DIFFERENTIAL EQUATIONS

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Introduction

When phenomenon in Physical Sciences/Engg/Finance etc. are modeled mathematically, Differential Equations arise.
What are Differential Equations:

- Equations that involve dependent variables and their derivatives with respect to the independent variables.
- Typical solutions will be a class of functions perturbed by a constant.

Basic Terms

- In $y = f(x)$, x is called independent variable and y is called dependent variable.
- Derivative : $\frac{\mathrm{d}y}{\mathrm{d}x}$
- Partial derivative which is written as $\frac{\partial u}{\partial t}$ can arise when we have multiple independent variables.

Classification

- Ordinary Differential Equations
- Partial Differential Equations

Terms about Differential Equations

- Order: The order of a differential equation is the highest derivative that appears in the differential equation.
- Degree: The degree of a differential equation is the power of the highest derivative term.
- Linear DE : A DE when there are no multiplications among dependent variables and their derivatives.
- Non-linear DE : A DE that do not satisfy the definition of linearity is a non-linear DE.
- Quasi-linear DE: A non-linear DE, when there are no multiplications among all dependent variables and their derivatives in the highest derivative term

Terms about Differential Equations

- General Solution: Solutions obtained from integrating the differential equations are called general solutions. The general solution of a nth order ordinary differential equation contains n arbitrary constants resulting from integrating n times.
- Particular Solution: Particular solutions are the solutions obtained by assigning specific values to the arbitrary constants in the general solutions.
- Singular Solutions: Solutions that can not be expressed by the general solutions are called singular solutions.

First Order Differential Equations

- The general first order DE is of the form $y' = f(x, y)$
- The solution of a DE such as $y' = f(x)$ will be $y = \phi(x) = \int^{x} f(t)dt + c$
- A one parameter solution for an equation like $y' + ay = g(x)$ is given by $y = \phi(x) = e^{-ax} \int^{x} e^{at} g(t)dt + ce^{-ax}$

Second Order Differential Equations

- A typical first order DE is of the form $y'' = f(x, y, y')$
- The solution of a DE such as $y'' = g(x)$ will be $y = \phi(x) = c_1 + c_2x + \int^x [\int^t g(s)ds]dt$,
- The general second order linear equation has the form

$$P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)y = G(x), \qquad (1)$$

Famous examples of Second Order Equations

- Equation governing motion of a mass on a spring

$$m\frac{d^2u}{dt^2} + c\frac{du}{dt} + ku = F(t), \tag{2}$$

- Bessel's equation of order v

$$x^2y'' + xy' + (x^2 - v^2)y = 0, \tag{3}$$

- Legendre's equation of order α

$$(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0 \tag{4}$$

Numerical methods to solve DE

- Euler's method
- Midpoint method
- Runge-Kutta method

Euler's method

- Euler's method is very geometric. We go along the tangent line of the graph of the solution to find next approximation
- We start by setting $t_0 = a$ and choosing a step size h. We inductively define the iterations as follows

$$y_{k+1} = y_k + hf(t_k, y_k), t_{k+1} = t_k + h,$$

Partial Differential Equations

- A DE involving an unknown function (or functions) of several independent variables and its (or their) partial derivatives with respect to those variables
- A simple PDE is $\frac{\partial}{\partial x}u(x, y) = 0$. The solution would be $u(x, y) = f(y)$ for any arbitrary function $f(y)$.

An example of PDE

- A DE involving an unknown function (or functions) of several independent variables and its (or their) partial derivatives with respect to those variables
- A simple PDE is $\frac{\partial}{\partial x}u(x, y) = 0$. The solution would be $u(x, y) = f(y)$ for any arbitrary function $f(y)$.

Heat Equation in one space dimension

- An equation for conduction of heat in one dimension is of the form

$$u_t = \alpha u_{xx} \tag{5}$$

where $u(t, x)$ is temperature, and α is a positive constant.

THANK YOU

Thanks for Coming: Questions and Discussion