Effective and Robust Preconditioners for SPD Matrices via Structured Incomplete Factorization

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For symmetric positive definite (SPD) dense and sparse matrices, we present a framework for designing effective and robust black-box preconditioners via structured incomplete factorization (SIF). In a scaling-and-compression strategy, off-diagonal blocks are first scaled and then compressed into low-rank approximations. ULV-type factorizations are then computed. A resulting prototype preconditioner is always positive definite. Generalizations to practical hierarchical multilevel preconditioners are given. Systematic analysis of the approximation error, robustness, and effectiveness is shown for both the prototype preconditioner and the multilevel generalization. In particular, we show how local scaling and compression control the approximation accuracy and robustness, and how aggressive compression leads to efficient preconditioners that can significantly reduce the condition number and improve the eigenvalue clustering. Analysis is also given for some discretized model problems.

The SIF preconditioners can be constructed via direct or randomized compression. The costs to apply the preconditioners are about $O(N)$, where N is the matrix size. Numerical tests on several ill-conditioned problems show the effectiveness and robustness even if the compression uses very small numerical ranks. In addition, significant robustness and effectiveness benefits can be observed as compared with a standard rank structured preconditioner based on direct off-diagonal compression. This is joint work with Zixing Xin.

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