# Time Complexity of Algorithms

#### You are expected to:

- use big-O notation to categorize an algorithm as constant, linear, quadratic, logarithmic and exponential time
- given two or more algorithms, rank them in terms of their time efficiency

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# Complexity of Algorithms

- In the coming lectures, we'll be discussing different implementations of collections and comparing them with respect to certain operations.
- We need to have a good way to define the performance of an algorithm (or a piece of code).
- In this section, we examine a means of analyzing the performance of an algorithm. Usually we are interested in the algorithm's
  - time complexity: time taken for an algorithm to run
  - · space complexity: amount of memory required by it
- In this course we mainly interested in time complexity.

# **Time Complexity**

- One approach to determining an algorithm's time complexity would be to count the number of CPU cycles (or CPU time) it takes the algorithm to to perform its operation
  - · tedious and not a very practical approach
  - · depends on the machine
- Instead we will count the number of simple statements (or steps) which are
  executed by the algorithm for a given input value n (time will be a function of
  n).
- By *simple statement* we mean a statement whose running time does not depend on n:
  - an assignment (without function calls )
  - a comparison between variables, etc.

For instance, a loop that executes n times would contribute n times the number of steps of the body

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# **Big-O Notation**

- We are not interested in an exact count of steps. Instead we want to know how fast the time grows as n grows. So, we use the following approximation
- **Definition:** Let T and f be a functions of n. We say that T is O(f(n)) (pronounced "big-O f(n)" or "O f(n)") if:

```
T(n) \ll c f(n) for any n > n_0
```

where c and  $n_0$  are constants.

• **Example:** Suppose that T is the time taken for an algorithm to sort an array of length n and that:

```
T(n) \le c n^2
```

for all n then we say that the algorithm is  $O(n^2)$ .

# Example 1

```
int count = 0;
int sum = 0;
while( count < N )
{
    sum += count;
    count++;
}
System.out.print( " The sum is : ");
System.out.println( sum );</pre>
```

- The time complexity of it depends on N.
- So T(N) = 2 + (3)N + 2= 4 + 3N
- and
   T(N) <= 4N + 3N
   <= 7N
   <= c N (c=7)</li>
- Therefore T(N) is O(n)

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# **Linear Algorithms**

- Algorithms like the previous one are called "linear algorithms"
- This means that the time taken to execute the algorithm T(n) for large values of n is O(n)
- It also means that the time for the algorithm grows linearly as n grows
- Let's suppose that we double n. How does this affect the time taken to execute the algorithm?

# Example 2

```
int count = 0;
int sum = 0;
while( count < N )
{
    int index = 0;
    while( index < N )
    {
        sum += index * count;
        index++;
    }
    count++;
}</pre>
```

```
• T(N) = 2 + N(2 + N(3) + 1)
= 2 + N(3N+3)
= 3N^2 + 3N + 2
```

- Then
   T(N) <= 3N<sup>2</sup> + 3N<sup>2</sup> + 2N<sup>2</sup>
   or
   T(N) <= 8N<sup>2</sup>
- Therefore:
   T(N) is O(N²)

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# Quadratic Algorithms and More

- Algorithms like the previous one are called "quadratic algorithms"
- This means that the time taken to execute the algorithm T(n) for large values of n is  $O(n^2)$
- It also means that the time for the algorithm grows by n² as n grows
- In general we are interested in the following algorithm types:

Algorithm Type	<u>T(n)</u>
Constant	0(1)
Linear	O(n)
Logarithmic	O(log n)
07/06/10 <b>Polynomial</b>	O (nk) where k is an int constant
Exponential	O(k <sup>n</sup> ) where k is an int constant

# Complexity Example (Sorting)

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#### **Selection Sort**

- Sorts an array by repeatedly finding the smallest element of the unsorted tail region and moving it to the front
- · Slow when run on large data sets
- Example: sorting an array of integers

11 9 17 5 12

# **Sorting an Array of Integers**

· Find the smallest and swap it with the first element

```
5 9 17 11 12
```

- Find the next smallest. It is already in the correct place
- 5 9 11 17 12 • Repeat
- 5 | 9 | 11 | 12 | 17 |
   Vynen the unsorted portion is of length 1, we are done

```
5 9 11 12 17
```

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#### ch14/selsort/SelectionSorter.java

```
ch14/selsort/SelectionSorter.java (cont.)
      for (int i = 0; i < a.length - 1; i++)
        int minPos = minimumPosition(i);
        swap(minPos, i);
      }
   }
     Finds the smallest element in a tail range of the array.
     @param from the first position in a to compare
     @return the position of the smallest element in the
     range a[from] . . . a[a.length - 1]
  private int minimumPosition(int from)
     int minPos = from;
     for (int i = from + 1; i < a.length; i++)
        if (a[i] < a[minPos]) minPos = i;
     return minPos;
                                                      Continued 13
```

```
ch14/selsort/SelectionSorter.java (cont.)

/**

    Swaps two entries of the array.
    @param i the first position to swap
    @param j the second position to swap

*/
private void swap(int i, int j)

{
    int temp = a[i];
    a[i] = a[j];
    a[j] = temp;
}

private int[] a;

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```

#### ch14/selsort/SelectionSortDemo.java

```
01: import java.util.Arrays;
02:
03: /**
04:
      This program demonstrates the selection sort algorithm by
      sorting an array that is filled with random numbers.
06: */
07: public class SelectionSortDemo
08: {
09:
       public static void main(String[] args)
10:
11:
          int[] a = ArrayUtil.randomIntArray(20, 100);
12:
          System.out.println(Arrays.toString(a));
13:
14:
          SelectionSorter sorter = new SelectionSorter(a);
15:
          sorter.sort();
16:
17:
          System.out.println(Arrays.toString(a));
18:
19: }
20:
21:
```

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#### File ArrayUtil.java

#### **Typical Output:**

```
[65, 46, 14, 52, 38, 2, 96, 39, 14, 33, 13, 4, 24, 99, 89, 77, 73, 87, 36, 81] [2, 4, 13, 14, 14, 24, 33, 36, 38, 39, 46, 52, 65, 73, 77, 81, 87, 89, 96, 99]
```

#### Question

Why do we need the temp variable in the swap method? What would happen if you simply assigned a[i] to a[j] and a[j] to a[i]?

**Answer:** Dropping the temp variable would not work. Then a[i] and a[j] would end up being the same value.

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#### Question

What steps does the selection sort algorithm go through to sort the sequence 6 5 4 3 2 1?

#### Answer:

1 5	4	3	2	6
-----	---	---	---	---

#### **Analyzing the Performance of the Selection Sort Algorithm**

- In an array of size n, count how many times an array element is visited
  - To find the smallest, visit n elements + 2 visits for the swap
  - To find the next smallest, visit (n 1) elements + 2 visits for the swap
  - The last term is 2 elements visited to find the smallest + 2 visits for the swap

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#### Analyzing the Performance of the Selection Sort Algorithm

- The number of visits:
  - n + 2 + (n 1) + 2 + (n 2) + 2 + ... + 2 + 2
  - This can be simplified to  $n^2/2 + 5n/2 3$
  - 5n/2 3 is small compared to  $n^2/2 so$  let's ignore it
  - Also ignore the 1/2 it cancels out when comparing ratios

#### **Analyzing the Performance of the Selection Sort Algorithm**

- The number of visits is of the order  $n^2$
- Using big-Oh notation: The number of visits is  $O(n^2)$
- Multiplying the number of elements in an array by 2 multiplies the processing time by 4
- Big-Oh notation "f(n) = O(g(n))" expresses that f grows no faster than g
- To convert to big-Oh notation: locate fastest-growing term, and ignore constant coefficient

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#### Question

If you increase the size of a data set tenfold, how much longer does it take to sort it with the selection sort algorithm?

**Answer:** It takes about 100 times longer.

# Learning Goals Review

#### You are expected to:

- use big-O notation to categorize an algorithm as constant, linear, quadratic, logarithmic and exponential time
- given two or more algorithms, rank them in terms of their time efficiency

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# Java Collections: the List Interface

You will be expected to:

- program to the generic List interface including read and use the List API (e.g., use Lists in ways similar to arrays)
- program using the ListIterator interface ( be able to read and use the ListIterator API)
- explain the difference between Iterator and ListIterator
- compare and contrast ArrayList and LinkedList implementations of the List interface

#### Reading:

►2<sup>nd</sup> Ed: 20.1

> 3<sup>rd</sup>/4<sup>th</sup> Eds: 15.1

### The List Interface

- A *list* is an ordered collection that can contain duplicates. Lists are also called *sequences*. *Example*: ArrayListS.
- The List interface extends collection and adds methods for:
  - positional access: can access
    - current position (using an iterator)
    - i-th element  $(0 \le i < size)$
  - · positional search
    - · returns the position of a given object
  - · special iteration
    - defines special iterators for moving forwards or backwards
  - · subrange operations

07/06/10 • create sub-lists

· add/delete elements at a given position

List

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# The List Interface (cont'd)

```
public interface List<E> extends Collection<E> {
   // Positional Access
   E get(int index);
   E set(int index, E element); // Optional
   void add(int index, E element); // Optional
   E remove(int index);
                               // Optional
   boolean addAll(int index, Collection c); // Optional
   // Search
   int indexOf(Object o);
   int lastIndexOf(Object o);
   // Iteration
   ListIterator<E> listIterator();
   ListIterator<E> listIterator(int index);
   // Sublist
   List<E> subList(int fromIndex, int toIndex);
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                                                        26
```

List

# The List Interface (cont'd)

- New methods in addition to those inherited from Collection:
  - add(i,e) adds at position i
  - addAll(i,c) adds the given collection c starting at position i
  - remove (i) removes object at position i
- Methods whose behaviour is specified to be different than in the Collection interface (overridden):
  - add (e) adds e at the end of the list
- addAll(c) adds collection c at the end of the  $_{07/06/10}$  list

• remove (o) - removes first occurrence of o

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# The List Interface (cont'd)

- The sublist method returns a view of this list between fromIndex (inclusive) and toIndex (exclusive).
- Any non-structural changes to the sublist are reflected in this list and vice versa.
- You must not make structural changes (i.e., add or remove) to the original underlying list while using the sublist.
- Structural changes to the sublist *are* reflected in the backing list.

# The List Interface (cont'd)

• Example: clear / remove all the items from a list between index 1 (inclusive) and 4 (exclusive):

```
myList.subList( 1, 4 ).clear();
```

- **Example**: a function that swaps two list elements:
  - public static <T> void swap(List<T> list, int i, int j)

Why do we have  ${}_{\tiny{\mbox{\scriptsize T}}}$  at the start of the method declaration?

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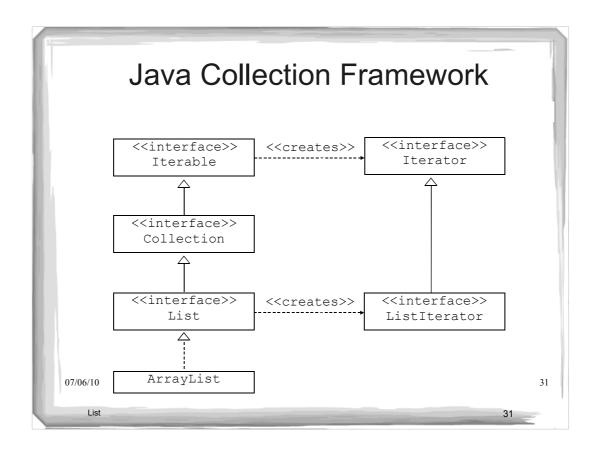
#### ListIterator

- In addition to a general Iterator, a list can also create a more specialized ListIterator.
- A ListIterator is an example of a bi-directional iterator. You can traverse the list either forwards or backwards.

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List



# The ListIterator Interface

```
public interface ListIterator<E>
                      extends Iterator<E>
  boolean hasPrevious();
  E previous();
  int nextIndex();
  int previousIndex();
  void set(E o);  // Optional
  void add(E o);  // Optional
 }
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                                                   32
  List
```

#### ListIterator

- · Can create an iterator that is positioned at
  - the first element
  - · a specified position
- next() returns items in the order they are in the list
- List iterators can move forwards or backwards
  - nextIndex() returns the index of the next item
  - previousIndex() returns the index of the previous element
- Best way is to think that the iterator points between the items: † item 0 † item 1 † item 2 † item 3 † 4 33 item 3 1 2 3 3 4 33

ListIterator (cont')

- The remove and set method affect the last element that was returned by a call to next or previous.
  - Cannot be called if remove() or add() have been called since last call to next() or previous(). Throws IllegalStateException.
- The add method adds a new element after the one that will be returned by a call to previous and before the one that will be returned by next.
  - Cannot be called if add() has been called since last call to next() or previous(). Throws IllegalStateException.
- The restrictions we discuss for general iterators are
   <sub>07/0</sub>also applicable to ListIterator's. See the Java API for
   more details on restrictions.

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# ListIterator (cont')

- After performing an add, a subsequent call to previous will return the element just added and a subsequent call to next is unaffected.
- Example:

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List

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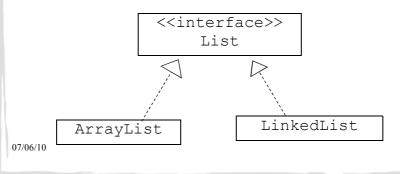
# In-Class Exercise I

- 1. Write a method that takes a List<String> parameter and prints out each item in the list
- 2. Write a second method that takes a List<String> parameter and prints out each item in reverse
- Indicate the time complexity of your methods

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# **List Implementations**

- The Java collections framework provides the following implementations of a List interface (among others):
  - ArrayList
  - LinkedList



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List

# List Implementations

- We've already considered the ArrayList. As you can probably guess, if we were to look under the hood, we'd find that its implementation is based on an array.
- The LinkedList class provides a different implementation of the List interface.

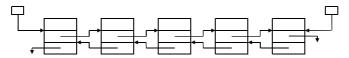
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List

### The LinkedList Class

 Rather than using an array to store data, the LinkedList class stores data in a collection of nodes that are chained or linked together similar to the one shown below:



One advantage of this implementation is that the list creates only as many nodes as are needed to store the data in the list. The number of nodes increases/decreases every time we add/remove data.

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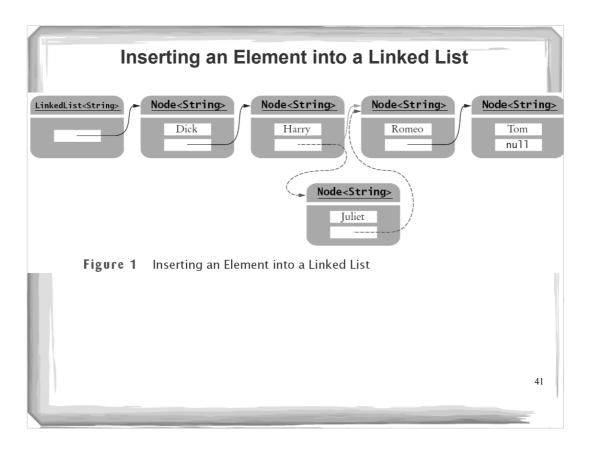
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List

a

#### **Using Linked Lists**

- A linked list consists of a number of nodes, each of which has a reference to the next node
- Adding and removing elements in the middle of a linked list is efficient
- Visiting the elements of a linked list in sequential order is efficient
- Random access is not efficient



#### Java's LinkedList class

- · Generic class
  - Specify type of elements in angle brackets: LinkedList<Product>
- Package: java.util
- Easy access to first and last elements with methods

void addFirst(E obj)

void addLast(E obj)

E getFirst()

E getLast()

E removeFirst()

E removeLast()

#### **List Iterator**

- ListIterator **type**
- · Gives access to elements inside a linked list
- Encapsulates a position anywhere inside the linked list
- Protects the linked list while giving access

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# A List Iterator | Node | Node

# A Conceptual View of the List Iterator Initial ListIterator position After calling next After inserting J Figure 3 A Conceptual View of the List Iterator

#### **List Iterator**

- Think of an iterator as pointing between two elements
  - Analogy: like the cursor in a word processor points between two characters
- The listIterator method of the LinkedList class gets a list iterator

LinkedList<String> employeeNames = . . .; ListIterator<String> iterator = employeeNames.listIterator();

#### **List Iterator**

- Initially, the iterator points before the first element
- The next method moves the iterator

```
iterator.next();
```

- next throws a NoSuchElementException if you are already past the end of the list
- hasNext returns true if there is a next element

```
if (iterator.hasNext())
iterator.next();
```

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#### **List Iterator**

The next method returns the element that the iterator is passing

```
while iterator.hasNext()
{
    String name = iterator.next();
    Do something with name
}
```

Shorthand:

```
for (String name : employeeNames)
{
    Do something with name
```

Behind the scenes, the for loop uses an iterator to visit all list elements

#### **List Iterator**

- LinkedList is a doubly linked list
  - · Class stores two links:
    - o One to the next element, and
    - o One to the previous element
- To move the list position backwards, use:
  - hasPrevious
  - previous

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## Adding and Removing from a LinkedList

- The add method:
  - Adds an object after the iterator
  - Moves the iterator position past the new element

iterator.add("Juliet");

#### Adding and Removing from a LinkedList

- The remove method
  - · Removes and
  - Returns the object that was returned by the last call to next or previous

```
//Remove all names that fulfill a certain condition
while (iterator.hasNext())
{
String name = iterator.next();
if (name fulfills condition)
  iterator.remove(); }
```

- Be careful when calling remove:
  - It can be called only once after calling next or previous
  - You cannot call it immediately after a call to add
  - If you call it improperly, it throws an IllegalStateException

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#### **Sample Program**

- ListTester is a sample program that
  - Inserts strings into a list
  - · Iterates through the list, adding and removing elements
  - Prints the list

```
ch15/uselist/ListTester.java
01: import java.util.LinkedList;
02: import java.util.ListIterator;
03:
04: /**
05: A program that tests the LinkedList class
06: */
07: public class ListTester
08: {
09:
       public static void main(String[] args)
10:
11:
          LinkedList<String> staff = new LinkedList<String>();
12:
          staff.addLast("Dick");
          staff.addLast("Harry");
13:
          staff.addLast("Romeo");
14:
15:
          staff.addLast("Tom");
16:
17:
          //\ | in the comments indicates the iterator position
18:
19:
          ListIterator<String> iterator
20:
                = staff.listIterator(); // |DHRT
          iterator.next(); // D|HRT
iterator.next(); // DH|RT
21:
22:
                                                                          Continued
```

```
ch15/uselist/ListTester.java (cont.)
23:
24:
          // Add more elements after second element
25:
          iterator.add("Juliet"); // DHJ|RT
26:
27:
          iterator.add("Nina"); // DHJN|RT
28:
          iterator.next(); // DHJNR|T
29:
30:
31:
          // Remove last traversed element
32:
33:
          iterator.remove(); // DHJN|T
34:
35:
          // Print all elements
36:
37:
          for (String name : staff)
38:
            System.out.print(iterator.next() + " ");
39:
          System.out.println();
40:
          System.out.println("Expected: Dick Harry Juliet Nina Tom");
41:
42: }
                                                                              54
```

#### ch15/uselist/ListTester.java (cont.)

#### **Output:**

Dick Harry Juliet Nina Tom Expected: Dick Harry Juliet Nina Tom

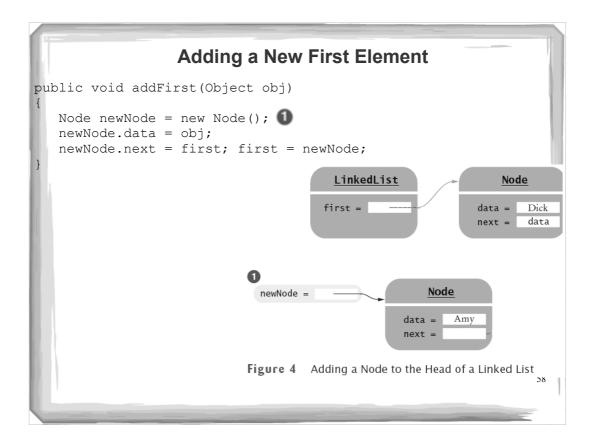
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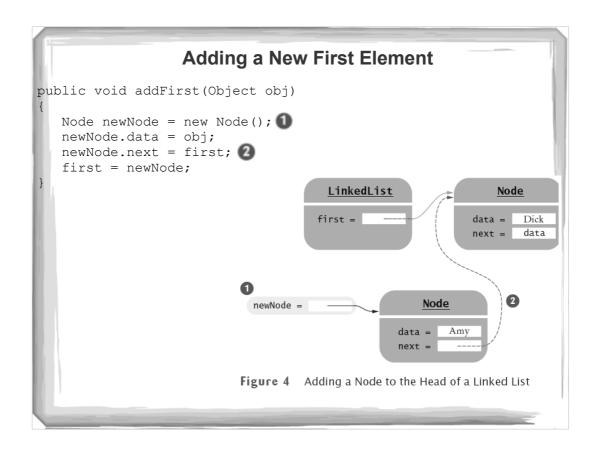
# LinkedList

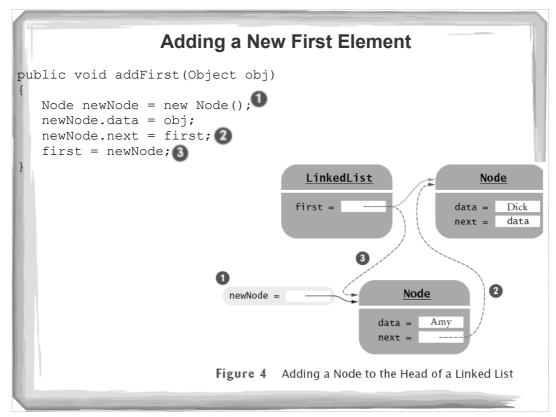
- Let's look at how LinkedList is actually implemented, since it differs from ArrayList
- This is actually a simplified version of LinkedList from Big Java

### **Adding a New First Element**

- When a new node is added to the list
  - It becomes the head of the list
  - · The old list head becomes its next node







#### **Removing the First Element**

- When the first element is removed
  - The data of the first node are saved and later returned as the method result
  - The successor of the first node becomes the first node of the shorter list
  - The old node will be garbage collected when there are no further references to it

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#### **Removing the First Element** public Object removeFirst() if (first == null) throw new NoSuchElementException(); Object obj = first.data; first = first.next; return obj; LinkedList Node Node first = data = Amy data = Dick next =

Figure 5 Removing the First Node from a Linked List

next =

#### **Linked List Iterator**

- We define LinkedListIterator: private inner class of LinkedList
- Implements a simplified ListIterator interface
- Has access to the first field and private Node class
- Clients of LinkedList don't actually know the name of the iterator class
  - They only know it is a class that implements the ListIterator interface

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#### LinkedListIterator

```
The LinkListIterator class
public class LinkedList
{
    . . .
    public ListIterator listIterator()
    {
        return new LinkedListIterator();
    }

    private class LinkedListIterator implements
        ListIterator
    {
        public LinkedListIterator()
        {
            position = null;
            previous = null;
        }
}
```

Continued

#### LinkedListIterator (cont.)

```
private Node position; private Node previous;
```

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#### The Linked List Iterator's next Method

- position: reference to the last visited node
- Also, store a reference to the last reference before that
- next method: position reference is advanced to position.next
- Old position is remembered in previous
- If the iterator points before the first element of the list, then the old position is null and position must be set to first

#### The Linked List Iterator's next Method

```
public Object next()
{
   if (!hasNext())
      throw new NoSuchElementException();
   previous = position; // Remember for remove
   if (position == null)
      position = first;
   else position = position.next;
   return position.data;
}
```

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#### The Linked List Iterator's hasNext Method

- The next method should only be called when the iterator is not at the end of the list
- The iterator is at the end
  - if the list is empty (first == null)
  - if there is no element after the current position (position.next == null)

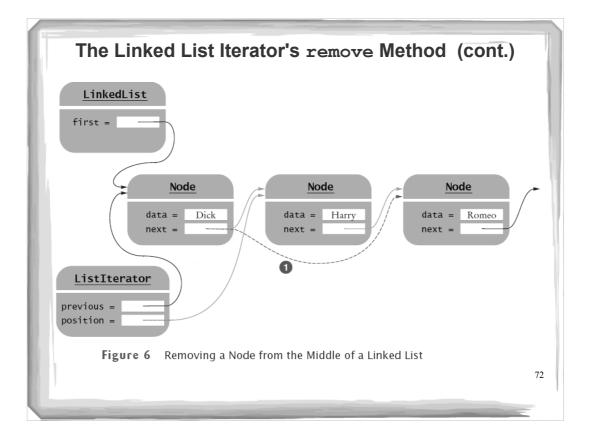
#### The Linked List Iterator's hasNext Method

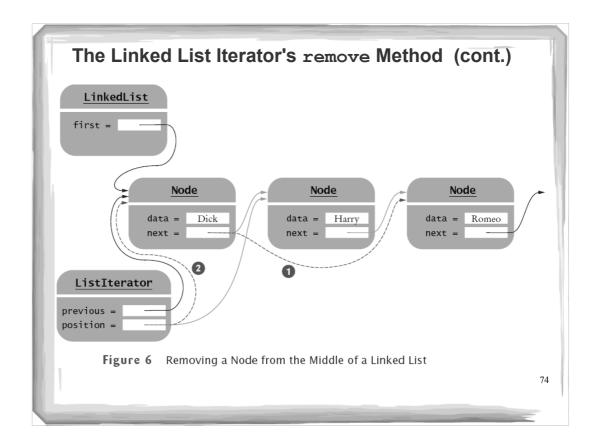
```
private class LinkedListIterator implements ListIterator
{
    . . .
    public boolean hasNext()
    {
        if (position == null)
            return first != null;
        else
            return position.next != null;
    }
    . . .
```

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#### The Linked List Iterator's remove Method

- If the element to be removed is the first element, call removeFirst
- Otherwise, the node preceding the element to be removed needs to have its <code>next</code> reference updated to skip the removed element
- If the previous reference equals position:
  - · this call does not immediately follow a call to next
  - throw an IllegalArgumentException
- It is illegal to call remove twice in a row
  - remove sets the previous reference to position





#### The Linked List Iterator's set Method

- Changes the data stored in the previously visited element
- The set method

```
public void set(Object obj)
{
   if (position == null)
      throw new NoSuchElementException();
   position.data = obj;
}
```

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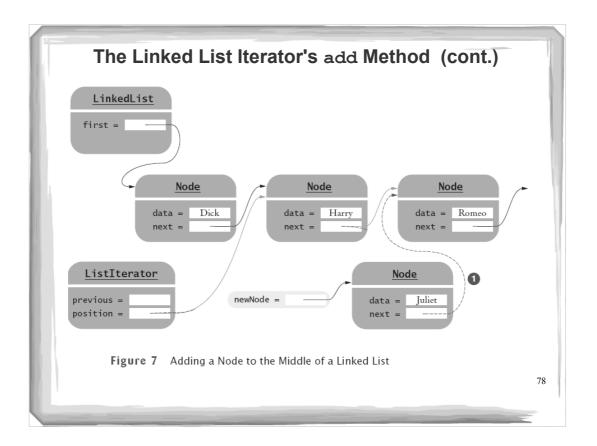
#### The Linked List Iterator's add Method

- The most complex operation is the addition of a node
  - add inserts the new node after the current position
  - Sets the successor of the new node to the successor of the current position

```
The Linked List Iterator's add Method

public void add(Object obj)
{
   if (position == null)
   {
      addFirst(obj);
      position = first;
   }
   else
   {
      Node newNode = new Node();
      newNode.data = obj;
      newNode.next = position.next;
      position.next = newNode;
      position = newNode;
   }
   previous = position;
}

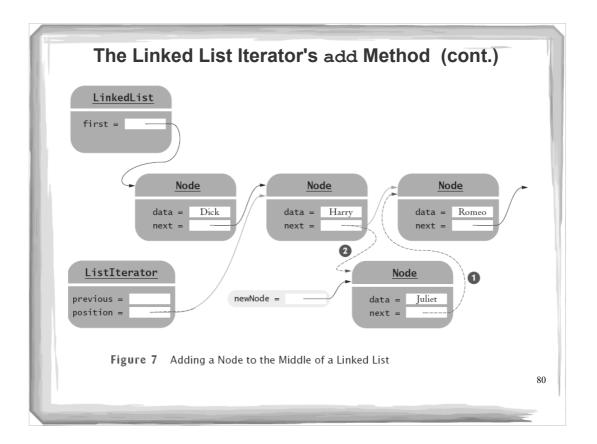
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```



```
The Linked List Iterator's add Method

public void add(Object obj)
{
    if (position == null)
    {
        addFirst(obj);
        position = first;
    }
    else
    {
        Node newNode = new Node();
        newNode.data = obj;
        newNode.next = position.next;
        position.next = newNode;
        position = newNode;
    }
    previous = position;
}

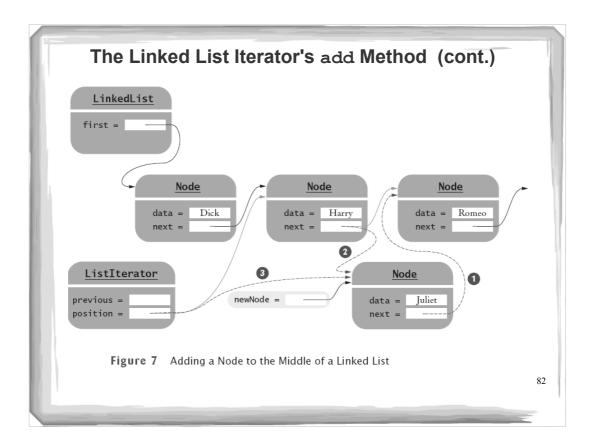
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```



```
The Linked List Iterator's add Method

public void add(Object obj)
{
   if (position == null)
   {
      addFirst(obj);
      position = first;
   }
   else
   {
      Node newNode = new Node();
      newNode.data = obj;
      newNode.next = position.next;
      position.next = newNode;
      position = newNode;
   }
   previous = position;
}

Continued
```



# The LinkedList Class

• Let's look at some differences between ArrayList and LinkedList. Let's consider the time complexity of some common operations in the worst case:

	LinkedList	ArrayList
get( int index )	0(n)	0(1)
add( int i, E e )	0(n)	0(n)
add(Ee)	0(1)	0(1)
remove( int index )	0(n)	0(n)
contains ( Object o )	0(n)	0(n)
oHidatIterator -> add	0(1)	0(n)
ListIterator -> remove	0(1)	O(n)

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#### **Efficiency of Operations for Arrays and Lists**

Operation	ArrayList	LinkedList
Random access	O(1)	O(n)
Linear traversal step	O(1)	O(1)
Add/remove an element	O(n)	O(1)

```
List Example

Reverse a list

public static <E > List E > reverse (List E > list) {

List E > new list = new ArrayList E > 0;

Iherator E > irr = list.iterator();

while (irrhasNext()) {

new list.add(0, irrnext());
}

return new list;
}

What is its complexity?
```

# List Example

■ A better way to write this method:

```
public static <E > List<E > reverse (List<E > list) {
```

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List

## In-Class Exercise II

- Write a method that takes an ArrayList<String>, and visits each item in order, printing out the item and then removing it
- Write a method that takes a LinkedList<String>, and visits each item in reverse order, printing out the item and then removing it
- Indicate the complexity of each method

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# Learning Goals Review

You will be expected to:

- program to the generic List interface including read and use the List API (e.g., use Lists in ways similar to arrays)
- program using the ListIterator interface (be able to read and use the ListIterator API)
- explain the difference between Iterator and ListIterator
- compare and contrast ArrayList and LinkedList implementations of the List interface

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# Midterm Exam

- Class contracts
  - Preconditions
  - Postconditions
  - Invariants

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# Midterm Exam

- Exceptions
  - Throwing
  - Catching
  - Propagating
  - Defining

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# Midterm Exam

- Testing
  - Unit testing
  - Blackbox testing
  - Equivalence classes
  - Test cases: typical values, boundary values

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# Midterm Exam

- Good and bad design
  - High cohesion, low coupling
  - Open-closed principle
  - Liskov substitution principle
  - Weakening precondition, strengthening postcondition

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# Midterm Exam

- Java collections
  - Interfaces: Iterable, Collection, List, Iterator
  - Classes: ArrayList
- Generic programming
  - Generic classes (defining and using)
  - Generic methods (defining and using)
  - Type parameters
  - Bounded wildcards

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