Problems: – $\begin{bmatrix} 0 & 1 \\ 1/2 & 0 \end{bmatrix}$

$A\vec{x} \le \begin{pmatrix} \vdots \\ 0 \\ \vdots \end{pmatrix} \leftarrow$ trouble

$A\vec{x} \le \begin{pmatrix} -3 \\ -1 \end{pmatrix} \leftarrow$ worse

Ch 3: "Pitfalls"

Proving same form of the simplex method will work...

2 Phase method...

Idea: Generally I use blackboard ...

Try this...

$[x_1 \;\; 1-x_1] \begin{bmatrix} 33 & 1 \\ -1 & 33 \end{bmatrix} \ge [v \;\; v] \ldots$

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LP! max $\vec{c}^T \cdot \vec{x}$ s.t.

$A\vec{x} \le \vec{b}$

$\vec{x} \ge 0$

success, problems

max $4x_1 + 7x_2$ s.t.

$x_1 \le 5$

$x_2 \le 8$

$x_1 + x_2 \le 11$

...

the $\vec{x}$ that is feasible that maximizes the objective is called the <u>optimal solution</u>

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There is no <u>feasible</u> solution

$(x_1 \le -8$ and $x_1 \ge 0)$.

This L.P. is called <u>infeasible</u>.

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2 Phase method!

(1) Phase 1: ~~Is~~ Are there ~~any~~ any feasible solutions. (No)

(max $7x_1$) s.t.

$x_1 \le 10$

$x_1 \le -7$

$x_1 \le -8$

$x_1 \ge 0$

In a LP: max $\vec{c} \cdot \vec{x}$ s.t.

$A\vec{x} \le \vec{b}$

$\vec{x} \ge 0$

an $\vec{x}$ s.t. is called a "<u>feasible</u> solution"

$x_4 = -8 - x_1 + x_0$

$x_0 - 8 - x_1 = x_4$

$\boxed{x_0 = 8 + x_1 + x_4}$

$x_3 = -7 - x_1 \quad + x_0$

$= -7 - x_1 + (8 + x_1 + x_4)$

$\boxed{x_3 = 1 \quad + x_4}$

$x_2 = 10 - x_1 \quad + x_0$

$= 10 - x_1 + 8 + x_1 + x_4$

$\boxed{x_2 = 18 \quad + x_4}$

minimize x_0 = max $-x_0$

forget z, new obj: $w = -x_0$

Form dictionary

$z = \quad 7x_1$

$x_2 = 10 - x_1 \quad (\geq 0)$

$x_3 = -7 - x_1 \quad (\geq 0)$

$x_4 = -8 - x_1 \quad (\geq 0)$

$(x_1, x_2, x_3, x_4 \geq 0)$

1-Phase: forget z,

$\to x_2 = 10 - x_1 \ (+x_0)$

$\to x_3 = -7 - x_1 \ + x_0$

$x_4 = -8 - x_1 \ + x_0$

$x_0 \rightsquigarrow 8 \qquad x_4 \to 0$

Pivot x_0 into basis "to feasibility"

So $\quad w = -8 - (x_1 + x_4)$

$x_1 + x_4 \geq 0$

$x_1 + (-8 - x_1) \geq 0$

$-8 \geq 0$

if there were a feasible solution...

$x_1 \geq 0$

$x_4 \geq 0$ i.e. $x_1 \leq -8$

then

$x_1 \geq 0 \quad -8 - x_1 \geq 0$

but $\quad -8 \not\geq 0$

$x_0 = 8 + x_1 + x_4$

$w = \qquad -x_0$

$\boxed{w = -8 - x_1 - x_4}$

$x_2 = 18 \qquad + x_4$

$x_3 = 1 \qquad + x_4$

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Simplex: maximum possible $w = -x_0$ given $x_0, x_1, x_2, x_3, x_4 \geq 0$, max $w = -8$.

Proof: $w = -8 - x_1 - x_4$ can't be any greater than -8:

$x_1 \geq 0$, $x_4 = -8 - x_1 \geq 0$,

$z = \qquad 7x_1$ ← Forget

$x_2 = 10 - x_1$

$x_3 = -2 + x_1$

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$x_2 = 10 - x_1 + x_0$

$x_3 = -2 + x_1 + x_0$

Pivot to feasibility

$x_0 - 2 + x_1 = x_3$

$x_0 = 2 - x_1 + x_3$

$x_2 = 10 - x_1 \qquad + x_0$

$= 10 - x_1 + (2 - x_1 + x_3)$

$= 12 - 2x_1 + x_3$

First possibility in 2-phase method:

- Phase 1 shows (proves) that the LP is infeasible

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max $\quad z = 7x_1$

$x_1 \leq 10$

$x_1 \geq 2$

$x_1 \geq 0$

$-x_1 \leq -2$

22top

$$\overline{\text{non-basic} \rightsquigarrow 0}$$

$$w = \cancel{-x_0}$$

$$x_1 = 2 + x_3 \cancel{+x_0}$$

$$x_2 = 8 - x_3 \cancel{+2x_0}$$

$$x_1 = 2 + x_3$$

$$x_2 = 8 - x_3$$

$$\max \quad w = -x_0$$

$$= -2 + x_1 - x_3$$

$$x_0 = 2 - x_1 + x_3$$

$$x_2 = 12 - 2x_1 + x_3$$

x_1 enters . . . x_0 leave

$$x_1 = \underbrace{2 - x_0 + x_3}$$

$$w = -2 + (x_1) - x_3$$

$$w = -x_0$$

$$x_2 = 12 + x_3 - 2(2 - x_0 + x_3)$$

$$= 8 + 2x_0 - x_3$$