A Finite Element Solution of the Thermoacoustic Equations for Modeling of Trace Gas Sensors

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Trace gas sensors are utilized in a wide range of applications, such as monitoring of air quality, breath analysis for medical diagnosis, and industrial process control. These sensors operate by a sinusoidally modulated laser source which excites the trace gas to generate a pressure-temperature wave that is detected by a quartz tuning fork. To date, mathematical modeling of trace gas sensors has focused on either the acoustic or the thermal components of the wave only. A model that fully couples temperature, pressure and fluid velocity, as derived by Morse and Ingard, has been considered for simple domains only. In this paper we present the first 3-dimensional finite element model of the thermoacoustic equations in the presence of the tuning fork, coupled with temperature diffusion in the sensor. We employ a perfectly matched layers (PML) method to truncate our computational domain, and benchmark it against the analytic solution in the free-space. As the resulting discretized system is poorly conditioned, we present a parallelizable block preconditioning approach for Krylov subspace solvers. In order to precondition the highly indefinite Helmholtz block with PML, we implement a novel approach based on based on domain decomposition, where we couple an algebraic multigrid solver in the computational domain to a direct solver in the PML region.

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