422 big picture: Where are we?

**Deterministic**
- Logics
  - First Order Logics
- Ontologies
  - Full Resolution
  - SAT

**Stochastic**
- Belief Nets
  - Approx. : Gibbs
- Markov Chains and HMMs
  - Forward, Viterbi….
  - Approx. : Particle Filtering
- Undirected Graphical Models
  - Markov Networks
  - Conditional Random Fields
- Markov Decision Processes and Partially Observable MDP
  - Value Iteration
  - Approx. Inference

**StarAI** (statistical relational AI)
- Hybrid: Det + Sto
- Prob CFG
- Prob Relational Models
- Markov Logics

**Applications of AI**

**Representation**
- Reasoning Technique
Combining Symbolic and Probabilistic R&R systems

(a) Probabilistic Context-Free Grammars
   • Weights are conditional prob. on rewriting rules
   • Applications: NLP parsing & Hierarchical Planning

(b) Markov Logics: weighted FOL
   \[ P(\text{world}) \propto \exp\left(\sum \text{weights of formulas it satisfies}\right) \]

(c) Probabilistic Relational models
   • Probs specified on relations
A customer $C_1$ will / will not recommend a book $B_1$ depending on the book quality, and the customer honesty and kindness.

When you have two customers and two books.....
Lecture Overview

- Motivation and Representation
- Semantics of Probabilistic Relational Models (PRMs)
  - Classes and Relations
  - Attributes and Reference Slots
  - Full Relational Schema and its Instances
  - Fixed vs. Probabilistic Attributes
  - Relational Skeleton and its Completion Instance
  - Inverse Slot and Slot chain
Motivation for PRMs

• Most real-world data are stored in relational DBMS

• Combine advantages of relational logic & Bayesian networks:
  - natural domain modeling: objects, properties, relations;
  - generalization over a variety of situations;
  - compact, natural probability models.

• Integrate uncertainty with relational model:
  - properties of domain entities can depend on properties of related entities;
  - uncertainty over relational structure of domain.
Limitations of Bayesian Networks

A Bayesian networks (BNs) represents a pre-specified set of attributes/variables whose relationship to each other is fixed in advance.
How PRMs extend BNs?

1. PRMs conceptually extend BNs to allow the specification of a probability model for classes of objects rather than a fixed set of simple attributes.

2. PRMs also allow properties of an entity to depend probabilistically on properties of other related entities.
Lecture Overview

• Motivation and Representation
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Mapping PRMs from Relational Models

- The representation of PRMs is a direct mapping from that of relational databases

- A relational model consists of a set of classes $X_1, \ldots, X_n$ and a set of relations $R_1, \ldots, R_m$, where each relation $R_i$ is typed
University Domain Example - Classes and relations

Indicates many-to-many relationship

Indicates one-to-many relationship

Professor

Student

Course

Registration

CPSC 422, Lecture 33
Mapping PRMs from Relational Models: attributes

• Each class or entity type (corresponding to a single relational table) is associated with a set of attributes $\mathcal{A}(X_i)$ (at least one of which is a primary key)

```
Course
  Name
    Rating
    Difficulty
```
Mapping PRMs from Relational Models: reference slot

- Each class or entity type is also associated with a set of *reference slots* \( R(X) \)
- correspond to attributes that are *foreign keys* (key attributes of another table)
- \( X.\rho \), is used to denote reference slot \( \rho \) of \( X \).

Course, Instructor

<table>
<thead>
<tr>
<th>Course</th>
<th>Professor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Name</td>
</tr>
<tr>
<td>Instructor</td>
<td>Popularity</td>
</tr>
<tr>
<td>Rating</td>
<td>Teaching-Ability</td>
</tr>
<tr>
<td>Difficulty</td>
<td></td>
</tr>
</tbody>
</table>
University Domain Example – Full Relational Schema

- **Primary keys are indicated by a blue rectangle.**
- **Indicates many-to-many relationship.**
- **Dashed lines indicate the types of objects referenced.**
- **Underlined attributes are reference slots of the class.**

Diagram:

- **Professor**
  - Name
  - Popularity
  - Teaching-Ability

- **Student**
  - Name
  - Intelligence
  - Ranking

- **Course**
  - Name
  - Instructor
  - Rating
  - Difficulty

- **Registration**
  - RegID
  - Course
  - Student
  - Grade
  - Satisfaction

Primary keys are indicated by a blue rectangle. Dashed lines indicate the types of objects referenced. Underlined attributes are reference slots of the class.
Book Recommendation Domain - Full Relational Schema

Book
- Title
- Quality

Recommendation
- Name
- Book
- Customer
- Rating

Customer
- Name
- Honesty
- Kindness

R₁ \((1:M)\), R₂ \((M:M)\)

C. \(R₁(1:M) \Rightarrow R₂(1:M)\)

D. \(R₁(M:M) \Rightarrow R₂(M:M)\)
Book Recommendation Domain -
Full Relational Schema

Book
Title
Quality

Recommendation
Name
Book
Customer
Rating

Customer
Name
Honesty
Kindness

CPSC 422, Lecture 33
PRM Semantics: Attribute values

- Each attribute $A_j \in \mathcal{A}(X_i)$ takes on values in some fixed domain of possible values denoted $V(A_j)$. We assume that value spaces are finite.
- Attribute $A$ of class $X$ is denoted $X.A$
- E.g., $V(\text{Student.Intelligence})$ might be \{ high, low \}
PRM Semantics: Instance of Schema

- An *instance* $I$ of a schema/model specifies a set of objects $x$, partitioned into classes; such that there is
  - a value for each attribute $x.A$
  - and a value for each reference slot $x.\rho$
University Domain Example - An Instance of the Schema

One professor is the instructor for both courses.

Jane Doe is registered for only one course, Phil101, while the other student is registered for both courses.
There are two professors instructing a course

There are three students in the Phil201 course

University Domain Example – Another Instance of the Schema

- **Professor**
  - Name: Prof. Vincent
  - Popularity: high
  - Teaching Ability: medium

- **Course**
  - Name: Phil201
  - Difficulty: low
  - Rating: high

- **Registration**
  - RegID: #5639
  - Grade: A
  - Satisfaction: 3

- **Student**
  - Name: Jane Doe
  - Intelligence: high
  - Ranking: average

- **Registration**
  - RegID: #5723
  - Grade: A
  - Satisfaction: 3

- **Student**
  - Name: John Doe
  - Intelligence: high
  - Ranking: average

- **Professor**
  - Name: Prof. Vincent
  - Popularity: high
  - Teaching Ability: high

- **Course**
  - Name: Phil201
  - Difficulty: low
  - Rating: high

- **Registration**
  - RegID: #5723
  - Grade: A
  - Satisfaction: 3

- **Student**
  - Name: John Doe
  - Intelligence: high
  - Ranking: average

- **Professor**
  - Name: Prof. Vincent
  - Popularity: high
  - Teaching Ability: high

- **Course**
  - Name: Phil201
  - Difficulty: low
  - Rating: high

- **Registration**
  - RegID: #5723
  - Grade: A
  - Satisfaction: 3

- **Student**
  - Name: John Doe
  - Intelligence: high
  - Ranking: average
PRM Semantics: fixed vs. prob. attributes

• Some attributes, such as Name or Social Security Number, are fully determined. Such attributes are labeled as fixed. Assume that they are known in any instantiation of the schema.

• The other attributes are called probabilistic. We may be uncertain about their value.
University Domain Example - fixed vs. probabilistic attributes

Professor
Name
Popularity
Teaching-Ability

Student
Name
Intelligence
Ranking

Course
Name
Instructor
Rating
Difficulty

Registration
RegID
Course
Student
Grade
Satisfaction

Which ones are fixed? Which are probabilistic?
University Domain Example - fixed vs. probabilistic attributes

Fixed attributes are shown in regular font

Probabilistic attributes are shown in italic regular font
PRM Semantics: Skeleton Structure

• A *skeleton structure* $\sigma$ of a relational schema is a partial specification of an instance of the schema. It specifies
  - set of objects for each class,
  - values of the fixed attributes of these objects,
  - relations that hold between the objects
• The values of probabilistic attributes are left unspecified

• A *completion* $I$ of the skeleton structure $\sigma$ extends the skeleton by also specifying the values of the probabilistic attributes
University Domain Example – Relational Skeleton

Professor
Name
Prof. Gump
Popularity
???
Teaching-Ability
???

Course
Name
Phil101
Difficulty
???
Rating
???

Registration
RegID
#5639
Grade
???
Satisfaction
???

Student
Name
Jane Doe
Intelligence
???
Ranking
???

Registration
RegID
#5639
Grade
???
Satisfaction
???

Registration
RegID
#5639
Grade
???
Satisfaction
???
University Domain Example - The Completion Instance I

Professor
Name Prof. Gump
Popularity high
Teaching-Ability medium

Student
Name Jane Doe
Intelligence high
Ranking average

Course
Name Phil101
Difficulty low
Rating high

Registration
Name #5639
Grade A
Satisfaction 3

University Domain Example – The Completion Instance I

CPSC 422, Lecture 33
University Domain Example - Another Relational Skeleton

PRMs allow multiple possible skeletons

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University Domain Example - The Completion Instance I

PRMs also allow multiple possible instances and values
PRM Semantics: inverse slot

• For each reference slot $\rho$, we define an inverse slot, $\rho^{-1}$, which is the inverse function of $\rho$.
A *slot chain* \( \tau = \rho_1 \ldots \rho_m \) is a sequence of reference slots that defines functions from objects to other objects to which they are indirectly related.

Student.\(\text{Registered-In}\).Course.\(\text{Instructor}\) can be used to denote......a student's set of instructors
Slot chains will allow us...

To specify probabilistic dependencies between attributes of related entities
Learning Goals for today’s class

You can:

• Explain the need for Probabilistic relational model
• Explain how PRMs generalize BNs
• Define a Full Relational Schema and its instances
• Define a Relational Skeleton and its completion Instances
• Define an inverse slot and an slot chain
Next class on Wed

Finish Probabilistic Relational Models

- Probabilistic Model
- Dependency Structure
- Aggregation
- Parameters
- Class dependency Graph
- Inference

Keep working on Assignment-4
Due Dec 1