Farm: Parent

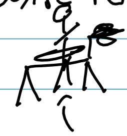
Child 1: 1/2 of the mules

Child 2: 1/3 " " "

Child 3: 1/9 " " "

Parent(s) die, leaving 17 mules

White Person



mule

take my mule:...

18 Ch 1 takes 9

Ch 2 takes 6

Ch 3 takes 2

17

1 left over

Math 340, Oct. 7; Ch 3:

2-Phase method, Perturbation method

use x_0 auxilliary variable

either: LP not feasible (phase 1 stops)

x_0 is back on RHS, non-basic

$\rightsquigarrow$ x_0 disappears (phase 1 "succeeds")

$\rightsquigarrow$ phase 2

Perturbation method:

add into LP $\varepsilon, \varepsilon^2, \varepsilon^3, \ldots$

$\varepsilon_1 \gg \varepsilon_2 \gg \varepsilon_3 \ldots$

eventually these disappear

$x_0 = 2$ or ≥ 2

Ask for more... can

$x_0, x_1, x_2, x_3 \geq 0$ but also $x_0 = 0$??

Pivot to feasibility

$x_3 = -2 - x_1 + x_0$

$-x_0 = -2 - x_1 - x_3$

$$\boxed{\begin{aligned} x_0 &= 2 + x_1 + x_3 \\ x_2 &= 10 - x_1 \end{aligned}}$$

see if x_0, now 2, can be 0

Minimize x_0, $w = -x_0$ maximize

2-Phase:

max $7x_1$ s.t.

$x_1 \leq 10$

$x_1 \leq -2$

$x_1 \geq 0$

$x_2 = 10 - x_1$

$x_3 = -2 - x_1$

$z = 7x_1$

forget...

Is there an x_0 s.t.

$x_2 = 10 - x_1$

$\boxed{x_3 = -2 - x_1 + x_0}$

$x_1, x_2, x_3 \geq 0$, $x_0 \geq 0$

add x_0 here too

YES!!

Dictionary

$x_2 = 10 - x_1$

$x_3 = -2 + x_1$

$(z = 7x_1)$ forget

$\boxed{x_2 = 10 - x_1}$ not

$\boxed{x_3 = -2 + x_1 + x_0}$ feasible

$w = -x_0$

pivot to feasibility:

$-x_0 = -2 + x_1 - x_3$

$x_0 = 2 - x_1 + x_3$

$x_2 = 10 - x_1$

$-x_0 = w = -2 + x_1 - x_3$

$x_0 = 2 + x_1 + x_3$

$x_2 = 10 - x_1$

$w = -x_0$

$-x_0 = w = -2 - x_1 - x_3$

Gives a proof that x_0's minimum value, keeping $x_1, x_2, x_3 \geq 0$, is 2, and 0 is impossible

$\Rightarrow$ the LP is infeasible.

max $7x_1$ s.t.

$x_1 \leq 10$, $-x_1 \leq -2$, $x_1 \geq 0$ $=$ $x_1 \geq 2$, $x_1 \geq 0$

Phase I:

$x_1 = 2 - x_0 + x_3$

$x_2 = 8 + x_0 - x_3$

$w = -x_0$

$x_1 + 3x_2 - 9x_3 \geq -7$

$x_1 \geq 2$ $\quad$ $x_3 + 2x_1 \leq 13$

$x_2 \leq 10$ $\quad$ ...

x_0 is zero ☺ x_0 = mule

$x_1 = 2 + x_3$

$x_2 = 8 - x_3$ feasible!!

w line $\Rightarrow$ x_1 increase

x_1 enters the LHS (basis)

$x_0 \geq 0 \Rightarrow x_1$ at most 2

$x_2 \geq 0 \Rightarrow x_1$ at most 10

Pivot on x_1 enters, x_0 leaves

$x_0 = 2 - x_1 + x_3$

$\boxed{x_1 = 2 - x_0 + x_3}$

$x_2 = 10 - x_1$

$= 10 - (2 - x_0 + x_3)$

$= 8 + x_0 - x_3$

$x_2 = 8 - x_3$

$x_3 = 8 - x_2$

$z = 14 + 7x_3$

$= 14 + 7(8 - x_2)$

$\rightarrow = \boxed{70 - 7x_2}$

$x_1 = 2 + x_3$

$= 10 - x_2$ } doesn't matter so much

z can be 70

(if $x_2 = 0$, $x_3 = 8$, $x_1 = 10$)

and z can't be any bigger.

Phase 2 begins: Phase 1 gives

$\boxed{x_1 = 2 + x_3}$

$\boxed{x_2 = 8 - x_3}$

originally: max $7x_1$

$z = 7x_1 = 7(2 + x_3)$

$\boxed{z = 14 + 7x_3}$

feasible dictionary !!!!!

increase x_3 since $z = 14 + 7x_3$

x_3 enters: $x_1 \geq 0$ x_3 can be anything

$x_2 \geq 0$ $x_3 \leq 8$

$$[x_1 \;\; 1-x_1]\begin{bmatrix} 33 & 1 \\ -1 & 33 \end{bmatrix} \geq [v \;\; v]$$

max v s.t.

$$x_1 \leq 1$$

$$v \leq 33x_1 + (-1)(1-x_1)$$

$$v \leq 1 \cdot x_1 + 33(1-x_1)$$

($v = v_1 - v_2$ if we don't know v will be non-neg)

max $\vec{c} \cdot \vec{x}$ s.t.

$$A\vec{x} \leq \vec{b}$$

$$\vec{x} \geq 0$$

$$\vec{x}_{slack} = \vec{b} - A\vec{x}_{orig}$$

corresponds to BFS

$\vec{x}_{orig} \to \vec{0}$, $\vec{x}_{slack} \to \vec{b}$

max v s.t.

$$x_1 \leq 1$$

$$v \leq 33x_1 + (-1)(1-x_1)$$

$$34x_1 - 1$$

$$v \leq -32x_1 + 33$$

max v s.t.

$$x_1 \leq 1$$

$$-34x_1 + v \leq -1$$

$$32x_1 + v \leq 33$$