$$\begin{bmatrix} 33 & 1 \\ -1 & 33 \end{bmatrix} \quad \text{value} > 0$$

if not $V = V_1 - V_2 - \ldots$

=

$x_2 = 1 - x_1$

$x_3 = -1 + 34x_1 - V + x_0$

$x_4 = 33 - 32x_1 - V$

faster — by hand only to add $x_0$ where necessary

1st: Pivot $x_0$ into basis, feasible

$-x_0 = -1 + 34x_1 - V - x_3$

$x_0 = 1 - 34x_1 + V + x_3$

Ch 3: ① Perturbation — Pivots that degenerate

② 2 Phase method — Can't start pivoting

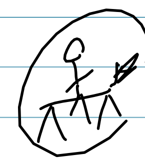
① $x_3 = 0 - x_1$

② $x_3 = -1 - 3x_1 + 3x_2 \ldots$

Mules

disappears

Extra Auxilliary mule



$34x_1 = 1 - x_0 + V + x_3$

$x_1 = \frac{1}{34} - \frac{1}{34}x_0 + \frac{1}{34}V + \frac{1}{34}x_3$

$w = \quad -x_0$ (principle)

$x_2 = 1 - x_1$

$= 1 - \left(\frac{1}{34} - \frac{1}{34}x_0 + \frac{1}{34}V + \frac{1}{34}x_3\right)$

$= \frac{33}{34} + \frac{1}{34}x_0 - \frac{1}{34}V - \frac{1}{34}x_3$

$x_4 = 33 - 32x_1 - V$

$= 33 - 32\left(\frac{1}{34} - \frac{1}{34}x_0 + \frac{1}{34}V + \frac{1}{34}x_3\right) - V$

$= \left(33 - \frac{32}{34}\right) + \frac{33}{34}x_0 - \frac{66}{34}V - \frac{32}{34}x_3$

$\frac{545}{17}$

max $w = -x_0$

$w = -1 + 34x_1 - V - x_3$

$x_0 = 1 - 34x_1 + V + x_3$

$x_2 = 1 - x_1$

$x_4 = 33 - 32x_1 - V$

$x_1$ enters

$x_0 \geq 0 \Rightarrow x_1 \leq \frac{1}{34}$

$x_2 \geq 0 \Rightarrow x_1 \leq 1$

$x_4 \geq 0 \Rightarrow x_1 \leq \frac{33}{32}$

=

$x_1$ leaves?

$x_1 = 1 - x_2 \rightsquigarrow x_0 = 1 - 34x_1 + \ldots$

$= -33$

$$\begin{bmatrix} 33 & 1 \\ -1 & 33 \end{bmatrix}$$

$x_1 = 0$

$x_2 = 1 - x_1 = 1$

initially

Phase 2 begins ...

max $z = v$ — old objective

increase $v$, hold $x_3 = 0$

what leaves the basis, i.e. what turns to zero soonest

$x_2 \ge 0$ imposes $v \le 33$

$x_4 \ge 0$ imposes

$$v \le \frac{17}{33} \cdot \frac{545}{17} = \frac{545}{33}$$

$w = \quad -x_0$

$$x_1 = \frac{1}{34} - \frac{1}{34}x_0 + \frac{1}{34}v + \frac{1}{34}x_3$$

$$x_2 = \frac{33}{34} + \frac{1}{34}x_0 - \frac{1}{34}v - \frac{1}{34}x_3$$

$$x_4 = \frac{545}{17} + \frac{32}{34}x_0 - \frac{66}{34}v - \frac{32}{34}x_3$$

$x_0$, the mule of the wise person goes away

BFS $x_0 = 0$, $v = x_3 = 0$ $\quad x_0 = 0$ !!!!

$\downarrow$

$$x_1 = \frac{1}{34} + \frac{1}{34}v + \frac{1}{34}x_3$$

$$x_2 = \frac{33}{34} - \frac{1}{34}v - \frac{1}{34}x_3$$

$$x_4 = \frac{545}{17} - \frac{66}{34}v - \frac{32}{34}x_3$$

$$x_2 = \frac{33}{34} - \frac{1}{34}v - \frac{1}{34}x_3$$

$$= \frac{33}{34} - \frac{1}{34}v$$

More trouble ...

max $v$

$$[x_1 \;\; 1-x_1]\begin{bmatrix} 0 & 1 \\ 1/2 & 0 \end{bmatrix} \ge [v \;\; v]$$

$$v \le \frac{1}{2}(1 - x_1)$$

$$v \le 0 + x_1$$

$x_4$ leaves:

$$\frac{66}{34}v = \frac{545}{17} - x_4 - \frac{32}{34}x_3$$

$$z = v$$

$$= \frac{34}{66}\left(\frac{545}{17} - x_4 - \frac{32}{34}x_3\right)$$

$$x_1 = \frac{1}{34} + \frac{1}{34}v + \frac{1}{34}x_3$$

$$v = \frac{545}{33}$$

$\max \quad x_1$

$x_1 - x_2 \leq 0$ 

$x_2 \leq 1$

and

$x_1, x_2 \geq 0$

Clearly we want $x_1 \leq x_2 \leq 1$ to be as big as possible

$x_1 = x_2 = 1$ ...

Perturbation Method

If

$x_3 = 0 + x_1 - x_2$

and $x_2$ enters

$x_2 = 0 + x_1 - x_3$

Toy problems ...

Consider $\max \quad x_1$

st. $\underline{x_1 \leq x_2}$ and $x_2 \leq 1$

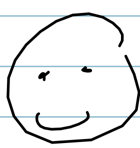
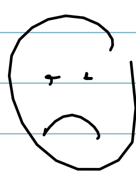
$x_1$ enters the basis ...

$x_3$ "leaves right away ..."

$x_1 = 0 - x_3 + x_2$

$x_4 = 1 \qquad - x_2$

$z = 0 - x_3 + x_2$

 OR 

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$z = \qquad x_1$

$x_3 = 0 - x_1 + x_2$

$x_4 = 1 \qquad - x_2$

BFS $x_1, x_2$ (the decision vars i.e. "original") are 0

$x_1 = 0, x_2 = 0$

$x_3 = 0, x_4 = 1, z = 0$