# Domain Splitting, Local Search

CPSC 322 – CSP 4

Textbook §4.6, §4.8

February 4, 2011

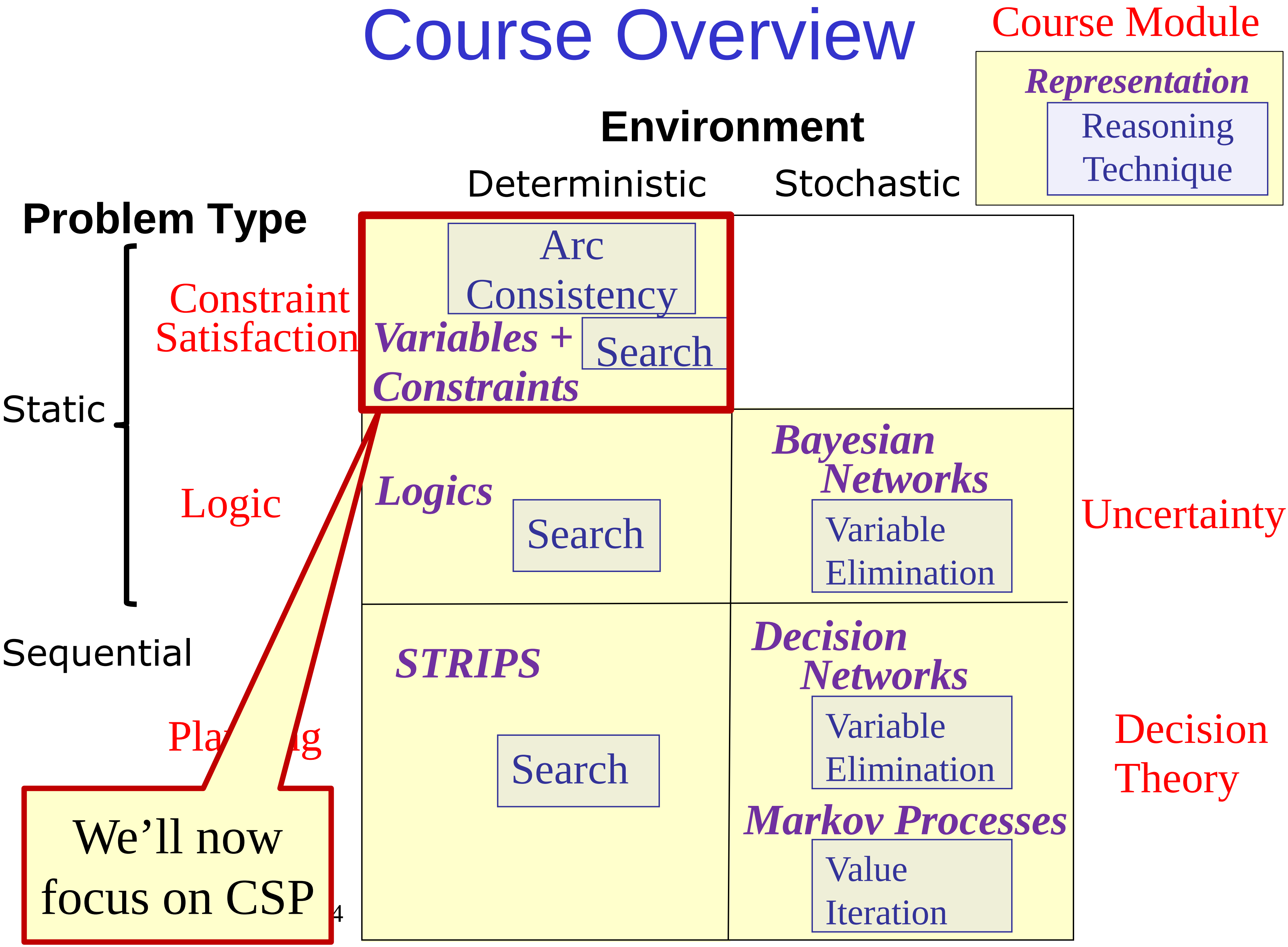
# Discussion of feedback

- Pace
  - 2 “fine”, 1 “could go faster”
  - 2: recap too long, 3: “sometimes rushed later (as a consequence)”
- Coloured card questions
  - Some more explanation would be good
  - More consistent: get everyone to vote”
- Which parts are most important?
  - Definitions + algorithms. Examples are for illustration
- Hard concepts:
  - Arc consistency: today + work in AIspace + practice exercise
  - Alternative formulation of CSP as graph search: after class

# Discussion of feedback

- Midterm: review & sample questions?
  - Midterm date confirmed: Mon, Feb 28, 3pm (1 to 1.5 hours, TBD)
  - Sample midterm has been on WebCT for ~2 weeks
    - Topics: everything up to (including all of) CSP
    - The topic of its question 2 (Planning based on CSP) won't be on the midterm
  - Should we do a midterm review session?
- More explanation of practice exercises?
  - I'll show where they are in WebCT
  - If you have trouble with them, please come to office hours
- How will what we learn eventually be applied in making an intelligent agent?
  - Game AI: lots of search
  - Reasoning under constraints is core to making intelligent decisions
    - With CSPs, we're right in the middle of it!

# Course Overview



# Lecture Overview

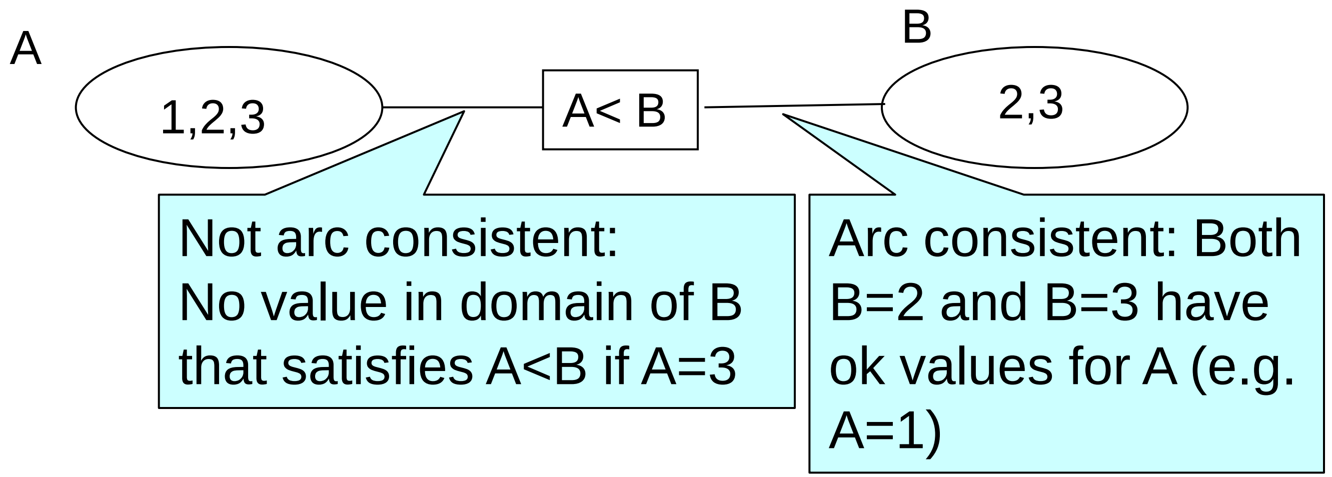
- Arc consistency
  - Recap
  - Complexity analysis
  - Domain Splitting
- Intro to Local Search

# Arc Consistency

Definition:

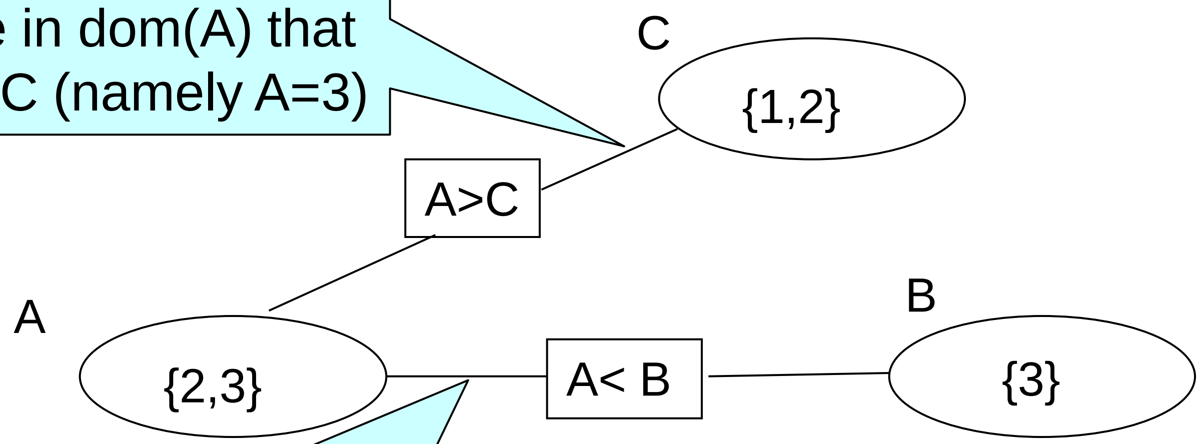
An arc <x, r(x,y)> is arc consistent if for each value x in dom(X) there is some value y in dom(Y) such that r(x,y) is satisfied.

A network is arc consistent if all its arcs are arc consistent.



# Arc Consistency

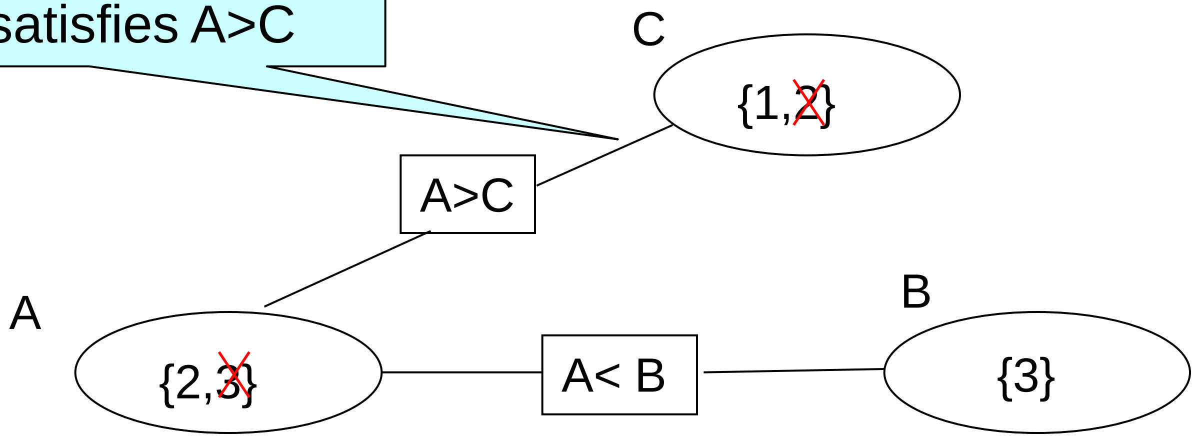
Arc consistent:
For each value in dom(C),
there is one in dom(A) that
satisfies A>C (namely A=3)




Not arc consistent:
No value in domain of B
that satisfies A<B if A=3

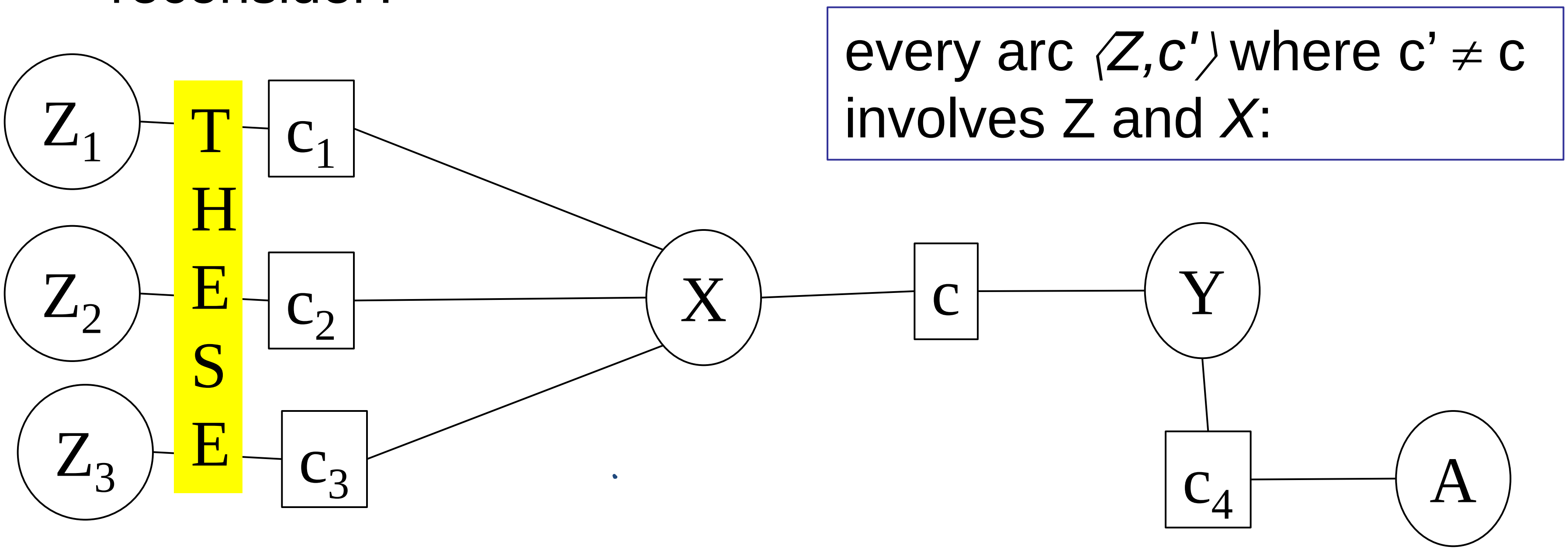
# Arc Consistency

Not arc consistent anymore:
For C=2, there is no value in
dom(A) that satisfies A>C



# Which arcs need to reconsidered?

- When we reduce the domain of a variable X to make an arc ⟨X,c⟩ arc consistent, which arcs do we need to reconsider?




- You do not need to reconsider other arcs
  - If an arc ⟨X,c'⟩ was arc consistent before, it will still be arc consistent
  - Nothing changes for arcs of constraints not involving X

# Lecture Overview

- Arc consistency
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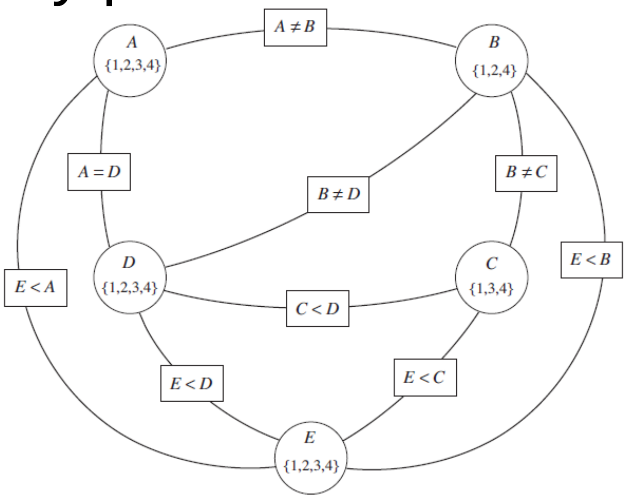
# Arc Consistency Algorithm: Complexity

- Worst-case complexity of arc consistency procedure on a problem with N variables
  – let ***d*** be the max size of a variable domain
  – let ***c*** be the number of constraints




- How often will we prune the domain of variable V? O(d) times
- How many arcs will be put on the ToDoArc list when pruning domain of variable V?
  - O(degree of variable V)
  - In total, across all variables: sum of degrees of all variables = …
    - 2*number of constraints, i.e. 2*c
- Together: we will only put O(dc) arcs on the ToDoArc list
- Checking consistency is $O(d^2)$ for each of them

- Overall complexity: $O(cd^3)$
- Compare to $O(d^N)$ of DFS!! Arc consistency is MUCH faster

# Lecture Overview

- Arc consistency
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# Can we have an arc consistent network with no solution?

YES

NO

- Example: vars A, B, C with domain {1, 2} and constraints $A \neq B$, $B \neq C$, $A \neq C$

- Or see AIspace CSP applet Simple Problem 2

# Domain splitting (or case analysis)

- Arc consistency ends: Some domains have more than one value $\rightarrow$ may or may not have a solution
  A. Apply Depth-First Search with Pruning or
  B. Split the problem in a number of disjoint cases:

  CSP with dom(X) = $\{x_1, x_2, x_3, x_4\}$ becomes

  $CSP_1$ with dom(X) = $\{x_1, x_2\}$ and
  $CSP_2$ with dom(X) = $\{x_3, x_4\}$

- Solution to CSP is the union of solutions to $CSP_i$

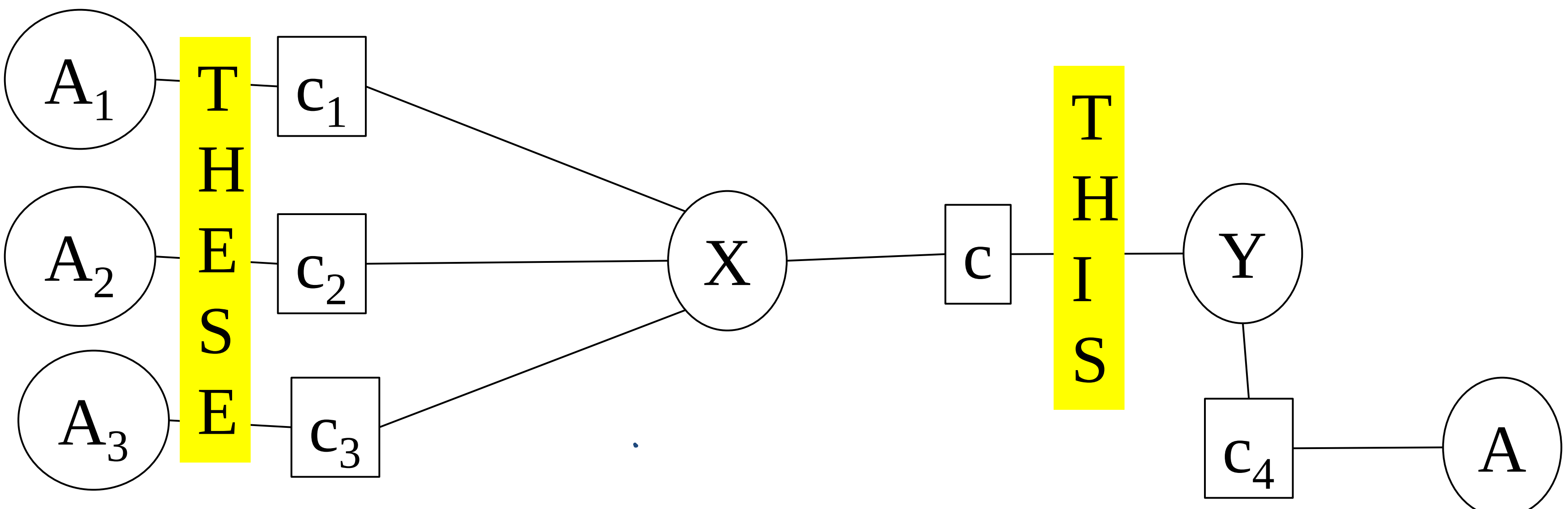
# Domain splitting

- Each smaller CSP is easier to solve
  - Arc consistency might already solve it
- For each subCSP, which arcs have to be on the ToDoArcs list when we get the subCSP by splitting the domain of X?

arcs <Z, r(Z,X)>

arcs <Z, r(Z,X)> and <X, r(Z,X)>

All arcs



# Domain splitting in action

- Trace it on “simple problem 2”

# Searching by domain splitting

CSP, apply AC

If domains with multiple values

Split on one

$CSP_1$, apply AC

If domains with multiple values
Split on one

$CSP_n$, apply AC

If domains with multiple values…..Split on one

How many CSPs do we need to keep around at a time?
With depth m and b children at each split: O(bm). It's a DFS

# Learning Goals up to here

- Define/read/write/trace/debug the arc consistency algorithm. Compute its complexity and assess its possible outcomes

- Define/read/write/trace/debug domain splitting and its integration with arc consistency

# Lecture Overview

- Arc consistency
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  - Complexity analysis
  - Domain Splitting
- Intro to Local Search

# Local Search: Why

- Solving a CSP is NP hard
  - Search space for many CSPs is huge
  - Exponential in the number of variables
  - Even arc consistency with domain splitting isn't enough

- Alternative: local search
  - Algorithms that often find a solution quickly
  - But that cannot prove that there is no solution

- Useful method in practice
  - Best available method for many constraint satisfaction and constraint optimization problems
  - Extremely general!
    - Works for problems other than CSPs
    - E.g. arc consistency only works for CSPs

# Local Search

- Idea:
  - Consider the space of complete assignments of values to variables (all possible worlds)
  - Neighbours of a current node are similar variable assignments
  - Move from one node to another according to a function that scores how good each assignment is

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 1 | 4 | 8 | 3 | 4 | 3 | 5 |
| 7 | 9 | 3 | 6 | 2 | 8 | 1 | 4 | 7 |
| 4 | 6 | 5 | 7 | 1 | 2 | 8 | 5 | 6 |
| 3 | 3 | 7 | 3 | 1 | 4 | 1 | 9 | 3 |
| 8 | 5 | 7 | 8 | 2 | 2 | 9 | 7 | 8 |
| 5 | 4 | 4 | 3 | 7 | 8 | 7 | 6 | 2 |
| 4 | 8 | 7 | 1 | 2 | 8 | 5 | 3 | 6 |
| 1 | 1 | 7 | 5 | 9 | 3 | 4 | 2 | 8 |
| 7 | 5 | 8 | 4 | 8 | 6 | 7 | 3 | 5 |

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| 2 | 8 | 1 | 4 | 8 | 3 | 4 | 3 | 5 |
| 7 | 9 | 3 | 6 | 2 | 8 | 1 | 4 | 7 |
| 4 | 6 | 5 | 7 | 1 | 2 | 8 | 5 | 6 |
| 3 | 3 | 7 | 3 | 1 | 4 | 1 | 9 | 3 |
| 8 | 5 | 7 | 8 | 2 | 2 | 9 | 7 | 8 |
| 5 | 4 | 4 | 3 | 7 | 8 | 7 | 6 | 2 |
| 4 | 8 | 7 | 1 | 2 | 8 | 5 | 3 | 6 |
| 1 | 1 | 7 | 5 | 9 | 3 | 4 | 2 | 8 |
| 7 | 5 | 8 | 4 | 8 | 6 | 7 | 3 | 5 |

# Local Search Problem: Definition

Definition: A local search problem consists of a:

CSP: a set of variables, domains for these variables, and constraints on their joint values. A node in the search space will be a complete assignment to all of the variables.

Neighbour relation: an edge in the search space will exist when the neighbour relation holds between a pair of nodes.

Scoring function: this can be used to incorporate information about how many constraints are violated.
It can also incorporate information about the cost of the solution in an optimization context.

# Example: Sudoku as a local search problem

**CSP**: usual Sudoku CSP

- One variable per cell; domains {1,…,9};
- Constraints:
  each number occurs once per row, per column, and per 3x3 box

**Neighbour relation**: value of a single cell differs

**Scoring function**: number of constraint violations

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 1 | 4 | 8 | 3 | 4 | 3 | 5 |
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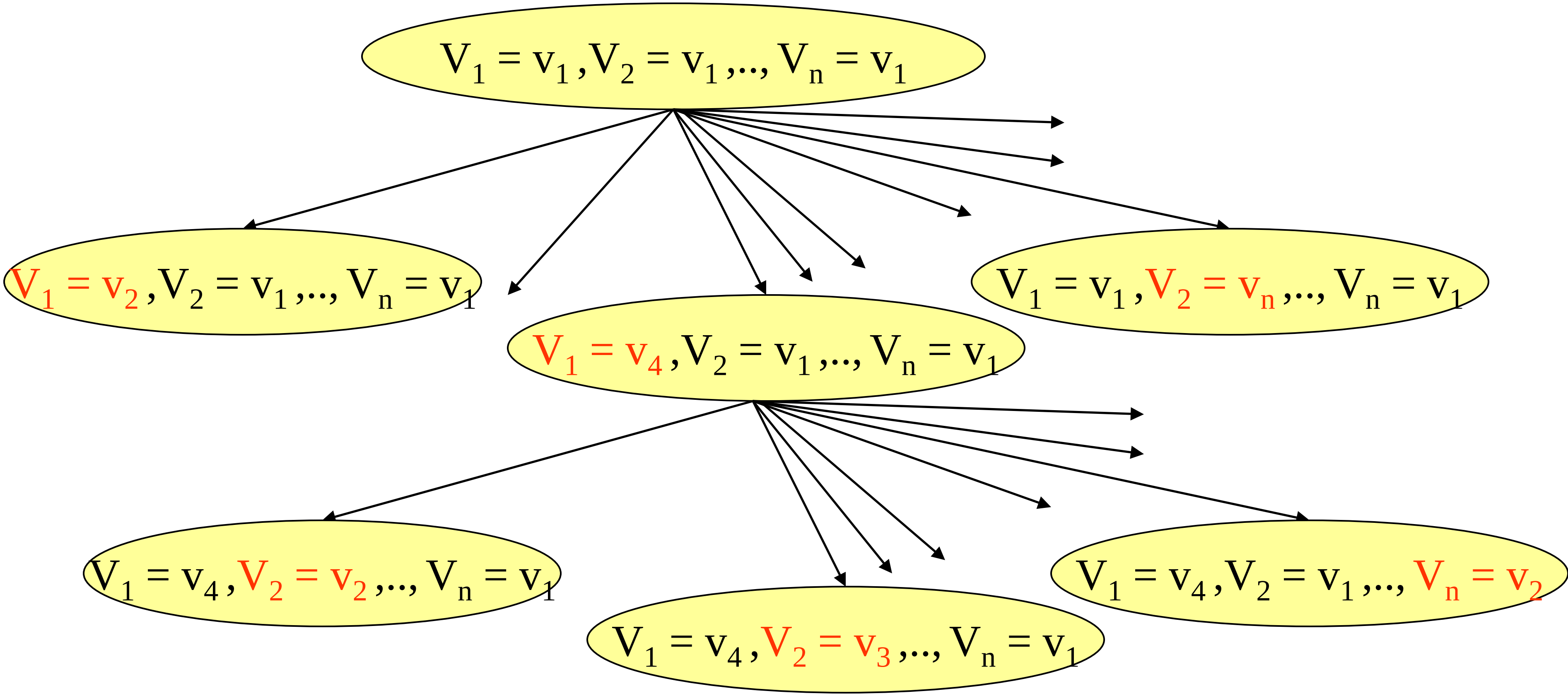
# Search Space




Only the current node is kept in memory at each step.

Very different from the search approaches we have seen so far!

Local search does NOT backtrack!

# Example: Hill climbing for Sudoku

Assign random numbers

between 1 and 9 to blank fields

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | 9 | 3 | 6 | 2 | 8 | 1 | 4 | |
| | 6 | | | | | | 5 | |
| | 3 | | | 1 | | | 9 | |
| | 5 | | 8 | | 2 | | 7 | |
| | 4 | | | 7 | | | 6 | |
| | 8 | | | | | | 3 | |
| | 1 | 7 | 5 | 9 | 3 | 4 | 2 | |
| | | | | | | | | |

# Example: Hill climbing for Sudoku

- Assign random numbers between 1 and 9 to blank fields
- Repeat
  - For each cell & each number: Evaluate how many constraint violations the assignment would yield
  - Choose the cell and number that leads to the fewest violated constraints; change it
- Until solved

| | | | | | | | | |
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| 1 | 1 | 7 | 5 | 9 | 3 | 4 | 2 | 8 |
| 7 | 5 | 8 | 4 | 8 | 6 | 7 | 3 | 5 |

# Example: Hill climbing for Sudoku

Example for one local search step:

Reduces #constraint violations by 3:

- Two 1s in the first column
- Two 1s in the first row
- Two 1s in the top-left box

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
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# General Local Search Algorithm

```
Procedure Local-Search(V,dom,C)
        Inputs
                V: a set of variables
                dom: a function such that dom(X) is the domain of variable X
                C: set of constraints to be satisfied Output
                complete assignment that satisfies the constraints
        Local
                A[V] an array of values indexed by V
        repeat
                for each variable X do
                        A[X] ←a random value in dom(X);

                while (stopping criterion not met & A is not a satisfying assignment)
                        Select a variable Y and a value V ∈dom(Y)
                        Set A[Y] ←V

                if (A is a satisfying assignment) then
                        return A

         until termination
```

You can be smart about selecting a variable and a new value for it: variable and value selection heuristics

Based on local information. E.g., for each neighbour, you can evaluate how many constraints are unsatisfied.

Hill climbing: select Y and V to minimize #unsatisfied constraints at each step

# Learning Goals for local search (started)

- Implement local search for a CSP.
  - Implement different ways to generate neighbors
  - Implement scoring functions to solve a CSP by local search through either greedy descent or hill-climbing.

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- Assignment 1 is due on Monday

- Local search practice exercise is on WebCT
- Programming assignment (part of assignment #2) is available on WebCT (due Wednesday, Feb 23rd)

- Coming up: more local search, Section 4.8