Robust Preconditioners for Optimality Systems - an Infinite-Dimensional Perspective

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In this talk we consider optimization problems in function space with objective functionals of tracking type and elliptic partial differential equations (PDEs) as constraints, like inverse problems for elliptic PDEs or optimal control problems with elliptic state equations. Such problems typically involve an additional regularization/cost term depending on a regularization/cost parameter.

The discretized optimality systems of such problems are typically ill-conditioned due to high-dimensional approximation spaces and/or small regularization/cost parameters. Preconditioners for these discretized optimality systems based on the concept of Schur complements have been frequently proposed in literature leading to robust convergence properties of associated preconditioned Krylov subspace methods.

We will show how to exploit this Schur complement based strategy already for the formulation of the optimality systems in function space. A crucial element is the variational formulation of the elliptic state equation. Besides the usual weak form also the strong and the very weak form are considered. The theoretical results for optimality systems using the strong or the very weak form are not restricted to convex computational domains. We discuss possible implications of this approach for preconditioning the corresponding discretized optimality systems.

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