CS 554m
controlled experiments I

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learning goals

be able to answer the following:

what is the experimental method?

what is an experimental hypothesis?

how do I plan an experiment?

why are statistics used?

within- & between-subject comparisons: how do they differ?

how do I compute a t-test?

what are the different types of t-tests?

Acknowledgement: Some of the material in this lecture is based on material prepared for similar courses by Saul Greenberg (University of Calgary)

a good portion of the material in these lectures on experimental design should be familiar from ugrad stats class, although perhaps presented here from a slightly different perspective

also, most of this material is well covered in today's readings:

Newman & Lamming, Ch 10

Lazar, Feng, & Hochheiser, Ch 2 - 4

Who has run an experiment?

material I assume you already know and will not be covered (some additional slides at end)

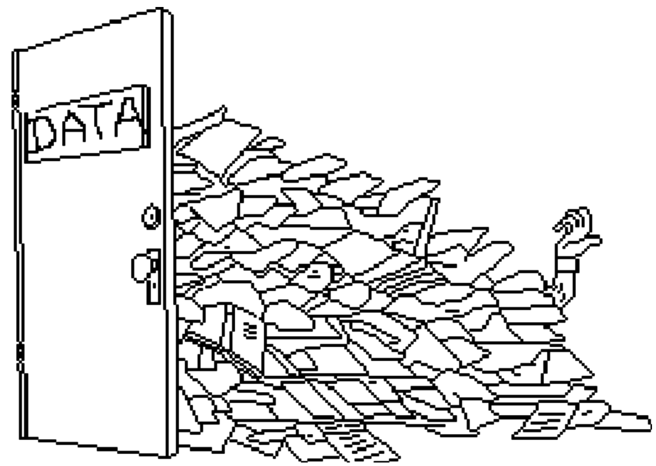
types of variables

samples & populations

normal distribution

variance and standard deviation

quantitative methods



1. user performance data collection
   - data is collected on system use

descriptive statistics

- frequency of request for on-line assistance
  - what did people ask for help with?
- frequency of use of different parts of the system
  - why are parts of system unused?
- number of errors and where they occurred
  - why does an error occur repeatedly?
- time it takes to complete some operation
  - what tasks take longer than expected?

- collect heaps of data in the hope that something interesting shows up
- often difficult to sift through data unless specific aspects are targeted (as in list above)

quantitative methods

2. controlled experiments

the traditional scientific method

- reductionist
  - clear convincing result on specific issues
- in HCI
  - insights into cognitive process, human performance limitations, ...
  - allows comparison of systems, fine-tuning of details ...

strives for

- lucid and testable hypothesis (usually a causal inference)
- quantitative measurement
- measure of confidence in results obtained (inferencial statistics)
- replicability of experiment
- control of variables and conditions
- removal of experimenter bias

desired outcome of a controlled experiment

statistical inference of an event or situation’s probability:

“Design A is better <*in some specific sense*> than Design B”

or, Design A meets a target:

“90% of incoming students who have web experience can complete course registration within 30 minutes”

steps in the experimental method

step 1: begin with a lucid, testable hypothesis

Example 1:

H_0: there is no difference in the number of cavities in children and teenagers using crest and no-teeth toothpaste

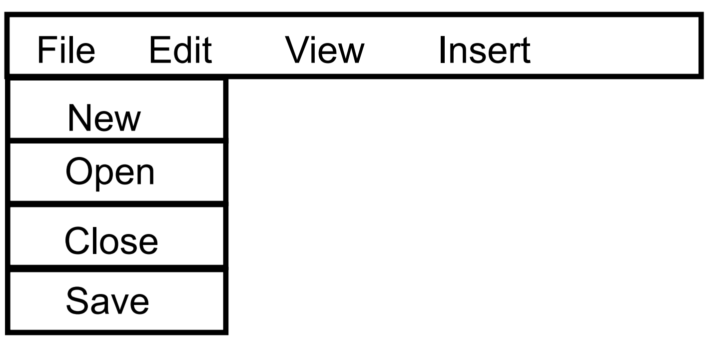
H_1: children and teenagers using crest toothpaste have fewer cavities than those who use no-teeth toothpaste

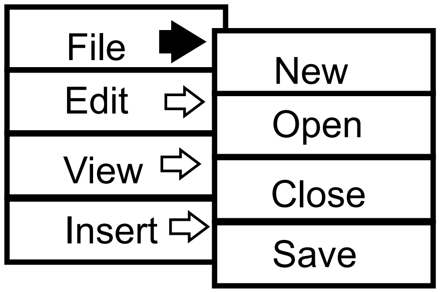
step 1: begin with a lucid, testable hypothesis

Example 2:

H_0: there is no difference in user performance (time and error rate) when selecting a single item from a pop-up or a pull down menu, regardless of the subject's previous expertise in using a mouse or using the different menu types

H_1: selecting from a pop-up menu will be faster and less error prone than selecting from a pull down menu





general: hypothesis testing

hypothesis = **prediction** of the outcome of an experiment.

framed in terms of **independent** and **dependent** variables:

a variation in the independent variable will cause a difference in the dependent variable.

aim of the experiment: prove this prediction

do by: *disproving* the "null hypothesis"

H_0: experimental conditions **have no effect** on performance (to some degree of **significance)** → **null hypothesis**

H_1: experimental conditions **have an effect** on performance (to some degree of **significance)** → **alternate hypothesis**

step 2: explicitly state the independent variables

Independent variables

- things you control/manipulate (independent of how a subject behaves) to produce different conditions for comparison
- two different kinds:
    - treatment manipulated (can establish cause/effect, true experiment)
    - subject individual differences (can never fully establish cause/effect)

in toothpaste experiment

- toothpaste type: Crest or No-teeth toothpaste *(treatment)*
- age: <= 12 years *or* > 12 years *(subject)*

in menu experiment

- menu type: pop-up or pull-down *(treatment)*
- menu length: 3, 6, 9, 12, 15 *(treatment)*
- expertise: expert or novice *(often subject, but can train an expert)*

step 3: carefully choose the dependent variables

Dependent variables

- things that are measured
- expectation that they depend on the subject's behaviour / reaction to the independent variable (but unaffected by other factors)

in toothpaste experiment:

in menu experiment:

step 4: consider possible nuisance variables & determine mitigation approach

- undesired variations in experiment conditions which **cannot be eliminated**, but which **may affect** dependent variable
  - critical to know about them
- experiment design & analysis must generally accommodate them:
  - treat as an additional experiment independent variable (if they can be controlled)
  - randomization (if they cannot be controlled)
- common nuisance variable: *subject* (individual differences)

in toothpaste experiment:

in menu experiment:

how to manage?

step 5: design the task to be performed

tasks must:

be externally valid

external validity = do the results generalize?

… will they be an accurate predictor of how well users can perform tasks as they would in real life?

for a large interactive system, can probably only test a small subset of all possible tasks.

exercise the designs, bringing out any differences in their support for the task

e.g., if a design supports website **navigation**, test task should **not** require subject to work within a **single page**

be feasible - supported by the design/prototype, and executable within experiment time scale

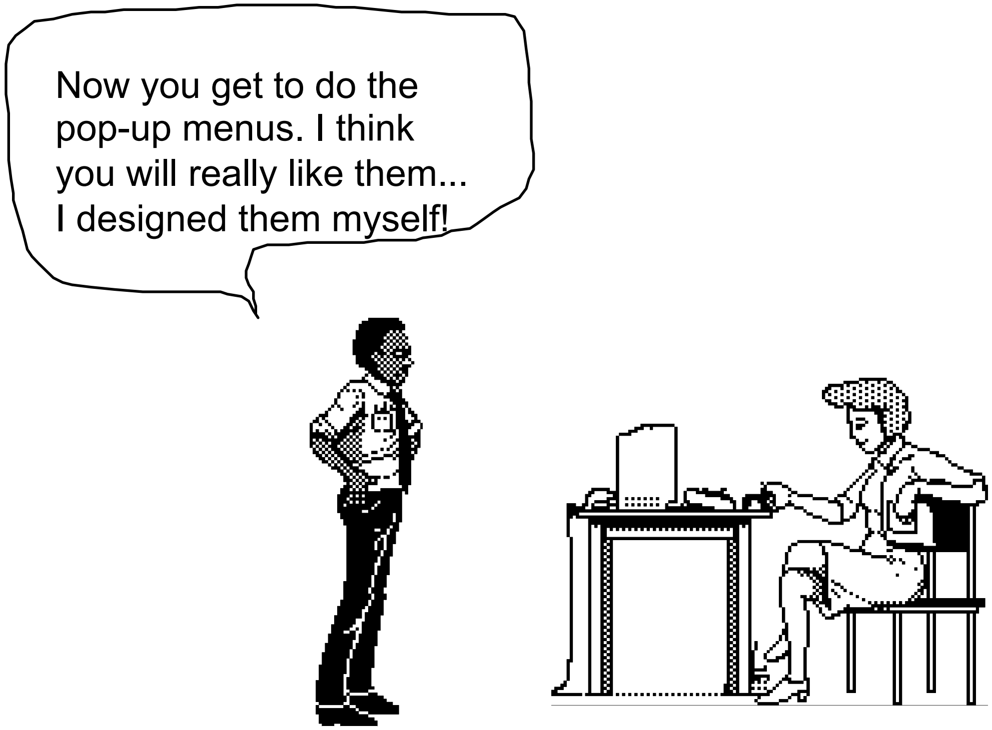
step 5: design the task to be performed

in toothpaste experiment:

in menu experiment:

step 6: design experiment protocol

- steps for executing experiment are prepared well ahead of time
- includes unbiased instructions + instruments (questionnaire, interview script, observation sheet)
- double-blind experiments, ...



step 7: make formal experiment design explicit

simplest: 2-sample (2-condition) experiment

based on comparison of **two sample means**:

- performance data from using Design A & Design B
  - e.g., new design & status quo design
  - e.g., 2 new designs

or, comparison of **one sample mean with a constant**:

- performance data from using Design A, compared to performance requirement
  - determine whether single new design meets key design requirement

step 7: make formal experiment design explicit

more complex: factorial design

in toothpaste experiment:

2 toothpaste types (crest, no-teeth)

x 2 age groups (<= 12 years *or* > 12 years)

in menu experiment:

2 menu types (pop-up, pull down)

x 5 menu lengths (3, 6, 9, 12, 15)

x 2 levels of expertise (novice, expert)

(more on this later)

step 8: judiciously select/recruit and assign subjects to groups

subject pool: *similar issues as for informal studies*

- match expected user population as closely as possible
- age, physical attributes, level of education
- general experience with systems similar to those being tested
- experience and knowledge of task domain

sample size: *more critical in experiments than informal studies*

- going for "statistical significance"
- should be large enough to be "representative" of population
- guidelines exist based on statistical methods used & required significance of results
- pragmatic concerns may dictate actual numbers
- "10" is often a good place to start

step 8: judiciously select/recruit and assign subjects to groups

- if there is too much variability in the data collected, you will not be able to achieve statistical significance (more later)
- you can reduce variability by controlling subject variability

how?

- recognize classes and make them an independent variable
  - e.g., older users vs. younger users
  - e.g., superstars versus poor performers
- use reasonable number of subjects and random assignment

Novice

Expert

step 9: apply statistical methods to data analysis

examples: t-tests, ANOVA, correlation, regression (more on these later)

confidence limits: the confidence that your conclusion is correct

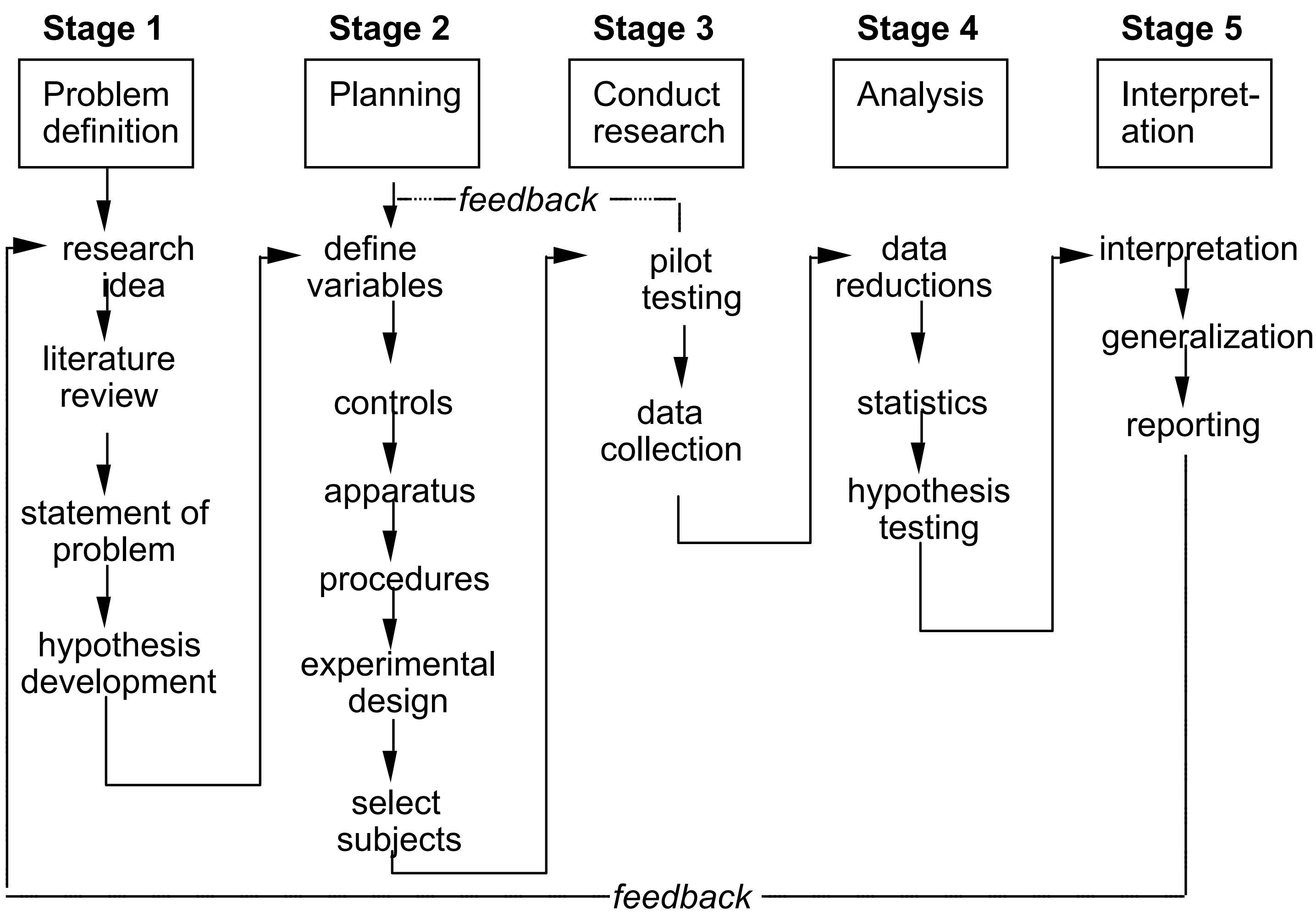
- “The hypothesis that mouse experience makes no difference is rejected at the .05 level” (i.e., null hypothesis rejected)
- this means:
  - a 95% chance that your finding is correct
  - a 5% chance you are wrong

step 10: interpret your results

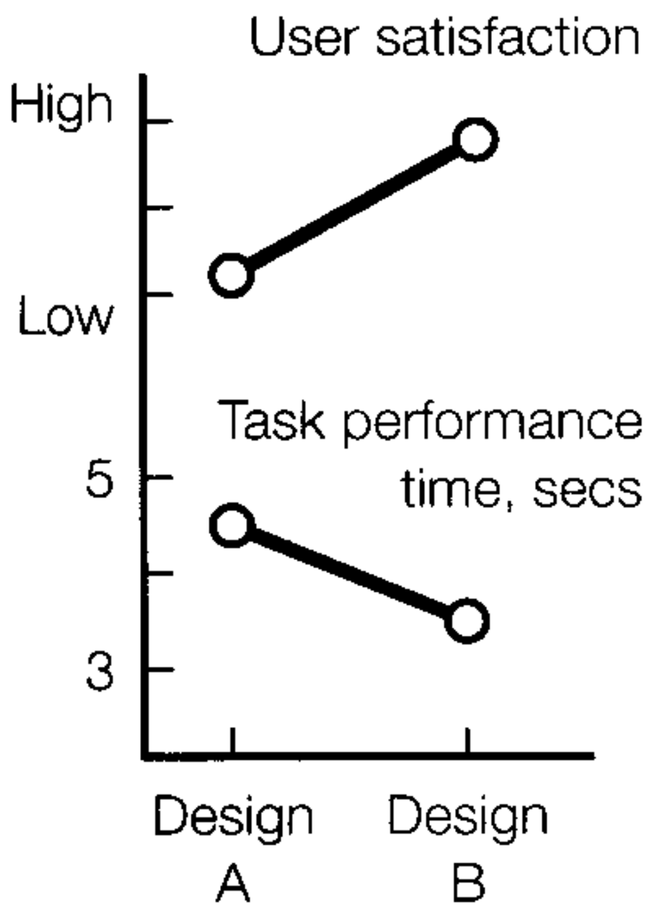
what *you* believe the results mean, and their implications

yes, there can be a subjective component to quantitative analysis

the planning flowchart



goal of experiment design



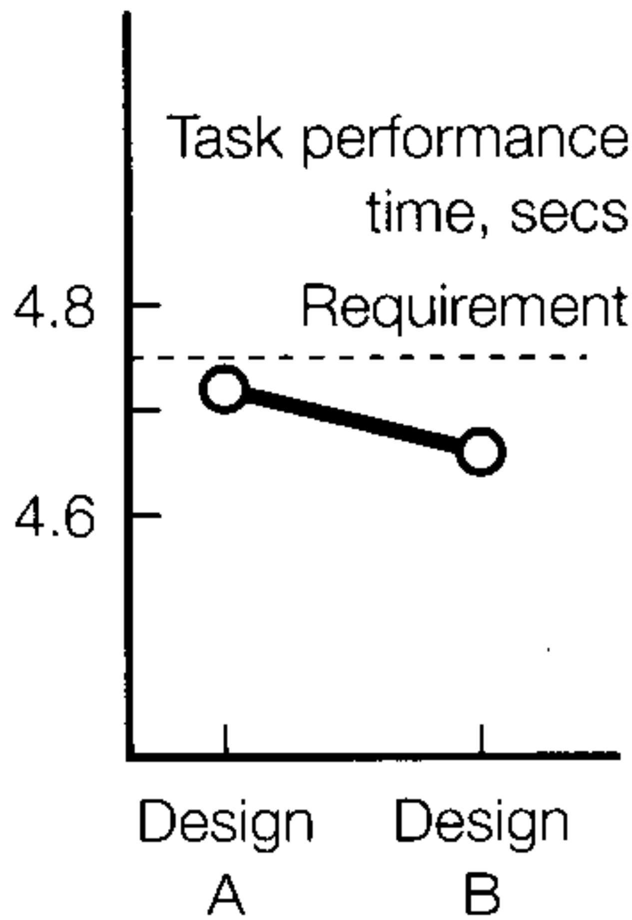
guard against ambiguous or misleading results

← a good (definitive) result

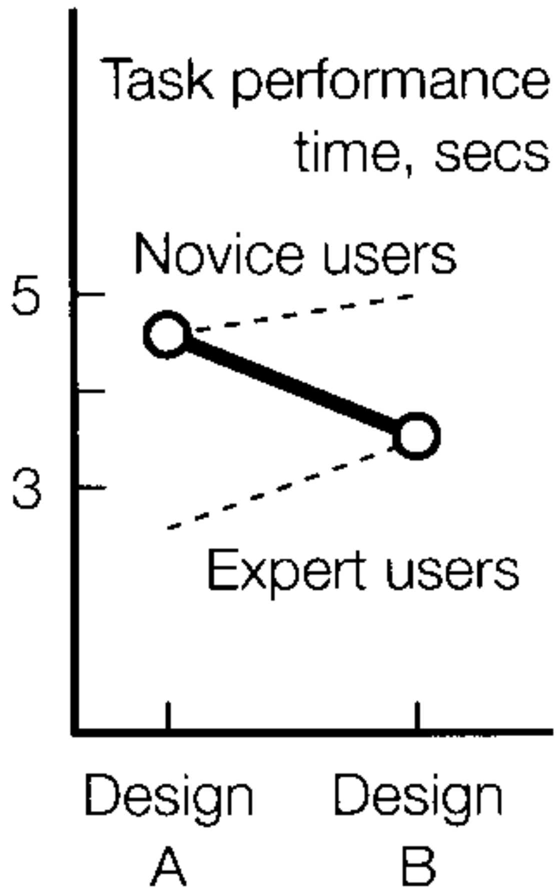
poor experiment design or results

less distinguishable results:

perhaps task was poorly chosen – or there's really no difference



poor experiment design



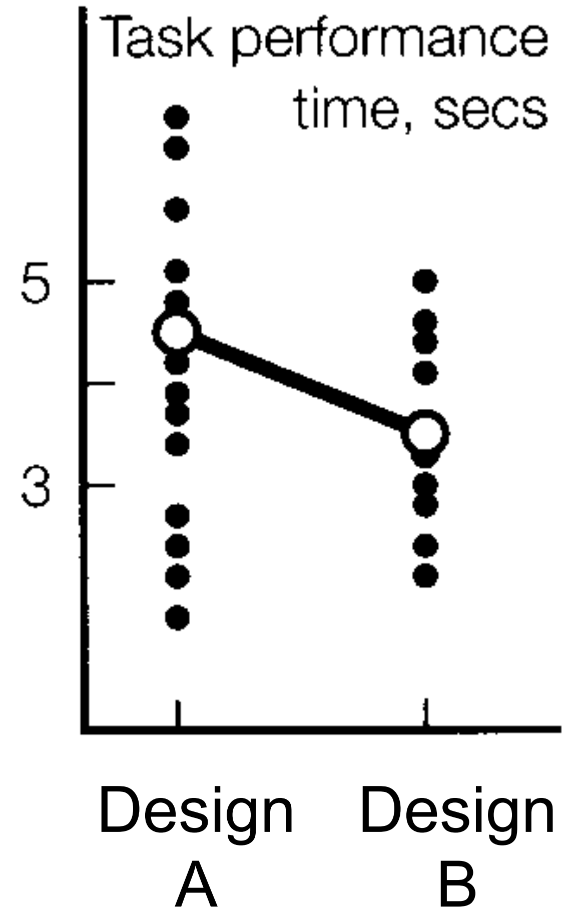
misleading results

e.g. subject assignment not controlled: one design tested on novices, other on experts, disguising actual trend

poor experiment design or results

large spread in values

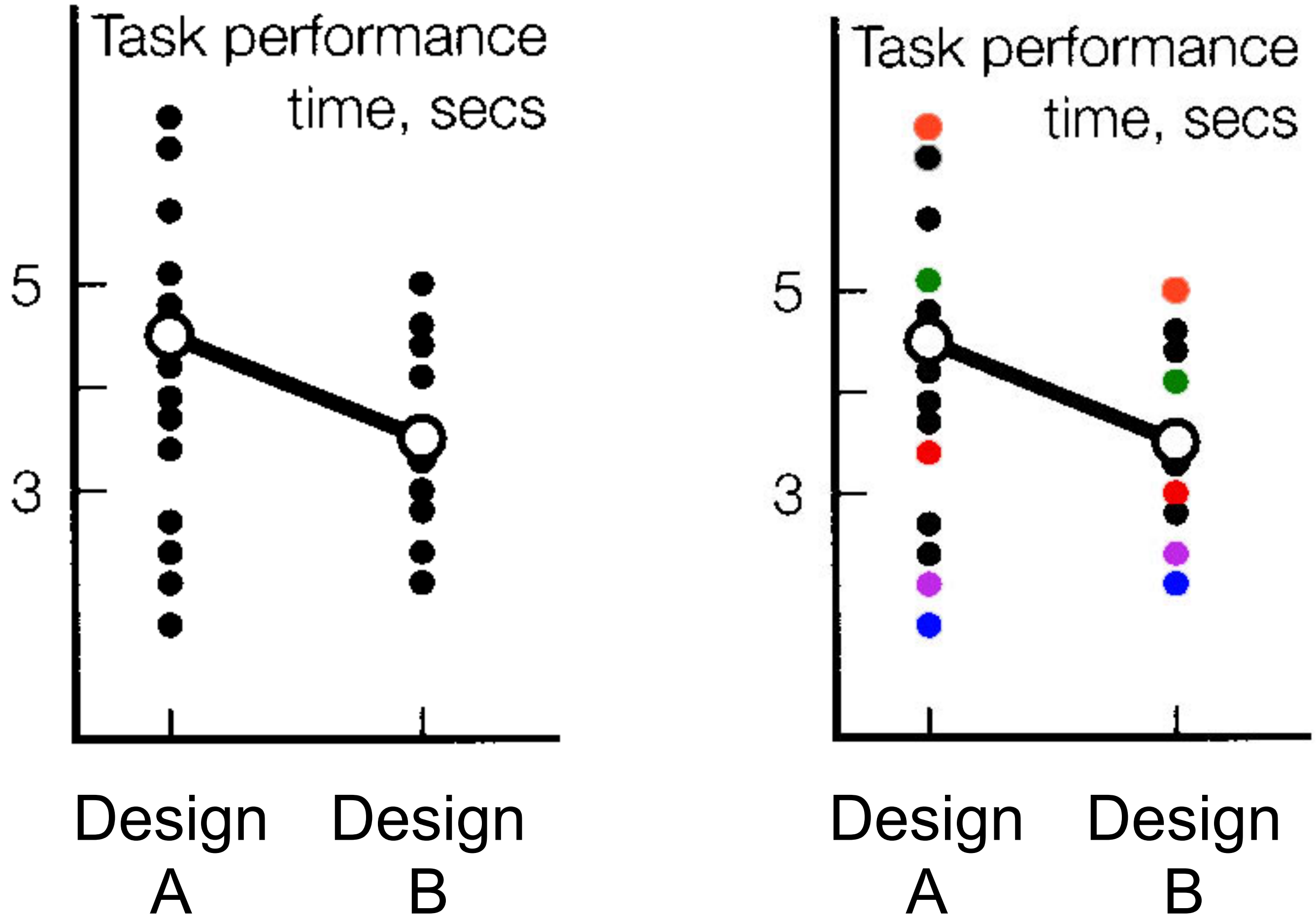
perhaps conditions were
not well controlled



as we have seen

individual (subject) differences may pose a **nuisance variable:**

variation in individual abilities can mask real differences in test conditions, if not analyzed properly



most common way to deal with:

account for each individual's performance in the two conditions (paired t-test, we'll cover more)

within/between subject comparisons

within-subject comparisons:

- **subjects exposed to multiple treatment conditions**

→ primary comparison internal to each subject

- allows control over subject variable
- greater statistical power, fewer subjects required
- not always possible (exposure to one condition might “contaminate” subject for another condition; or session too long)

between-subject comparisons:

- **subjects only exposed to one condition**

→ primary comparison is from subject to subject

- less statistical power, more subjects required
- why? because greater variability due to more individual differences

within/between subject comparisons

in toothpaste experiment

2 toothpaste types (crest, no-teeth) *between or within*

x 2 age groups (<= 12 years *or* > 12 years) *must be between*

in menu experiment :

2 menu types *(pop-up, pull down) between or within*

*x 5 menu lengths (*3, 6, 9, 12, 15*) should be within*

x 2 levels of expertise (novice, expert) *must be between* to summarize so far:

how a controlled experiment works

1. formulate an **alternate** and a **null** hypothesis:

   H_1: experimental conditions **have an effect** on performance

   H_0: experimental conditions **have no effect** on performance

2. through **experiment task**, try to demonstrate that the **null hypothesis is false** (reject it),

   for a particular level of **significance**

3. if successful, we can **accept** the alternate hypothesis,

   and state the probability ***p*** that we are wrong (the null hypothesis is true after all) → this is the result's **confidence level**

   e.g., selection speed is significantly faster in menus of length 5 than of length 10 ($p<.05$)

   → **5% chance we've made a mistake, 95% confident**

statistical analysis

what is a statistic?

- a number that describes a sample
- sample is a subset (hopefully representative) of the population we are interested in understanding

statistics are calculations that tell us

- mathematical attributes about our data sets (sample)
  - mean, amount of variance, ...
- how data sets relate to each other
  - whether we are “sampling” from the same or different populations
- the probability that our claims are correct
  - “statistical significance”

example: differences between means

given: two data sets measuring a condition

- e.g., height difference of males and females,
  time to select an item from different menu styles ...

question:

- is the difference between the means of the data statistically significant?

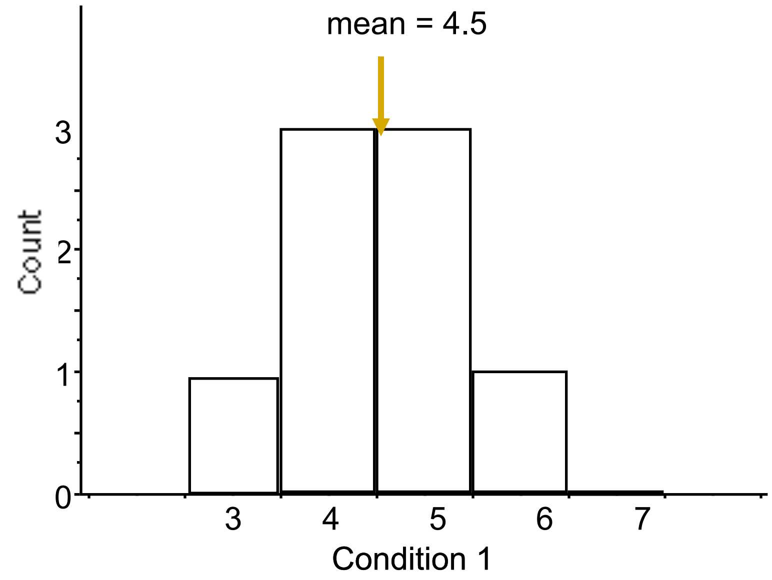
null hypothesis:

- there is no difference between the two means
- statistical analysis can only reject the hypothesis at a certain level of confidence
- *note: we never actually prove the null hypothesis true*

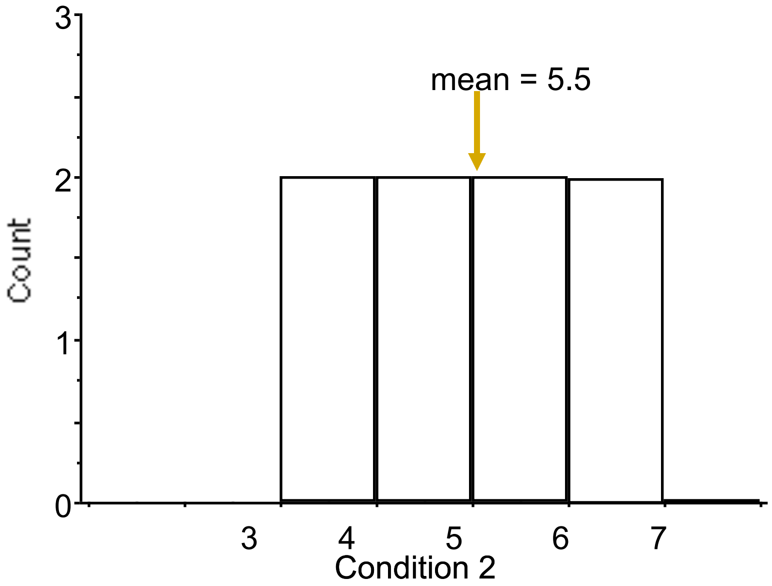
example:

Is there a *significant* difference between the means?

Condition one: 3, 4, 4, 4, 5, 5, 5, 6



Condition two: 4, 4, 5, 5, 6, 6, 7, 7

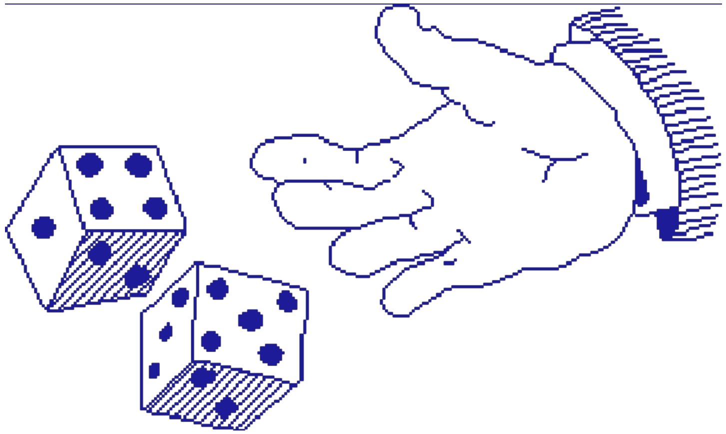


the problem with visual inspection of data

there is almost always variation in the collected data

differences between data sets may be due to:

- normal variation
  - e.g., two sets of ten tosses with different but fair dice
    - differences between data and means are accountable by expected variation
- real differences between data
  - e.g., two sets of ten tosses with loaded dice and fair dice
    - differences between data and means are not accountable by expected variation



t-test

a statistical test

allows one to say something about differences between two means at a certain confidence level

null hypothesis of the t-test:

no difference exists between the means

possible results:

- I am 95% sure that null hypothesis is rejected
  - there is probably a true difference between the means
- I cannot reject the null hypothesis
  - the means are likely the same

different types of t-tests

comparing two sets of independent observations ***(between subjects)***

usually different subjects in each group (number may differ as well)

Condition 1	Condition 2
S1–S20	S21–S43

paired observations ***(within subjects)***

usually single group studied under separate experimental conditions

data points of one subject are treated as a pair

Condition 1	Condition 2
S1–S20	S1–S20

different types of t-tests

non-directional vs directional alternatives

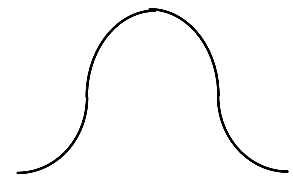
non-directional (two-tailed)

- no expectation that the direction of difference matters

directional (one-tailed)

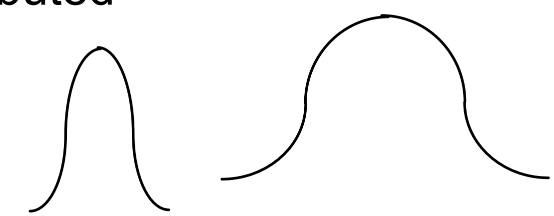
- only interested if the mean of a given condition is greater than the other

t-tests



Assumptions of t-tests

- data points of each sample are normally distributed
  - but t-test very robust in practice



- sample variances are equal
  - t-test reasonably robust for differing variances
  - deserves consideration

- individual observations of data points in sample are independent
  - must be adhered to *(can you think of examples where they are not?)*

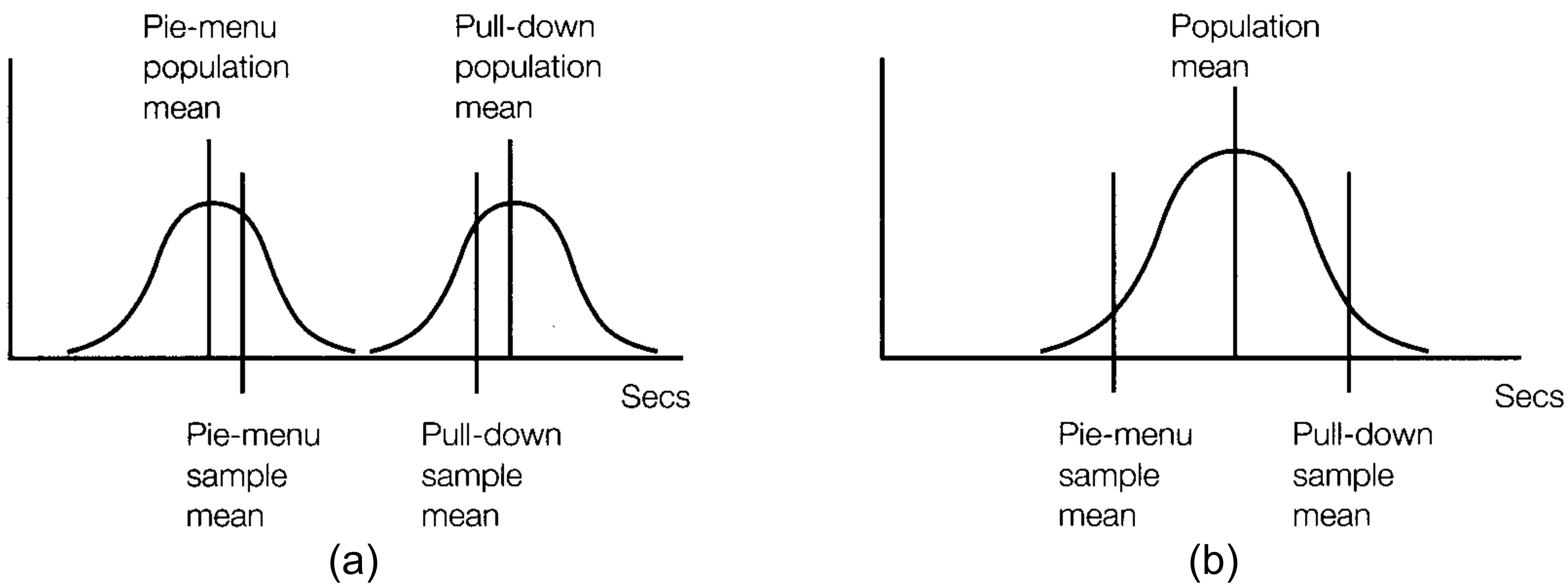
Significance level

- decide upon the level before you do the test!
- typically stated at the .05 or .01 level
- .10 can be considered a trend, but is controversial

what the t-test is testing

(a) the two samples come from two different populations;

(b) the two samples are part of the same population.



Which represents H_0 and which represents H_1?

two-tailed unpaired t-test

n: number of data points in the one sample ($N = n_1 + n_2$)

ΣX: sum of all data points in one sample

$\overline{X}$: mean of data points in sample

$\Sigma(X^2)$: sum of squares of data points in sample

s^2: unbiased estimate of population variation

t: t ratio

df = degrees of freedom = $n_1 + n_2 - 2$

N&L shows derivation of formula

How to maximize t?

Formulas

$$s^2 = \frac{\Sigma(X_1^2) - \frac{(\Sigma X_1)^2}{n_1} + \Sigma(X_2^2) - \frac{(\Sigma X_2)^2}{n_2}}{n_1 + n_2 - 2}$$

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\frac{s^2}{n_1} + \frac{s^2}{n_2}}}$$

<N&L derivation>

mean & sum of squares

mean $= \bar{X} = \dfrac{\sum X_i}{N}$

sum of squares $= SS = \sum (X_i - \bar{X})^2$

(same, faster) $= \sum X_i^2 - \dfrac{(\sum X_i)^2}{N}$

error in N&L pg. 231

degrees of freedom (df)

freedom of a set of values to vary independently of one another:

$$X = \{21, 20, 24\} \qquad \text{N=3}$$

$$\bar{X} = \frac{65}{3} = 21.6: \quad \leftarrow \bar{X} \text{ has N-1=2 df}$$

once you know the mean of N values, only N-1 can vary independently

sample variance & standard deviation

$$\text{sample variance} = s^2 = \frac{SS}{N-1}$$

$$\text{standard deviation} = sd = \sqrt{s^2}$$

</N&L derivation>

calculating *t*

compute **combined variance** for the two samples:

$$s^2 = \frac{SS_1 + SS_2}{n_1 + n_2 - 2}$$

← *note df computation*

compute **standard error of difference**, s_{ed} :

$$s_{ed} = \sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

compute *t*:

$$t = \frac{\left|\bar{X}_1 - \bar{X}_2\right|}{s_{ed}}$$

no, you won't have to memorize the formula for exams. but you *should* know how / when to use it.

Level of significance for two-tailed test

df	*.05*	*.01*
1	12.706	63.657
2	4.303	9.925
3	3.182	5.841
4	2.776	4.604
5	2.571	4.032
6	2.447	3.707
7	2.365	3.499
8	2.306	3.355
9	2.262	3.250
10	2.228	3.169
11	2.201	3.106
12	2.179	3.055
13	2.160	3.012
14	2.145	2.977
15	2.131	2.947

df	*.05*	*.01*
16	2.120	2.921
18	2.101	2.878
20	2.086	2.845
22	2.074	2.819
24	2.064	2.797

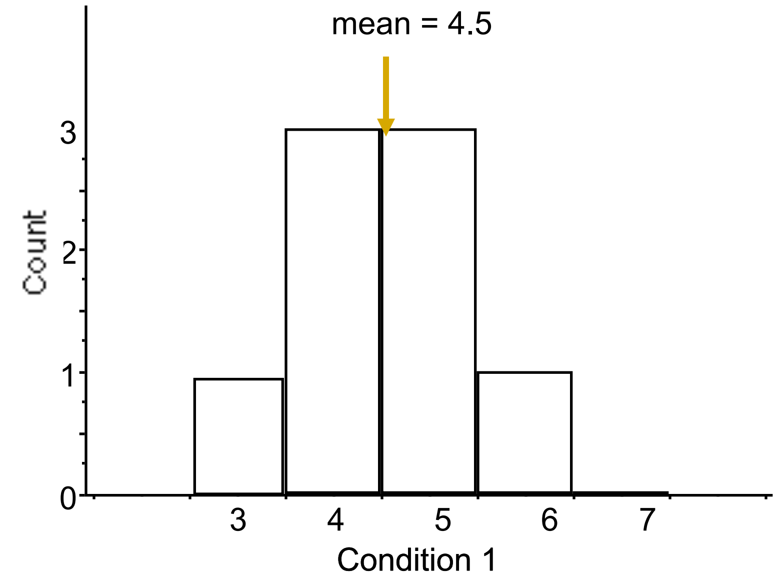
Critical value (threshold) that t statistic much reach to achieve significance.

How does critical value change based on *df* and confidence level?

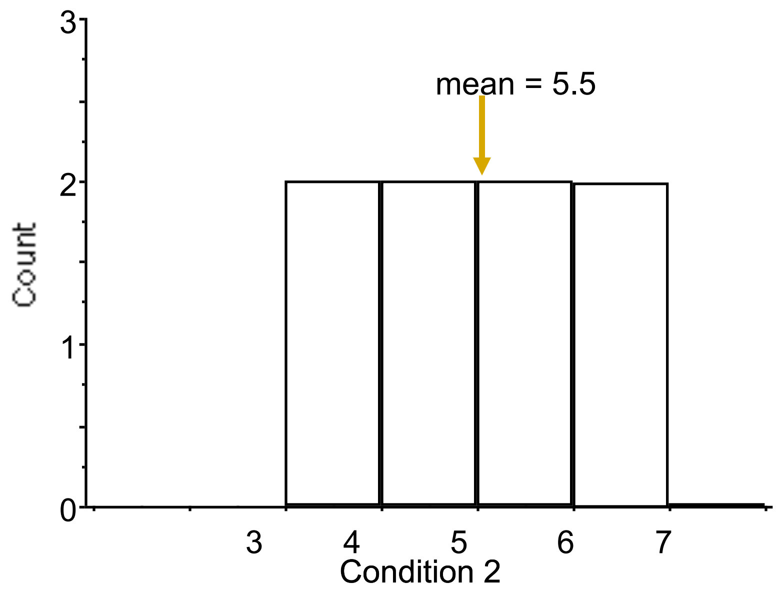
back to example:

Is there a *significant* difference between the means?

Condition one: 3, 4, 4, 4, 5, 5, 5, 6



Condition two: 4, 4, 5, 5, 6, 6, 7, 7



example calculation

x_1 = 3 4 4 4 5 5 5 6
x_2 = 4 4 5 5 6 6 7 7

hypothesis: there is no significant difference between the means at the .05 level

Step 1. Calculating s^2

	1	2
N	8	8
Σx	36	44
$\bar{x}$	4.5	5.5
$\Sigma(x^2)$	168	252
$(\Sigma x)^2$	1296	1936

df = 14

$$s^2 = \frac{\Sigma x_1^2 - (\Sigma x_1)^2/N_1 + \Sigma x_2^2 - (\Sigma x_2)^2/N}{N_1 + N_2 - 2}$$

$$= \frac{168 - 1296/8 + 252 - 1936/8}{8+8-2}$$

$$= 1.1429$$

example calculation

Step 2. Calculating *t*

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2/N_1 + s^2/N_2}}$$

$$= \frac{4.5 - 5.5}{\sqrt{2 \cdot (1.1429/8)}}$$

$$= \frac{-1}{.5345}$$

$$= -1.871$$

Step 3: Looking up critical value of *t*

- Use table for two-tailed *t*-test, at p=.05, df=14
- critical value = 2.145
- because t=1.871 < 2.145, there is no significant difference
- therefore, we cannot reject the null hypothesis
  i.e., there is no significant difference between the means

two-tailed unpaired t-test

Condition one: 3, 4, 4, 4, 5, 5, 5, 6

Condition two: 4, 4, 5, 5, 6, 6, 7, 7

What the results would look like in stats software.

Unpaired t-test

DF:	Unpaired t Value:	Prob. (2-tail):
14	-1.871	.0824 hint

probability that means are from the same underlying population

Group:	Count:	Mean:	Std. Dev.:	Std. Error:
one	8	4.5	.926	.327
two	8	5.5	1.195	.423

How does the outcome change for a confidence level of 0.10?

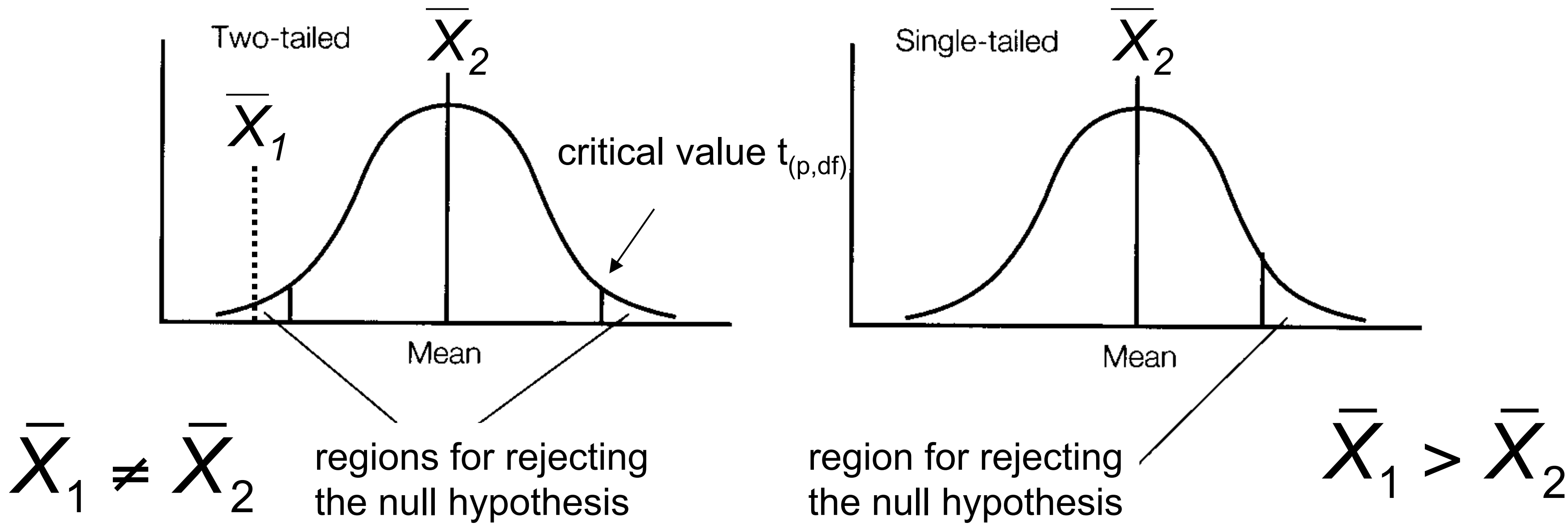
summary of the t-test

the point: establish a confidence level in the difference we've found between 2 sample means.

the process:

1. compute df
2. choose desired **significance, *p*** (aka α)
3. calculate value of the ***t* statistic**
4. compare it to the **critical value** of *t* given *p*, df: $t_{(p,df)}$
5. if $t > t_{(p,df)}$, can **reject null hypothesis at *p***

what does this look like graphically?



null hypothesis rejection area:

- two-tailed: divided equally between left/right
- single-tailed: all on one side

region(s) for rejecting the null hypothesis:

the area of the normal distribution that equals the chance you might be wrong

you now know

How to answer the following:

what is the experimental method?
what is an experimental hypothesis?
how do I plan an experiment?
why are statistics used?
within- & between-subject comparisons: how do they differ?
how do I compute a t-test?
what are the different types of t-tests?

additional slides:
material I assume you know

types of variables

samples & populations

normal distribution

variance and standard deviation

types of variables
(independent or dependent)

discrete: can take on **finite** number of levels

- e.g. a 3-color display can only render in red, green or blue;
- a design may be version A, or version B

continuous: can take any value (usually within bounds)

- e.g. a response time that may be any positive number (to resolution of measuring technology)

normal: one particular **distribution** of a continuous variable

populations and samples

statistical sample =
approximation of total possible set of, e.g.

- **people** who will ever use the system
- **tasks** these users will ever perform
- **state** users might be in when performing tasks

← the population

"**sample**" a representative fraction

- draw **randomly** from population
- if large enough and representative enough, the **sample mean** should lie somewhere near the **population mean**

confidence levels

"the **sample mean** should lie somewhere near the **population mean**"

how close?

how sure are we?

a confidence interval provides an **estimate of the probability** that the statistical measure is valid:

> "We are **95%** certain that selection from menus of five items is faster than that from menus of seven items"

how does this work?

important aspect of experiment design

establishing confidence levels:

normal distributions

fundamental premise of statistics:

predict behavior of a **population** based on a **small sample**

validity of this practice depends on the **distribution**

of the population and of the sample

many populations are **normally distributed**:

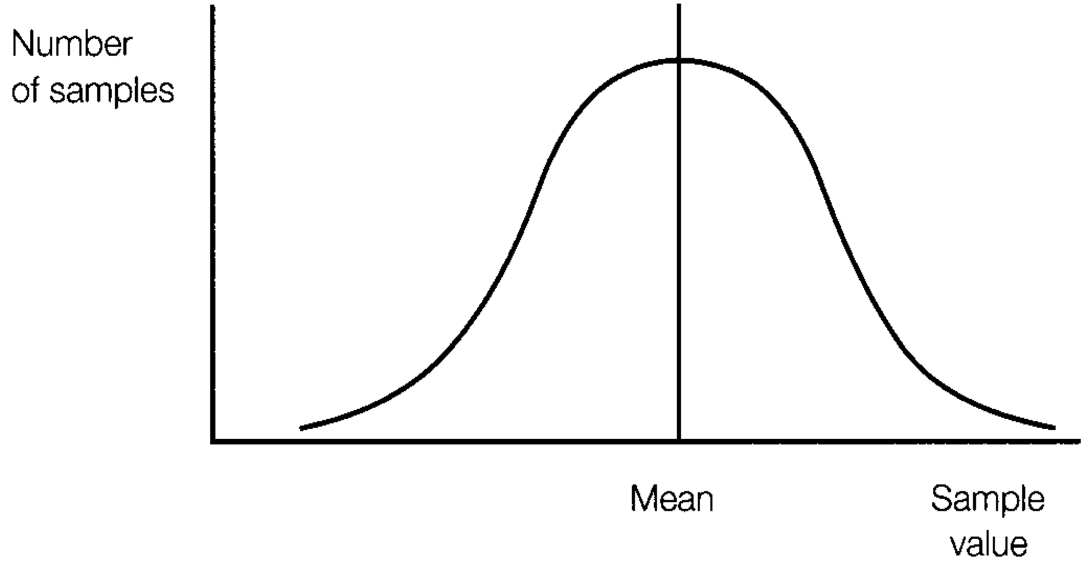
many statistical methods for **continuous dependent variables** are based on the assumption of normality

if **your sample is normally distributed**,
your **population is likely to be**,

and these statistical methods are valid,

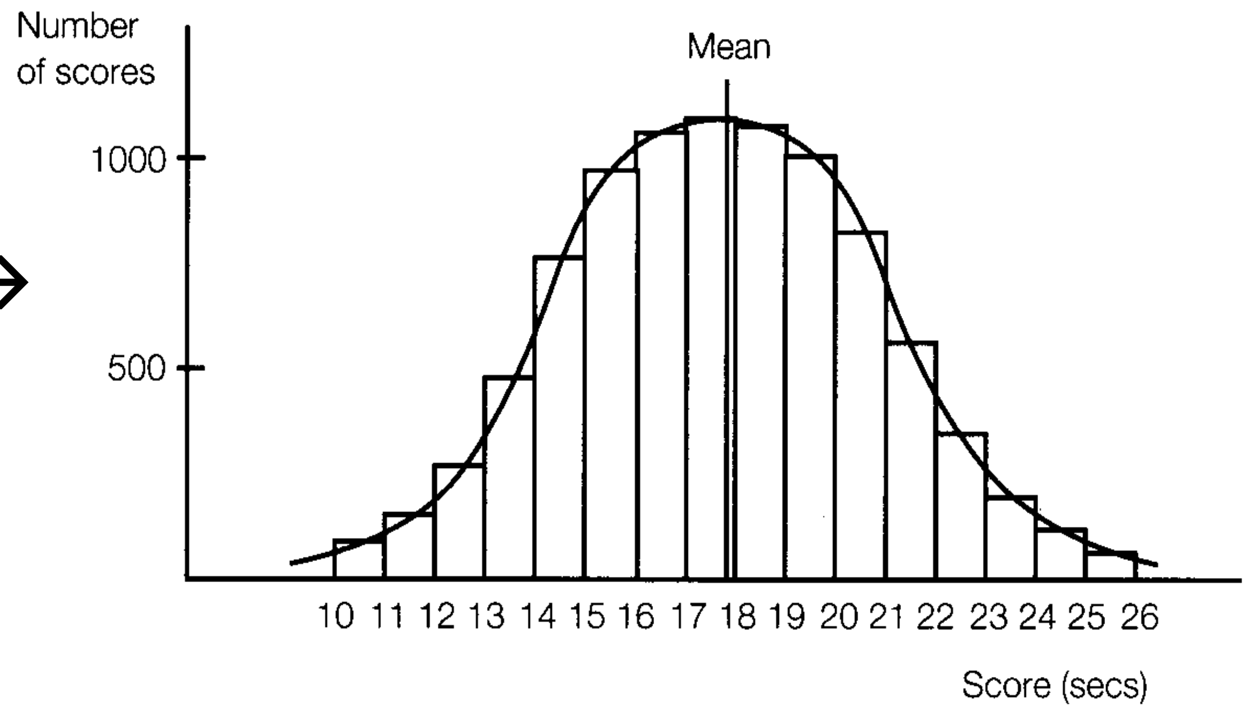
and everything is a lot easier.

population →



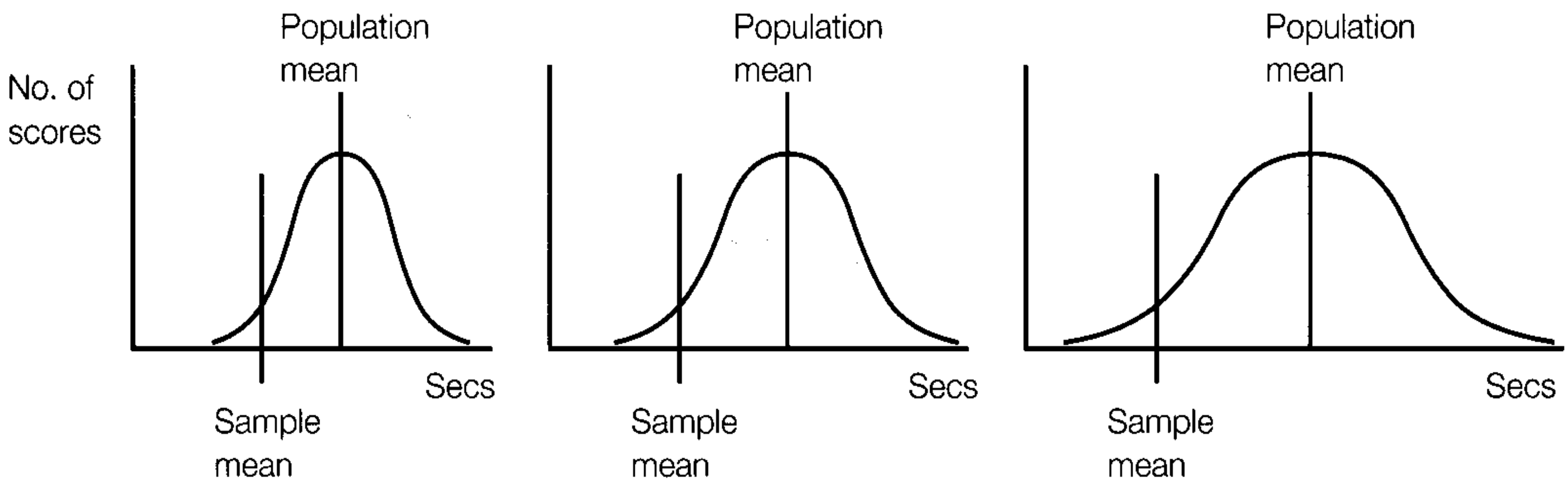
what's a normal distribution?

sample →



variance and standard deviation

all normal distributions are not the same:

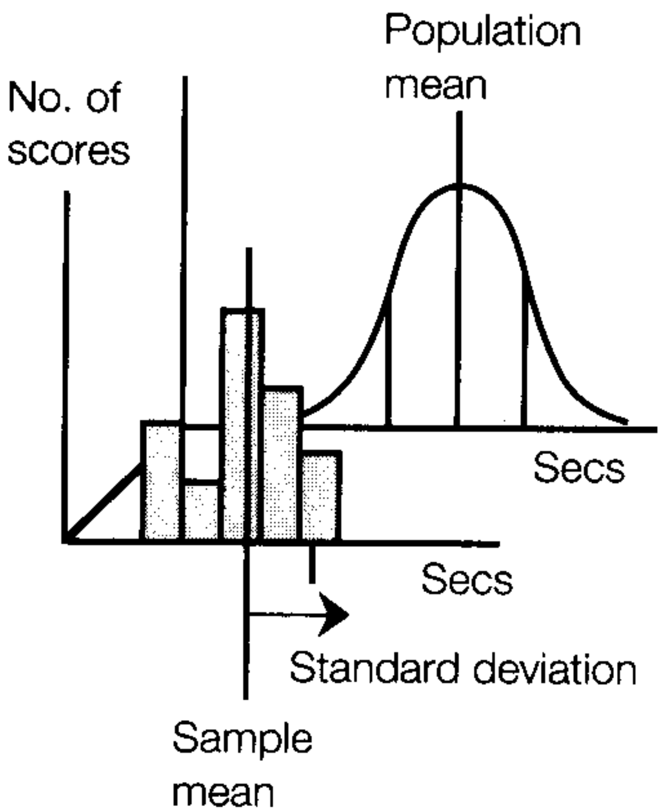


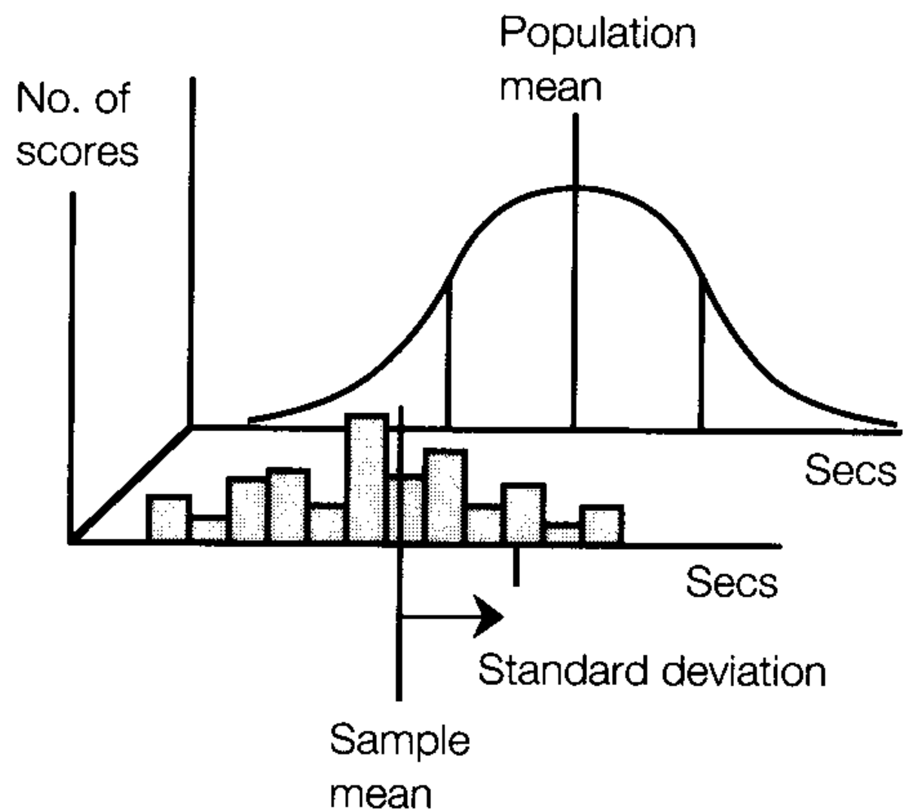
population variance is a measure of the distribution's "spread"

all normal population distributions still have the same shape

how do you get the population's variance?

estimate the population's (true) **variance**
from the (measured) sample's **standard deviation:**





what's the big deal?

if you know you're dealing with samples from a normal distribution,

and you have a good estimate of its variance
(i.e. your sample's std dev)

then, you know the **probability** that a given sample came from that population (vs. a different one).