Low-Rank Factorization Algorithms for Kernel Matrices

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Multilevel block low-rank factorization for dense matrices have been demonstrated to lead to many fast algorithms [5, 2, 4, 7, 3, 6, 1] to solve linear systems, compute eigenvalues, singular values, ... There are many algebraic algorithms available to obtain low-rank factorizations such as the rank-revealing LU or QR factorization, and randomized algorithms based on subsampling or projection on random directions. Broadly speaking, these algorithms need to strike a balance between generality (applicability to a wide class of matrices), and computational efficiency. For kernel matrices, e.g., matrices whose entries are given in the form

$$a_{ij} = K(x_i, x_j) \quad \text{or} \quad a_{ij} = \iint \phi_i(x)\,\phi_j(y)\,K(x,y)\,dxdy$$

it is possible to win on both fronts. In particular, we will demonstrate a novel approach which is robust and accurate (in particular the accuracy is independent of the point distribution x_i or the functions $\phi_i(x)$), and has low-computational cost.

If r is the minimal rank required to approximate a block of A with tolerance ε, the algorithm produces a rank r_0 approximation with computational cost $O(nr_0^2)$, where r_0 is very close to the optimal rank r, and n is the block size.

In this talk, the novel algorithm, which we call Skeletonized Interpolation, will be presented along with a numerical analysis that proves its rate of convergence, and various numerical benchmarks to validate the method and illustrate its efficiency.

References

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