Preconditioners for Coupled Magma/Mantle Dynamics

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McKenzie [1] derived a system of partial differential equations that model the creeping flow of high-viscosity mantle matrix and the porous flow of magma. This system of magma/mantle dynamics couples Stokes equations for the mantle with Darcy's law for the magma. In dimensionless form, this two-phase flow model is given by conservation of momentum and mass for the two-phase mixture,

$$-\nabla \cdot (\eta \mathbf{D}\mathbf{u}) + \nabla p = \nabla \left(\left(\zeta - \tfrac{1}{3}\eta \right) \nabla \cdot \mathbf{u} \right) + \phi \mathbf{e}_3, \tag{1a}$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \left(k \left(\nabla p - \mathbf{e}_3 \right) \right), \tag{1b}$$

and conservation of mass for the solid matrix,

$$\partial_t \phi - \nabla \cdot \left((1 - \phi) \mathbf{u} \right) = 0. \tag{2}$$

Here $\mathbf{u}$ is the matrix velocity, p is the dynamic pressure, ϕ is the porosity and $\mathbf{D}\mathbf{u} = \frac{1}{2}\left(\nabla \mathbf{u} + \nabla \mathbf{u}^T \right)$ is the total strain rate. The permeability, shear viscosity and bulk viscosity, respectively, k, η and ζ, are usually functions of porosity.

In solving this model numerically for time-dependent simulations, (2) is usually decoupled from (1): given a porosity, velocity and pressure are determined from (1) after which an update for the porosity is found from (2). Unfortunately, iterative methods for (1) perform badly in regions of low porosity [2]. This is in part due to the bulk viscosity being inversely proportional to the porosity. In this talk I will discuss how, by introducing the compaction pressure as a new unknown, we can circumvent some of the trouble associated with low values of porosity [3].

Furthermore, I will discuss the necessity of compatible finite element methods when using the approximate velocity obtained from (1) in the transport equation for porosity (2), and the effect this has on preconditioning.

References

[1] D. McKenzie, *The generation and compaction of partially molten rock*, J. Petrol., 25:713–765, 1984.

[2] S. Rhebergen, G. Wells, R. Katz, and A. Wathen, *Analysis of block-preconditioners for models of coupled magma/mantle dynamics*, SIAM J. Sci. Comput., 36:A1960–A1977, 2014.

[3] S. Rhebergen, G. Wells, A. Wathen, and R. Katz, *Three-field block preconditioners for models of coupled magma/mantle dynamics*, SIAM J. Sci. Comput., 37:A2270–A2294, 2015.

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