Butterfly-based Fast Direct Solvers for Highly Oscillatory Problems

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Fast direct integral equation (IE) solvers for oscillatory problems governed by the Helmholtz and Maxwells equations constitute an active area of research in applied mathematics and engineering. Direct solvers represent an attractive alternative to iterative schemes for problems that are inherently ill-conditioned and/or involve many excitations (right-hand sides). Present direct solvers leveraging hierarchical (semi-separable) matrix and related constructs rely on low-rank (LR) representations of blocks of discretized forward and inverse IE operators to rein in the CPU and memory requirements of traditional direct methods. Unfortunately, when applied to highly oscillatory problems, the computational complexity and memory requirements of these solvers suffer from a lack of LR compressibility of blocks of the discretized operators. Recently, we developed a family of butterfly-enhanced direct IE solvers that circumvents this bottleneck. Contrary to LR schemes, butterfly representations are well-suited for compressing highly oscillatory operators. Previous studies established their effectiveness for compressing forward Helmholtz and Maxwell IE operators as well as Fourier and various polynomial transforms. Here, we develop butterfly representation-based direct IE solvers for high-frequency Helmholtz and Maxwells equations. The solvers leverage randomized schemes to construct butterfly-compressed approximations of blocks obtained by adding, multiplying, and inverting previously butterfly-compressed matrices. The proposed randomized butterfly schemes constitute nontrivial generalizations of randomized LR compression schemes and the resulting constructs can be regarded as butterfly-based hierarchical (semi-separable) matrices. The CPU and memory complexities of these solvers are estimated and numerically validated to be $O(Nlog^2N)$ and at most $O(N^{1.5}logN)$, respectively. These complexities stand in stark contrast to those of LR-based direct IE solvers. The accuracy and efficiency of the proposed solvers are demonstrated via their application to the analysis of scattering from 2D and 3D, conducting and homogenous dielectric objects involving millions of spatial unknowns.

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