 1 + \varepsilon$	$x_1 \le 1 + \varepsilon^3$
$x_2 \le 1 + \varepsilon^2$	$x_2 \le 1 + \varepsilon^2$
$x_1+x_2 \le 2 + \varepsilon^3$	$x_1+x_2 \le 2 + \varepsilon$

Betty

Alice

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 & (1-x_1-x_2-x_3-x_4) \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 2 & 2 \\ 0 & 1 \\ 6 & 7 \end{bmatrix} \geq \begin{bmatrix} v & v \end{bmatrix}$$

decision

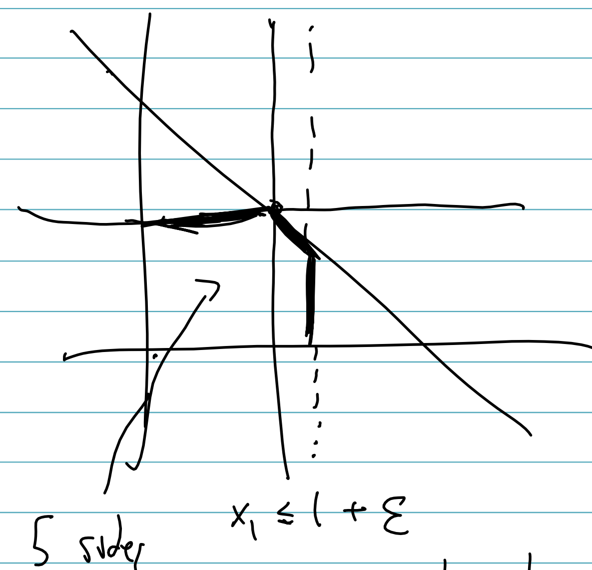
max v

$x_5 = 1-x_1-x_2-x_3-x_4 \geq 0$ $\quad x_5$

$1\cdot x_1+3x_2+2x_3+0\cdot x_4+6(1-x_1-\dots-x_4) \geq v$ $\quad x_6$

— — — — — — — $\geq v$ $\quad x_7$

$x_1,\dots,x_4 \geq 0$



$x_1 \le 1 + \varepsilon$

$x_2 \le 1$ + extremely small $\quad \varepsilon^2$

x_1+x_2 2 + more extremely small $\quad \varepsilon^3$

—

MIDTERM (until next week)