Many high-performance SLS methods are based on combinations of *simple (pure) search strategies (e.g.*, ILS, MA).

These hybrid SLS methods operate on two levels:

- Iower level: execution of underlying simple search strategies
- higher level: activation of and transition between lower-level search strategies.

**Key idea underlying Generalised Local Search Machines:** Explicitly represent higher-level search control mechanism in the form of a *finite state machine*. Example: Simple 3-state GLSM



- States z<sub>0</sub>, z<sub>1</sub>, z<sub>2</sub> represent simple search strategies, such as Random Picking (for initialisation), Iterative Best Improvement and Uninformed Random Walk.
- PROB(p) refers to a probabilistic state transition with probability p after each search step.

### Generalised Local Search Machines (GLSMs)

- States  $\cong$  simple search strategies.
- State transitions  $\cong$  search control.
- GLSM *M* starts in initial state.
- In each iteration:
  - ► *M* executes one search step associated with its current state *z*;
  - ► *M* selects a new state (which may be the same as *z*) in a nondeterministic manner.
- ► *M* terminates when a given termination criterion is satisfied.

# Modelling SLS Methods Using GLSMs

Uninformed Picking and Uninformed Random Walk



procedure step- $RP(\pi, s)$ input: problem instance  $\pi \in \Pi$ , candidate solution  $s \in S(\pi)$ output: candidate solution  $s \in S(\pi)$  s' := selectRandom(S); return s'end step-RP

procedure step- $RW(\pi, s)$ input: problem instance  $\pi \in \Pi$ , candidate solution  $s \in S(\pi)$ output: candidate solution  $s \in S(\pi)$  s' := selectRandom(N(s));return s'end step-RW

#### Uninformed Random Walk with Random Restart



 $R = \text{restart predicate, } e.g., \operatorname{countm}(k)$ 

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#### Iterative Best Improvement with Random Restart



```
procedure step-Bl(\pi, s)

input: problem instance \pi \in \Pi, candidate solution s \in S(\pi)

output: candidate solution s \in S(\pi)

g^* := \min\{g(s') \mid s' \in N(s)\};

s' := selectRandom(\{s' \in N(s) \mid g(s') = g^*\});

return s'

end step-Bl
```

Randomised Iterative Best Improvement with Random Restart



## Simulated Annealing



- Note the use of transition actions and memory for temperature T.
- The parametric state SA(T) implements probabilistic improvement steps for given temperature T.
- ► The initial temperature *T*<sub>0</sub> and function *update* implement the annealing schedule.



- The acceptance criterion is modelled as a state type, since it affects the search position.
- Note the use of transition actions for memorising the current candidate solution (pos) at the end of each local search phase.
- Condition predicates CP and CL determine the end of perturbation and local search phases, respectively; in many ILS algorithms, CL := Imin.



## Ant Colony Optimisation (1)

 General approach for modelling population-based SLS methods, such as ACO, as GLSMs:

Define search positions as *sets of candidate solutions*; search steps manipulate some or all elements of these sets.

*Example:* In this view, Iterative Improvement (II) applied to a population *sp* in each step performs one II step on each candidate solution from *sp* that is not already a local minimum.

(Alternative approaches exist.)

Pheromone levels are represented by memory states and are initialised and updated by means of transition actions.



- The condition predicate CC determines the end of the construction phase.
- ► The condition predicate *CL* determines the end of the local search phase; in many ACO algorithms, *CL* := lmin.

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