Math 340, Oct. 19

Midterm Friday!

Location TBA

Start 2:05pm or so

Topics!

1st topic
- Games (matrix games)
- General aspects
- 2×2 Matrix games

2nd topic
- Simplex method (Ch. 2)
- 2 Phase-Method (Ch. 3)

NOT ON MIDTERM: Ch. 3: - Degenerate Pivots, Cycling, and Perturbation Method
- Unbounded LP

Where to look:

"Learning Goals" webpage...

Sample Exam problems

Past Midterm, Finals

1st topic: Matrix games

2nd top: Simplex method

$1\frac{1}{2}$ top: Simplex method & matrix games

2

100

Linear program

$[x_1 \dots x_{100}]$

$\geq [v \; v]$

Last time:

In analogy to complex numbers:

$\alpha + i\beta, \quad \alpha, \beta \in \mathbb{R}$

$2+i5, \quad 3-\sqrt{17}\,i, \dots$

multiply, divide, add, ...

polynomials in ε

$A\vec{x} \leq \vec{b} \rightarrow \vec{x}_{slack} = \vec{b} - A\vec{x}_{dec}$

$$\vec{x}_{slack} = \vec{b} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \vdots \\ \varepsilon^m \end{bmatrix} - A\vec{x}_{dec}$$

to use $\begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \vdots \end{bmatrix}$ we only need to

1. compare 2 polynomials in ε
2. multiply/divide poly(ε) by real numbers

$$x_1 = 14 + \varepsilon - \varepsilon^2 - 4x_5 + \dots$$

$$4x_5 = 14 + \varepsilon - \varepsilon^2 - x_4$$

$$x_5 = \frac{14 + \varepsilon - \varepsilon^2}{4}$$

Only problem in simplex

$$x_6 = \text{zero} - 4x_3 + 7x_2 - x_1$$

x_3 enters

$$z = 117 + \_\_ + 2x_3 + \_\_$$

$$x_6 = \underbrace{(2\varepsilon^2 - \varepsilon^3)}_{>0} - 4x_3 + \_\_$$

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- Why do we never get a 0?
- Simplex method can always be made to work

$$2 + \varepsilon - \varepsilon^2 \quad \text{vs} \quad 2 + \varepsilon - 2\varepsilon^2 + 10^{10^{10^{10}}}\varepsilon^3$$

1. compare constant term
2. if tied, compare ε term
3. " " " ε^2 "

$$2 + \varepsilon - \varepsilon^2 > 2 + \varepsilon - 2\varepsilon^2 + \left(\begin{matrix}\text{any}\\ \text{real}\end{matrix}\right)\varepsilon^3$$

Perturbation, lexicographical method

$$\underbrace{\left[\, A \mid I \,\right] \begin{bmatrix} \vec{x}_{dec} \\ \vec{x}_{slack} \end{bmatrix}}_{} = \vec{b}$$

$$\underbrace{A_{big} \vec{x}_{big}}_{} = \vec{b}$$

$$\vec{x}_{big} = \begin{pmatrix} \vec{x}_{dec} \\ \vec{x}_{slack} \end{pmatrix} \rightsquigarrow \vec{x}_B, \vec{x}_N$$

$$A_{big}\vec{x}_{big} = A_{basic}\vec{x}_{basic} + A_{nonb}\vec{x}_{nonb}$$

Let's say:

$$\vec{x}_{slack} = \vec{b} + \begin{bmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ \vdots \end{bmatrix} - A\vec{x}_{dec}$$

is this good?

$$\vec{b} = \begin{bmatrix} 3 \\ 3 \\ 1 \\ 0 \\ 7 \end{bmatrix} \qquad \begin{bmatrix} 3 \\ 3 \\ 1 \\ 0 \\ 7 \end{bmatrix} + \begin{pmatrix} \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{pmatrix}$$

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Recall: $\vec{x}_{slack} = \vec{b} - A\vec{x}_{dec}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 17 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 1 \\ 7 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 8 \\ 20 \\ 50 \end{bmatrix}$$

$$\begin{matrix} x_2 = \\ x_3 = \\ x_5 = \end{matrix} \begin{bmatrix} \; \end{bmatrix}^{-1} \left(\begin{bmatrix} 8 \\ 20 \\ 50 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 5 & 1 \\ 7 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_4 \end{bmatrix} \right)$$

max whatever s.t.

$$x_1 + x_2 \le 8 \quad \leftarrow x_3$$

$$5x_1 + x_2 \le 20 \quad \leftarrow x_4$$

$$7x_1 + 17x_2 \le 50 \quad \leftarrow x_5$$

cols (x_2 x_3 x_5)

$$\left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 1 & 0 \\ 7 & 17 & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \hline x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 20 \\ 50 \end{bmatrix}$$

Thought experiment...

basic

$$\begin{pmatrix} x_2 \\ x_3 \\ x_5 \end{pmatrix} = \underbrace{\begin{bmatrix} \; \end{bmatrix}^{-1}}_{\text{invertible}} \left(\begin{bmatrix} 8 \\ 20 \\ 50 \end{bmatrix} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \end{bmatrix} - \begin{pmatrix} \text{whatever} \end{pmatrix} \vec{x}_{\text{Nonbasic}} \right)$$

say we want x_2 x_3 x_5 in basis

$$\begin{bmatrix} \; \end{bmatrix}^{-1} \left(\begin{bmatrix} \; \end{bmatrix} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \end{bmatrix} + \ldots \right)$$

vect of constraints

only source of epsilons...

you would need

$$\begin{bmatrix} \; \end{bmatrix}^{-1} \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \end{bmatrix} = \begin{bmatrix} 7\varepsilon - 20\varepsilon^3 \\ 0 \; (?!) \\ \ldots \end{bmatrix}$$

$$\begin{bmatrix} \; \end{bmatrix}^{-1} = \begin{bmatrix} \text{---} \\ 0\;0\;0 \\ \text{---} \end{bmatrix}$$ has 0 determinant

Remark: take, say, a 3×3 matrix, e.g. $= \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 17 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} \; \end{bmatrix}^{-1} \left(\underbrace{\begin{bmatrix} 8 \\ 20 \\ 50 \end{bmatrix}}_{\text{ignore}} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \varepsilon^3 \end{bmatrix} + \ldots \right)$$

Can this have a zero constant ???

$$x_3 = 0 + \quad x_1 \quad x_4$$

Idea

$z = 4x_1 + 20x_2$

x_1 enters or x_2 enters

Bland's rule $\begin{cases} \text{- never cycle} \\ \text{- unique simplex method} \end{cases}$

Not covering Bland's rule

Bottom Line:

$$A\vec{x}_{dec} + I\vec{x}_{slack} = \vec{b} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \vdots \end{bmatrix}$$

$$\vec{x}_{Basic} = A_{Basic}^{-1}\left(\vec{b} + \begin{bmatrix} \varepsilon \\ \varepsilon^2 \\ \vdots \end{bmatrix} - A_N \vec{x}_N\right)$$

can't have all 0 row's --

Hence no degenerate pivot

Bland's rule -- smallest subscript