Practice Final Exam

You should try all questions on the exam - write down your best ideas even if you don’t have a full solution.

You may use your class notes, lecture slides and handouts, and refer to the texts we used in the class: Kleinberg and Tardos, Erickson, Gusfield. But you should not try to find the answers to the questions from any other sources, e.g., on the internet. You’ll return the exam within 48 hours of the time you receive it.

1. Consider a pattern that can contain any number of wildcard symbols “=”, each of which can match the same arbitrary single character in some text. For example, pattern =HOC=SPOC=S appears in the texts WHUHOCUSPOCUSOT and ABRAHOCASPOCASCADABRA, but not in the text FRISHOCUSPOCESTIX. Design an algorithm that, given such a pattern and a text, finds the pattern in a text if it appears there. Your algorithm should run in time linear in the length of the pattern and the length of the text.

2. An instance of Min Cycle Cover is a complete undirected graph \( G = (V, E) \). A positive weight \( w(i, j) \) is associated with each edge \( \{i, j\} \) of \( E \). Your goal is to find a set \( S \) of edges of minimum cost so that every node is contained in exactly two edges, i.e., \( S \) forms a set of cycles that covers all nodes.

   (a) Formulate the problem as an integer linear program.

   (b) Propose a randomized rounding strategy that returns a feasible solution for the integer program. What can you say about its approximation ratio?

3. You are given a set of \( n \) boxes where box \( i \) has height \( h_i \), width \( w_i \), and depth \( d_i \). Box \( i \) may be placed on box \( j \) only if \( w_i < w_j \) and \( d_i < d_j \). Describe an efficient algorithm to find a stack of boxes that is as tall as possible. What is the running time of your solution?

4. A group of \( n \) people show up for the annual dog sale at the dog pound. There are \( d \) dogs for sale; each costs $10. Every person \( i \) tells the organizers the maximum number of dogs they can buy, \( b_i \), and which dogs they would be willing to buy, \( D_i \). For example, person 5 might be willing to take at most two (\( b_5 = 2 \)) of the dogs in the set \( D_5 = \{3, 5, 12, 14, 18\} \). (The dogs are identified by number.) Describe an efficient algorithm that the organizers can use to assign dogs to people so that the number of dogs sold is maximized.