1 Directed Questions

- What does it mean for an arc to be consistent? **Answer:** An arc \( <X, r(X, Y)> \) is arc consistent if for each value \( x \) in \( \text{dom}(X) \) there is some value \( y \) in \( \text{dom}(Y) \) such that \( r(x, y) \) is satisfied.

- How can we enforce consistency of an arc \( (X, r(X, Y)) \)? **Answer:** Remove all the values \( x \) in \( \text{dom}(X) \) for which there is no corresponding value \( y \) in \( \text{dom}(Y) \) that satisfies the constraint.

- What does it mean for a network to be arc consistent? **Answer:** All of its arcs are consistent.

- What are the possible outcomes of the arc consistency algorithm? **Answer:** At least one domain could be empty, in which case there is no solution. Each domain could have a single value, in which case there is a unique solution. Or some domains could have multiple values.

2 Arc Consistency

Consider the case where the arc consistency algorithm terminates and some domains have multiple values. Is there guaranteed to be a solution? Consider the CSP problem in Figure 1. **Answer:** No, there might not be a solution. In this example shown in Figure 1, no values are removed from any of the variable domains during arc consistency, but there is no solution to the problem.

![Figure 1: Sample Constraint Network](image)

3 Learning Goals

You can:
• Build a constraint network for a set of constraints.
• Verify whether a network is arc consistent.
• Define/read/write/trace/debug the arc consistency algorithm. Compute its complexity and assess its possible outcomes.
• Define/read/write/trace/debug domain splitting and its integration with arc consistency.