A compressive sensing perspective on simultaneous marine acquisition

Hassan Mansour[a], Haneet Wason[b], Tim T.Y. Lin[b], and Felix J. Hermann[b],

[a] Computer Science and Mathematics Departments - The University of British Columbia,

[b] Seismic Laboratory of Imaging and Modeling - The University of British Columbia

This paper was prepared for presentation at the Twelfth International Congress of the Brazilian Geophysical Society, held in Rio de Janeiro, Brazil, August 15-18, 2011.

Abstract

The high cost of acquiring seismic data in Marine environments compels the adoption of simultaneous-source acquisition - an emerging technology that is stimulating both geophysical research and commercial efforts. In this paper, we discuss the properties of randomized simultaneous acquisition matrices and demonstrate that sparsity-promoting recovery improves the quality of the reconstructed seismic data volumes. Simultaneous Marine acquisition calls for the development of a new set of design principles and post-processing tools. Leveraging established findings from the field of compressed sensing, the recovery from simultaneous sources depends on a sparsifying transform that compresses seismic data, is fast, and reasonably incoherent with the compressive sampling matrix. To achieve this incoherence, we use random time dithering where sequential acquisition with a single airgun is replaced by continuous acquisition with multiple airguns firing at random times and at random locations. We demonstrate our results with simulations of simultaneous Marine acquisition using periodic and randomized time dithering.

Introduction

Data acquisition in seismic exploration forms one of the bottlenecks in seismic imaging and inversion. It involves the collection and processing of massive data volumes, which can be up to 5-dimensional. Constrained by the Nyquist sampling rate, the increasing sizes of these data volumes pose a fundamental shortcoming in the traditional sampling paradigm as the size and desired resolution of the survey areas continue to grow.

Recently, "compressed sensing" (Donoho, 2006; Candés et al., 2006) has emerged as an alternate sampling paradigm in which randomized sub-Nyquist sampling is used to capture the structure of the data with the assumption that it is sparse or compressible in some transform domain. In seismic imaging, the data consists of wavefronts that exhibit structure across all 5 dimensions. In this paper, we rely on this dimensionality reduction feature of compressed sensing to develop a new simultaneous Marine acquisition technology where acquisition related costs are no longer determined by the stringent Nyquist sampling criteria. Our new sampling scheme replaces the high dimensional sequential shot record with a single long "supershot" in which a subset of randomly selected impulsive shots are superposed with random shifts in time. We then recovery the sequential shot record by solving a sparsity promoting inverse problem. We focus on finding appropriate measurement or sampling matrices which are incoherent with the sparsifying transform and demonstrate the effectiveness of these matrices in recovering seismic lines. Moreover, the proposed sampling scheme is physically realizable.

Compressed sensing overview

Compressive sensing (abbreviated as CS throughout the paper) is a process of reconstructing a signal utilizing the prior knowledge that it is sparse or compressible in some transform domain. The core idea of CS is a novel sampling technique, which under certain conditions can lead to smaller sampling rate compared to the conventional Nyquist sampling rate. CS is based on three key elements: randomized subsampling, sparsifying transforms and sparsity-promotion recovery by convex optimization.

One of the main advantages of CS is that it combines transformation and encoding in a single linear step, resulting in a direct application of this technology in seismic acquisition where the acquisition costs are quantified by the transform-domain sparsity of seismic data instead of the grid size. This scheme aims to design acquisition surveys in a way that renders the randomized subsampling related artifacts, caused by periodic missing traces or by crosstalk between simultaneous sources, harmless by turning them into incoherent Gaussian noise that can be easily removed during processing.

Suppose that the high resolution data is represented by the N-dimensional vector $f_0 \in \mathbb{R}^m$ which admits a sparse representation $x_0 \in \mathbb{R}^N$ in some transform domain characterized by the operator $S \in \mathbb{R}^{m\times N}$ with $N \geq m$. Note here that the signal f_0 and its sparse representation $x_0 = Sf_0$ can be of different dimensions as in the case of the redundant Curvelet transform (Candés et al., 2006a).

The sparse recovery problem involves solving an underdetermined system of equations

$$b = Ax_0, \tag{1}$$

where $b \in \mathbb{R}^n$, $n < N$ represents the compressively sampled data of n measurements, and $A \in \mathbb{R}^{n\times N}$ represents the sampling matrix. Note that A can be written as the product of a restriction/mixing operator $RM \in \mathbb{R}^{n\times m}$ and the sparsifying operator S such that $A = RMS^H$ and

$$Ax_0 = RMf_0.$$

Sparsity-promoting recovery can then be achieved by solving the basis pursuit (BP) convex optimization problem shown below

$$\tilde{x} = \arg\min_{x \in \mathbb{R}^N} \|x\|_1 \quad \text{subject to } b = Ax, \tag{2}$$

where $\tilde{x}$ represents the sparse approximation of x_0, and the ℓ_1 norm $\|x\|_1$ is the sum of absolute values of the elements of a vector x. The BP problem finds a sparse or (under some conditions) the sparsest solution that explains the data exactly.

By solving a sparsity-promoting problem (Candés et al., 2006; Donoho, 2006; Herrmann et al., 2007; Mallat, 2009), we reconstruct high-resolution data volumes from the randomized samples at the moderate cost of a minor oversampling factor compared to data volumes obtained after conventional compression (see e.g. Donoho et al., 1999, for wavelet-based compression). With sufficient sampling, this nonlinear recovery outputs a set of largest transform-domain coefficients that produces a reconstruction with a recovery error comparable with the error incurred during conventional compression. As in conventional compression this error is controllable, but in the case of CS this recovery error depends on the sampling ratio, i.e., the ratio between the number of samples taken and the number of samples of the high-resolution data. From a seismic perspective, this is the ratio between the size of the randomly mixed "supershot" and the size of the sequential shot record. Consequently, the survey time is reduced since shots are fired simultaneously and continuously and hence the costs of acquisition, storage, and possibly of data-driven processing are also reduced.

Next, we discuss the conditions that make unique recovery possible despite the fact that the linear system we are solving is under-determined, meaning that we have fewer equations than unknowns. Suppose that the vector x_0 is k-sparse, i.e., there are $k \ll N$ nonzero entries in x_0. It is possible to recovery x_0 exactly as long as any subset of k columns of the $n \times N$ matrix A is nearly orthogonal. By nearly orthogonal we mean that there exists a *restricted isometry constant* $0 < \delta_k < 1$ for which

$$(1-\delta_k)\|x_k\|_2^2 \leq \|A_\Lambda x_k\|_2^2 \leq (1+\delta_k)\|x_k\|_2^2, \tag{3}$$

where Λ is any subset of $\{1 \ldots N\}$ with cardinality $|\Lambda| \leq k$, A_Λ is the submatrix of A whose columns are indexed by Λ, and x_k is a k-dimensional vector.

Another measure of the orthogonality of the columns of A is the *mutual coherence* $\mu(A)$ between the columns of A. The mutual coherence, which provides an upper bound on $\delta_k \leq (k-1)\mu(A)$, is given by

$$\mu(A) = \max_{1 \leq i \neq j \leq N} |a_i^H a_j| / (\|a_i\|_2 \cdot \|a_j\|_2), \tag{4}$$

where a_i is the i^{th} column of A and the superscript H denotes the Hermitian transpose. Therefore, the mutual coherence is the largest absolute normalized inner product between different columns of A (Bruckstein et al., 2009). For successful (CS) recovery, the mutual coherence between the columns of the measurement matrix A should be small. When the mutual coherence is small, this means that the columns are closer to being orthogonal.

Compressed sensing and simultaneous Marine acquisition

Compressive sensing provides powerful tools for acquiring signals that have a sparse representation in some transform domain via sampling strategies/rates that are small compared to the conventional Nyquist sampling rate. Our focus here is specifically on the design of source subsampling schemes that favor recovery and on the selection of the appropriate sparsifying transform.

Seismic data permits sparse representation with multiscale and multidirection transforms that capture the "wavefront set" of the subsurface reflectors. By construction, curvelets are well adapted to data with wavefront-like features, hence, are well suited for representing seismic data parsimoniuosly as the elements of this tranform behave approximately as high-frequency asymptotic eigenfunctions of wave equations (see e.g. Smith, 1998; Candés and Demanet, 2005; Candés et al., 2006a; Herrmann et al., 2008). Therefore, we use curvelet transform as the sparsifying transform in our study.

During seismic data acquisition, the collected data volumes represent discretizations of analog finite-energy wavefields in two or more dimensions including time. We recover a sparse approximation $\tilde{f}$ of the discretized wavefield f from measurements $b = RMf$ by inverting the compressive sampling matrix

$$A := RMS^H \tag{5}$$

with the sparsity-promoting program

$$\tilde{f} = S^H \tilde{x} \quad \text{with} \quad \tilde{x} = \arg\min_x \|x\|_1 \quad \text{subject to} \quad Ax = b. \tag{6}$$

To perform this inversion, we use the SPGL1 solver (Berg and Friedlander, 2008).

Random time dithering

The design of the sampling operator RM is critical to the success of the recovery algorithm. In the simultaneous Marine acquisition scenario, the classic sequential acquisition with a single airgun is replaced with continuous acquisition with multiple airguns firing at random times and at random locations.

Suppose that we have m_s shots m_r receivers and every shot decays after m_t time samples. We first map the seismic line into a series of sequential shots f of total length $m = m_s m_t m_r$ and apply the sampling operator RM to reduce f to a single long "supershot" of length n that consists of a superposition of $n_s \ll m_s$ impulsive shots. To make the analysis feasible, we ignore for now any varying detector coverage with each source and assume the receiver spread to be constant throughout the entire survey. The resulting sampling operator is defined as follows

$$RM := [I \otimes T], \tag{7}$$

where $\otimes$ is the Kronecker product operator, I is an $m_r \times m_r$ identity matrix, and T is a random shot selector and time shifting linear operator. Applying the operator T turns the sequential source recordings into continuous recordings with n_s impulsive sources selected uniformly at random from $\{1 \ldots m_s\}$ source indices, and firing at random times selected uniformly at from $(1 \ldots (n_s - 1) \times m_t)$ time units. Consequently, the operator T subsamples the $m_s m_t$

samples to be recorded at each receiver to $n_{st} \ll n_s m_t$ samples. Figure 1 illustrates an example of a sampling operator T used in our simultaneous acquisition scheme.

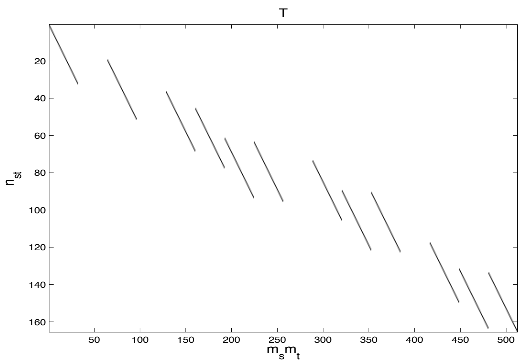

Figure 1: Example of the sampling operator T used in our proposed randomized time dithering acquisition scheme.

Next, we show that by randomizing the time dithering, i.e., when the time lag between individual shots is random, the measurement matrix A exhibits a smaller mutual coherence than constant time dithering. To illustrate this feature, we plot the Gramm matrix $G = A^H A$ of each of the random lag and constant lag operators in Figure 2. The Gramm matrix conveys information on the orthogonality of the columns of A. The faster the off-diagonal elements in G decay, the more orthogonal the columns of A, and consequently, the lower is the mutual coherence.

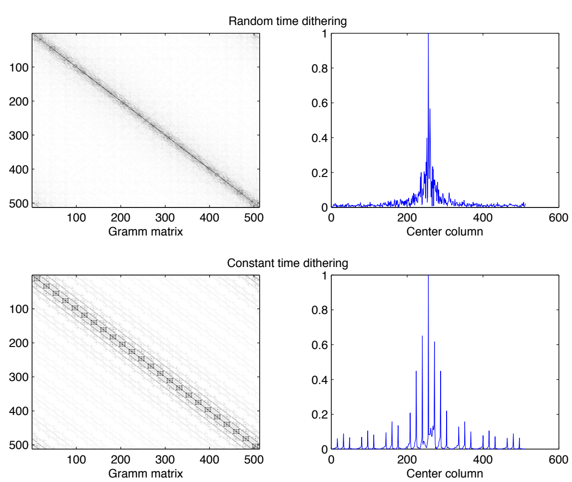

Figure 2: Gramm matrices of randomized and constant time dithering operators, top and bottom left, respectively. The top and bottom right plots show the center columns of the adjacent Gramm matrices.

Experimental results

We illustrate the effectiveness of our simultaneous source acquisition approach by studying its performance on a seismic line from the Gulf of Suez shown in Figure 3.

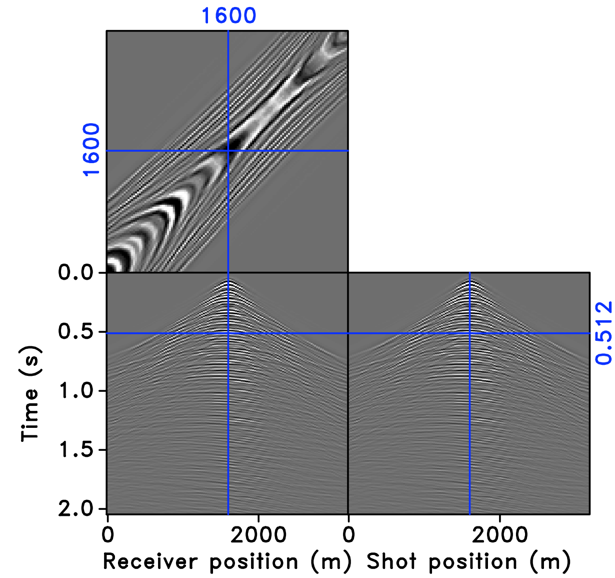

Figure 3: Fully sampled sequential data from conventional sequential acquisition with 128 shots, 128 receivers and 512 time samples.

We simulate 'Marine' acquisition by randomly selecting 128 shots from the total survey time $t = \delta \times (n_s - 1) \times t_0$ with a subsampling ratio $\delta = 0.5$. Figure 4 represents a subset of the 'supershot' obtained by applying the sampling operator RM to the data. Notice that this type of 'Marine' acquisition is physically realizable as long as the number of simultaneous sources involved is limited.

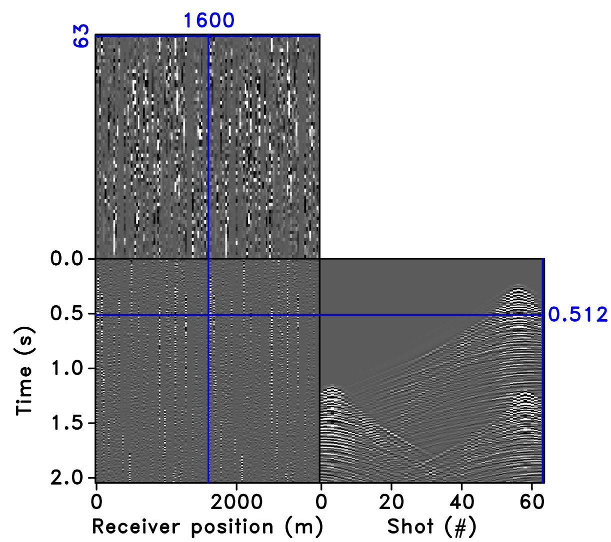

Figure 4: Compressively sampled 'Marine' data ($\delta = 0.5$).

Using the 3-D curvelet transform, which attains higher sparsity because it explores continuity of the wavefield along all three coordinate axes, the recovery results for 'Marine' acquisition is shown in Figure 5. The recovered image has an SNR of 11.1dB. We observe that accurate recovery is possible by solving an ℓ_1 minimization problem with only 200 iterations of SPGL1 (Berg and Friedlander, 2008).

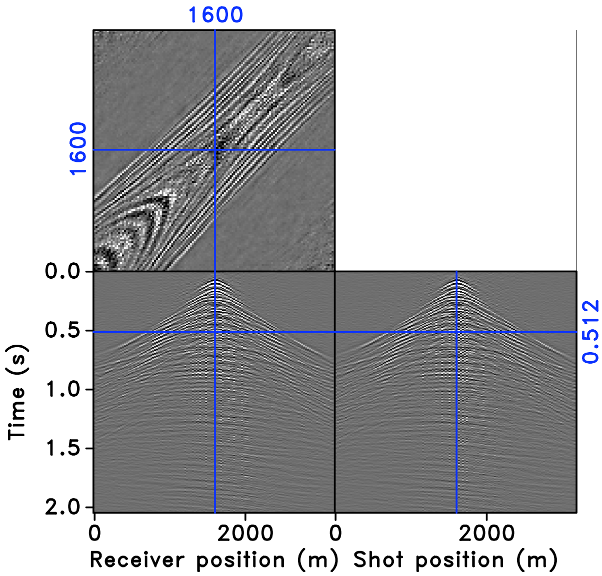

Figure 5: Recovery of 'Marine' data (SNR = 11.1dB).

Summary and Conclusions

In summary, following ideas from CS, seismic wavefields can be reconstructed from randomized subsamplings. Acquisition and processing costs are no longer determined by the resolution and size of the acquisition survey, rather, they scale with transform - domain sparsity of the wavefield, and a new paradigm for randomized processing and inversion. Recovery from simultaneous simulations depends on transform-domain sparsity wherein sparser signals, i.e., signals with a small number of significant transform domain coefficients permit better recovery. This new sampling paradigm can be successfully exploited in various problems in exploration seismology to effectively repulse the "curse of dimensionality", i.e., the exponential increase in volume on addition of extra dimensions to the data collection. Although our analysis is carried out under the assumption of fixed receiver spread, we hope to extend this framework to geometries that vary receiver spread with time, such as towed streamer surveys.

References

Berg, E. v., and M. P. Friedlander, 2008, Probing the Pareto frontier for basis pursuit solutions: Technical Report 2, Department of Computer Science, University of British Columbia.

Bruckstein, A. M., D. L. Donoho, and M. Elad, 2009, From Sparse Solutions of Systems of Equations to Sparse Modeling of Signals and Images: SIAM Review, 51, 34-81.

Candès, E., J. Romberg, and T. Tao, 2006, Stable signal recovery from incomplete and inaccurate measurements: Comm. Pure Appl. Math., 59, 1207-1223.

Candès, E. J., and L. Demanet, 2005, The curvelet representation of wave propagators is optimally sparse: Comm. Pure Appl. Math, 58, 1472-1528.

Candès, E. J., L. Demanet, D. L. Donoho, and L. Ying, 2006a, Fast discrete curvelet transforms: Multiscale Modeling and Simulation, 5, 861-899.

Donoho, D. L., 2006, Compressed sensing: IEEE Trans. Inform. Theory, 52, 1289-1306.

Donoho, P., R. Ergas, and R. Polzer, 1999, Development of seismic data compression methods for reliable, low-noise performance: SEG International Exposition and 69th Annual Meeting, 1903-1906.

Herrmann, F. J., P. P. Moghaddam, and C. C. Stolk, 2008, Sparsity- and continuity- promoting seismic imaging with curvelet frames: Journal of Applied and Computational Harmonic Analysis, 24, 150-173. (doi:10.1016/j.acha.2007.06.007).

Herrmann, F. J., U. Boeniger, and D. J. Verschuur, 2007, Non-linear primary-multiple separation with directional curvelet frames: Geophysical Journal International, 170, 781-799.

Herrmann, F. J., Y. A. Erlangga, and T. Lin, 2009, Compressive simultaneous full-waveform simulation: Geophysics, 74, A35.

Mallat, S. G., 2009, A Wavelet Tour of Signal Processing: the Sparse Way: Academic Press.

Romberg, J., 2009, Compressive sensing by random convolution: SIAM Journal on Imaging Sciences, 2, 1098-1128.

Smith, H. F., 1998, A Hardy space for Fourier integral operators.: J. Geom. Anal., 8, 629-653.

Acknowledgments

We would like to thank the authors of CurveLab (curvelet.org), a toolbox implementing the Fast Discrete Curvelet Transform, WaveAtom a toolbox implementing the Wave Atom transform (http://www.waveatom.org/), Madagascar (rsf.sf.net), a package for reproducible computational experiments, SPGL1(cs.ubc.ca/labs/scl/spgl1), SPOT (http://www.cs.ubc.ca/labs/scl/spot/), a suite of linear operators and problems for testing algorithms for sparse signal reconstruction, and pSPOT, SLIM's parallel extension of SPOT. The industrial sponsors of the Seismic Laboratory for Imaging and Modelling (SLIM) BG Group, BP, Chevron, ConocoPhillips, Petrobras, Total SA, and WesternGeco are also gratefully acknowledged.