Adaptive Multipreconditioning for Domain Decomposition

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Multipreconditioning is a technique that allows to simultaneously use several preconditioners within a Krylov subspace solver. It was first introduced in [1] for the conjugate gradient method. The idea is that at each iteration, instead of minimizing the error over one search direction (the preconditioned residual), the error is minimized over an $N$-dimensional space (spanned by the $N$ precondioned residuals, where $N$ is the number of preconditioners). Quite naturally, this significantly enlarged search space leads to robust solvers that can converge in a small number of iterations.

Domain decomposition methods are natural candidates for multipreconditioning. Indeed they all share the idea to split the domain into subdomains and then use a sum of local solves (one inside each of the subdomains) as a preconditioner. With multipreconditioning, no sum is performed and instead each local contribution to the preconditioner is kept separate and used to enlarge the search space. As an illustration, within multipreconditioned Additive Schwarz the search space at a given iteration (with residual $r$) is spanned by $\{R_i^T A_i^{-1} R_i r\}_{i=1,...,N}$ ($N$-dimensional) instead of $\left(\sum_{i=1}^{N} R_i^T A_i^{-1} R_i\right) r$ (unidimensional).

The drawback is of course that each iteration becomes more expensive. For this reason an adaptive multipreconditioned conjugate gradient algorithm was introduced in [3] where only some iterations of the Krylov subspace methods are multipreconditioned. In this talk I will discuss how to choose the adaptivity process and show some numerical results [2] that were obtained in collaboration with C. Bovet, P. Gosselet, and A. Parret-Fréaud on test cases that arise in aircraft engineering.

References


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