

Efficient Strong Hierarchical Matrix Preconditioning Strategies for Large Scale Elliptic Boundary Value Problems

Yuval Harness¹

In this talk we will present and discuss strategies for constructing efficient positive definite preserving fast hierarchical matrix preconditioning schemes for second order elliptic boundary value problems. The hierarchical matrix approximation approach offers unique advantages over other traditional multilevel methods, e.g., multigrid or algebraic domain decomposition. These include efficient parallelization schemes with provable convergence rates to some of the most challenging complex problems. Hierarchical matrices are, essentially, data-sparse approximations which rely on the fact that the matrices can be sub-divided into a hierarchy of smaller block matrices, and certain sub-blocks can be efficiently approximated as low-rank matrices. This, often, leads to highly efficient approximation of the given matrix. However, generally, the inverse of the hierarchical matrix approximation is not ensured to be equivalently efficient approximation of the inverse of the given matrix. The last observation is especially true when the minimal eigenvalue of the given matrix is very small, which can lead to a degraded hierarchical matrix preconditioner whose complexity and memory usage are high. A strong hierarchical preconditioning scheme that overcomes this fundamental problem in the symmetric positive definite case will be presented. The method is based on low-rank approximations in a properly chosen weighted norm that locally minimizes the condition number. The effectiveness of the method which outperforms other classical strong hierarchical techniques will be illustrated through numerical experiments. This work is a joint effort between Professor Darve's group at Stanford and the Inria Hiepacs team.

¹Inria, France (yharness@gmail.com)