Intelligent Systems (AI-2)

Computer Science cpsc422, Lecture 29

Nov, 20, 2017

Slide source: from Pedro Domingos UW

422 big picture: Where are we?

StarAI (statistical relational AI)

Hybrid: Det +Sto

Prob CFG

Prob Relational Models

Markov Logics

	Deterministic	Stochastic
Query	*Logics* *First Order Logics* *Ontologies* • Full Resolution • SAT	*Belief Nets* Approx. : Gibbs *Markov Chains and HMMs* Forward, Viterbi…. Approx. : Particle Filtering *Undirected Graphical Models* *Markov Networks* *Conditional Random Fields*
Planning		*Markov Decision Processes and Partially Observable MDP* • Value Iteration • Approx. Inference *Reinforcement Learning*

Applications of AI

Representation

Reasoning Technique

Lecture Overview

- Statistical Relational Models (*for us aka Hybrid*)
- Recap Markov Networks and log-linear models
- Markov Logic

Statistical Relational Models

Goals:

- Combine **(subsets of) logic** and **probability** into a single language (**R&R** system)
- Develop efficient **inference** algorithms
- Develop efficient **learning** algorithms
- Apply to real-world problems

L. Getoor & B. Taskar (eds.), *Introduction to Statistical Relational Learning,* MIT Press, 2007.

Plethora of Approaches

- Knowledge-based model construction [Wellman et al., 1992]
- Stochastic logic programs [Muggleton, 1996]
- Probabilistic relational models [Friedman et al., 1999]
- Relational Markov networks [Taskar et al., 2002]
- Bayesian logic [Milch et al., 2005]
- Markov logic [Richardson & Domingos, 2006]
- *And many others….!*

Prob. Rel. Models vs. Markov Logic

PRM

- Relational Skeleton
- Dependency Graph
- Parameters (CPT)

$\Rightarrow$ BNET

ML

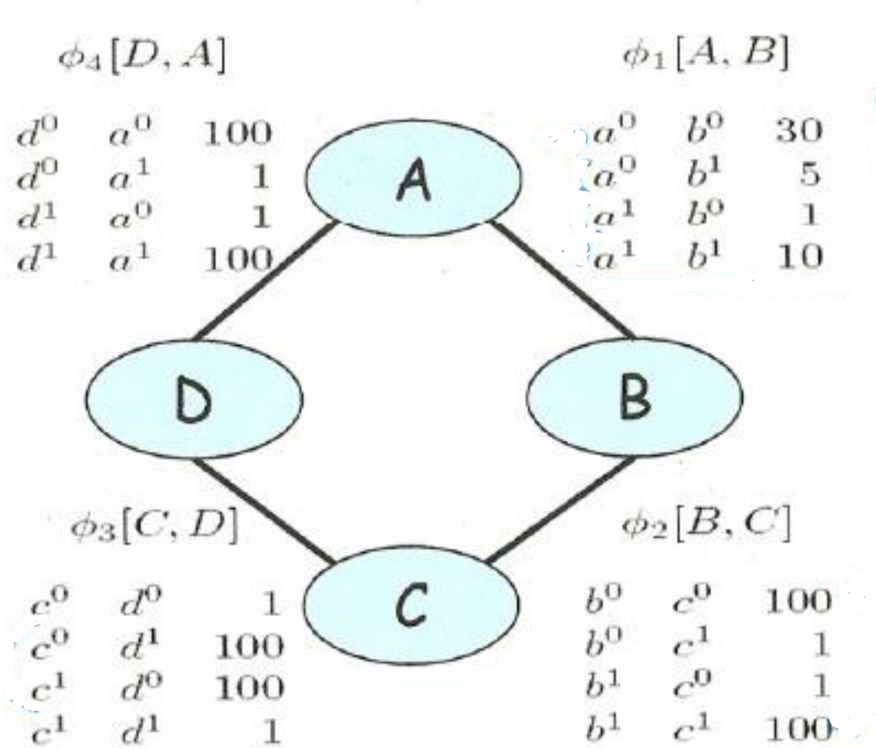
- weighted logical formulas
- set of constants

$\Rightarrow$ MARKOV LOGIC NETWORK

Lecture Overview

- Statistical Relational Models (*for us aka Hybrid*)
- **Recap Markov Networks and log-linear models**
- Markov Logic
  - Markov Logic Network (MLN)

Parameterization of Markov Networks



X set of random vars: A factor is

$\Phi(Val(X)) \to \mathbb{R}$

Factors define the local interactions (like CPTs in Bnets)

What about the global model? What do you do with Bnets?

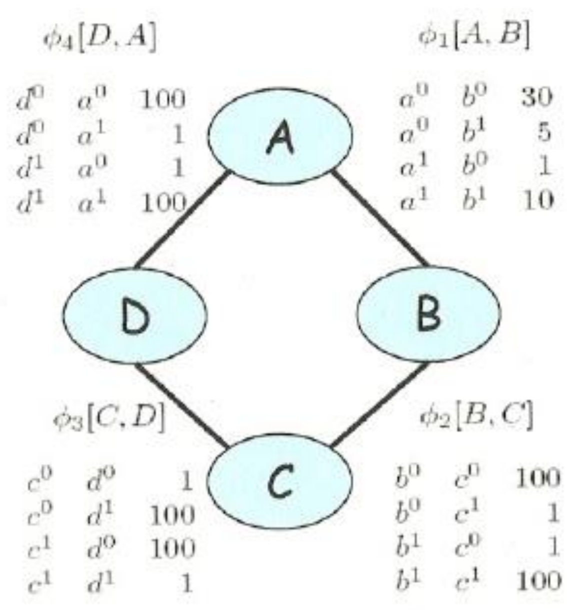
How do we combine local models?

As in BNets by multiplying them!

$$\tilde{P}(A,B,C,D) = \phi_1(A,B) \times \phi_2(B,C) \times \phi_3(C,D) \times \phi_4(A,D)$$

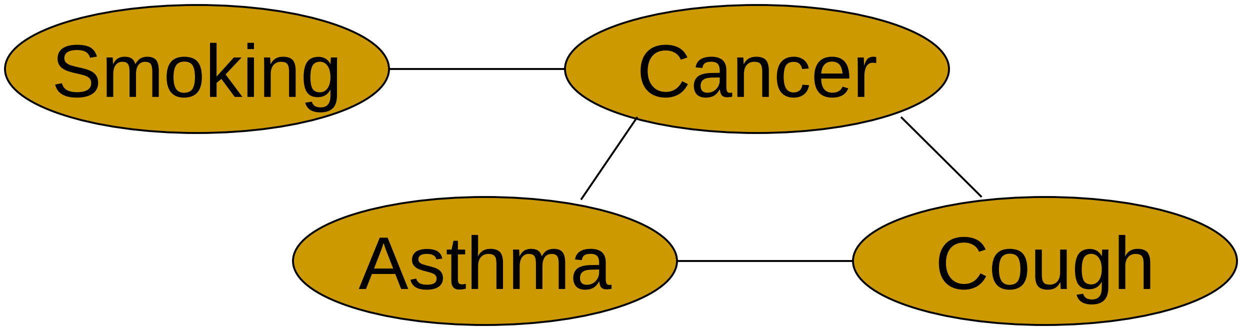
$$P(A,B,C,D) = \frac{1}{Z}\tilde{P}(A,B,C,D)$$

Assignment				Unnormalized	Normalized
a^0	b^0	c^0	d^0	300000	.04
a^0	b^0	c^0	d^1	300000	.04
a^0	b^0	c^1	d^0	300000	.04
a^0	b^0	c^1	d^1	30	4.1×10^{-6}
a^0	b^1	c^0	d^0	500	.
a^0	b^1	c^0	d^1	500	.
a^0	b^1	c^1	d^0	5000000	.69
a^0	b^1	c^1	d^1	500	.
a^1	b^0	c^0	d^0	100	.
a^1	b^0	c^0	d^1	1000000	.
a^1	b^0	c^1	d^0	100	.
a^1	b^0	c^1	d^1	100	.
a^1	b^1	c^0	d^0	10	.
a^1	b^1	c^0	d^1	100000	.
a^1	b^1	c^1	d^0	100000	.
a^1	b^1	c^1	d^1	100000	.



Markov Networks

- **Undirected** graphical models



- Factors/Potential-functions defined over cliques

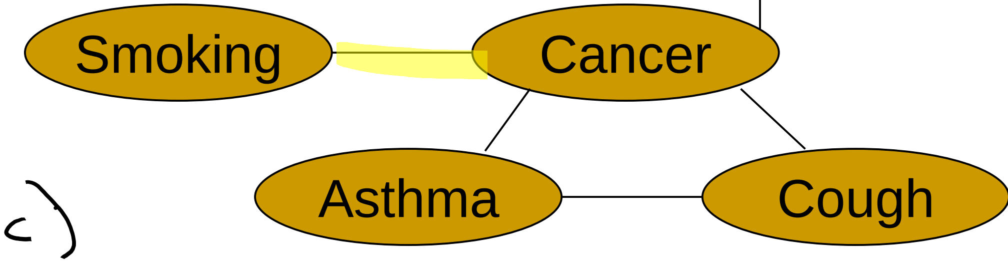
$$P(x) = \frac{1}{Z} \prod_c \Phi_c(x_c)$$

$$Z = \sum_x \prod_c \Phi_c(x_c)$$

Smoking	Cancer	Φ(S,C)
F	F	4.5
F	T	4.5
T	F	2.7
T	T	4.5

Markov Networks :log-linear model

$$P(x) = \frac{1}{Z}\prod_{c}\Phi_c(x_c)$$



- Log-linear model:

each $\Phi_c(x_c) = e^{w_c f_c(x_c)}$

$$w_1 = 0.51$$

$$f_1(\text{Smoking, Cancer}) = \begin{cases} 1 & \text{if } \neg \text{Smoking} \vee \text{Cancer} \\ 0 & \text{otherwise} \end{cases}$$

Smoking	Cancer	
1	1	$e^{.51}$
1	0	e^{0}
0	1	$e^{.51}$
0	0	$e^{.51}$

$$P(x) = \frac{1}{Z}\exp\left(\sum_i \underbrace{w_i}_{\text{Weight of Feature } i}\ \underbrace{f_i(x_i)}_{\text{Feature } i}\right)$$

Lecture Overview

- Statistical Relational Models (for us aka Hybrid)
- Recap Markov Networks
- **Markov Logic**

Markov Logic: Intuition(1)

- A logical KB is a set of **hard constraints** on the set of possible worlds

CONSTANT INDIVIDUALS = {a, b}

f $\boxed{\forall x\ Smokes(x) \Rightarrow Cancer(x)}$

$$\left.\begin{array}{l} Smokes(a) = T \\ Cancer(a) = F \\ Smokes(b) = F \\ Cancer(b) = F \end{array}\right\} \hat{w}$$

in FOL $\hat{w}$ is . . .

A. possible B. impossible

C. cannot tell

iclicker

Markov Logic: Intuition(1)

- A logical KB is a set of **hard constraints** on the set of possible worlds

$\hat{f}$ $\boxed{\forall x\ Smokes(x) \Rightarrow Cancer(x)}$

INDIVIDUALS = {a, b}

$$\left.\begin{array}{l} Smokes(a) = T \\ Cancer(a) = F \\ Smokes(b) = F \\ Cancer(b) = F \end{array}\right\} \hat{w}$$

in FOL $\hat{w}$ is impossible

- Let's make them **soft constraints**:
  When a world violates a formula,
  the world becomes less probable, not impossible

if $\hat{f}$ is True $\quad P(\hat{w})$ decreases
False in $\hat{w}$ $\quad P(\hat{w})$ increases

Markov Logic: Intuition (2)

- The more formulas in the KB a possible world satisfies the more it should be likely
- Give each formula a **weight**
- Adopting a **log-linear model**, by design, if a possible world satisfies a formula its **probability** should go up proportionally to **exp(the formula weight)**.

$$P(\text{world}) \propto \exp\left(\sum \text{weights of formulas it satisfies}\right)$$

That is, if a possible world satisfies a formula its **log probability** should go up proportionally to the formula weight.

$$\log(P(\text{world})) \propto \left(\sum \text{weights of formulas it satisfies}\right)$$

Markov Logic: Definition

- A Markov Logic Network (MLN) is
  - a set of pairs (F, w) where
    - F is a **formula** in first-order logic
    - w is a **real number**
  - Together with a set C of **constants**,
- It defines a **Markov network** with
  - One *binary node* for each **grounding** of each **predicate** in the MLN
  - One *feature/factor* for each **grounding** of each **formula** F in the MLN, with the corresponding weight w

Grounding: substituting vars with constants

(not required)consider Existential and functions

Table 2.2: Construction of all groundings of a first-order formula under Assumptions 2.2–2.4.

function Ground(F)
  input: F, a formula in first-order logic
  output: G_F, a set of ground formulas
for each existentially quantified subformula $\exists x\ S(x)$ in F
  $F \leftarrow F$ with $\exists x\ S(x)$ replaced by $S(c_1) \vee S(c_2) \vee \ldots \vee S(c_{|C|})$,
    where $S(c_i)$ is $S(x)$ with x replaced by c_i
$G_F \leftarrow \{F\}$
for each universally quantified variable x
  for each formula $F_j(x)$ in G_F
    $G_F \leftarrow (G_F \setminus F_j(x)) \cup \{F_j(c_1), F_j(c_2), \ldots, F_j(c_{|C|})\}$,
      where $F_j(c_i)$ is $F_j(x)$ with x replaced by c_i
for each formula $F_j \in G_F$
  repeat
    for each function $f(a_1, a_2, \ldots)$ all of whose arguments are constants
    $F_j \leftarrow F_j$ with $f(a_1, a_2, \ldots)$ replaced by c, where $c = f(a_1, a_2, \ldots)$
  until F_j contains no functions
return G_F

Example: Friends & Smokers

Smoking causes cancer.

Friends have similar smoking habits.

Example: Friends & Smokers

$$\forall x \; Smokes(x) \Rightarrow Cancer(x)$$

$$\forall x, y \; Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$$

Example: Friends & Smokers

1.5	$\forall x \; Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y \; Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Example: Friends & Smokers

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Two constants: **Anna** (A) and **Bob** (B)

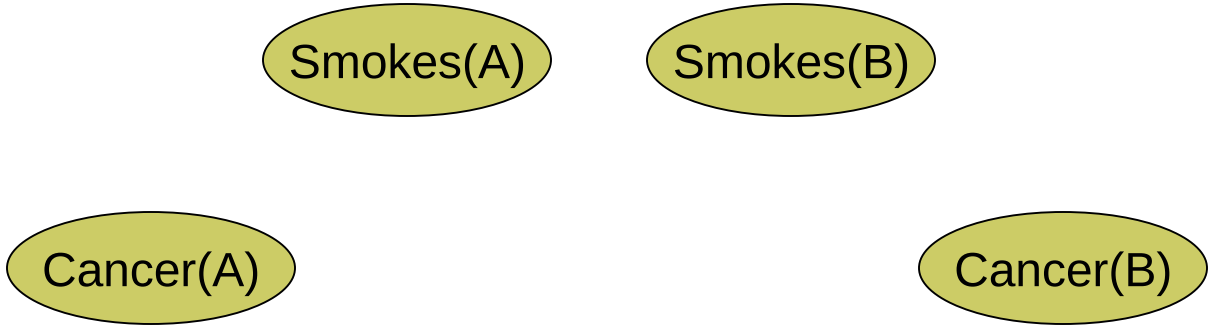
MLN nodes

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Two constants: **Anna** (A) and **Bob** (B)

- One *binary node* for each grounding of each predicate in the MLN

Grounding: substituting vars with constants



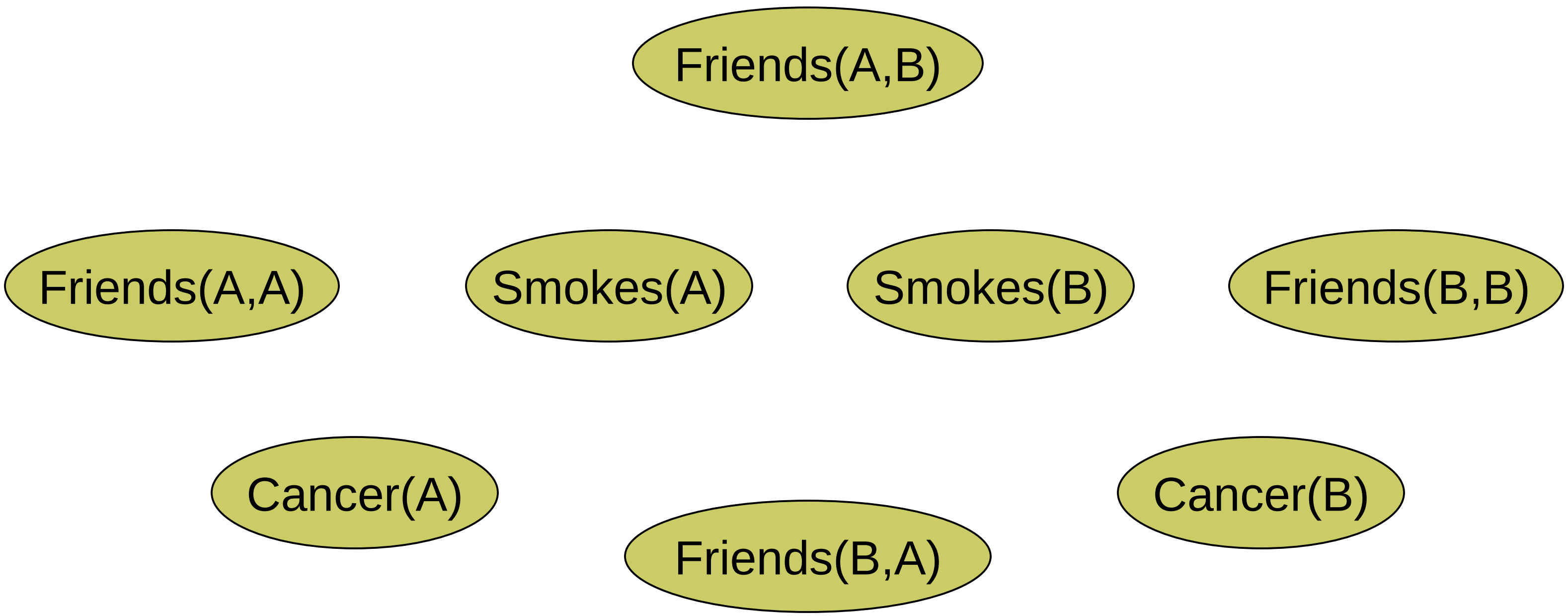
- Any nodes missing?

MLN nodes (complete)

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \big(Smokes(x) \Leftrightarrow Smokes(y)\big)$

Two constants: **Anna** (A) and **Bob** (B)

- One *binary node* for each grounding of each predicate in the MLN



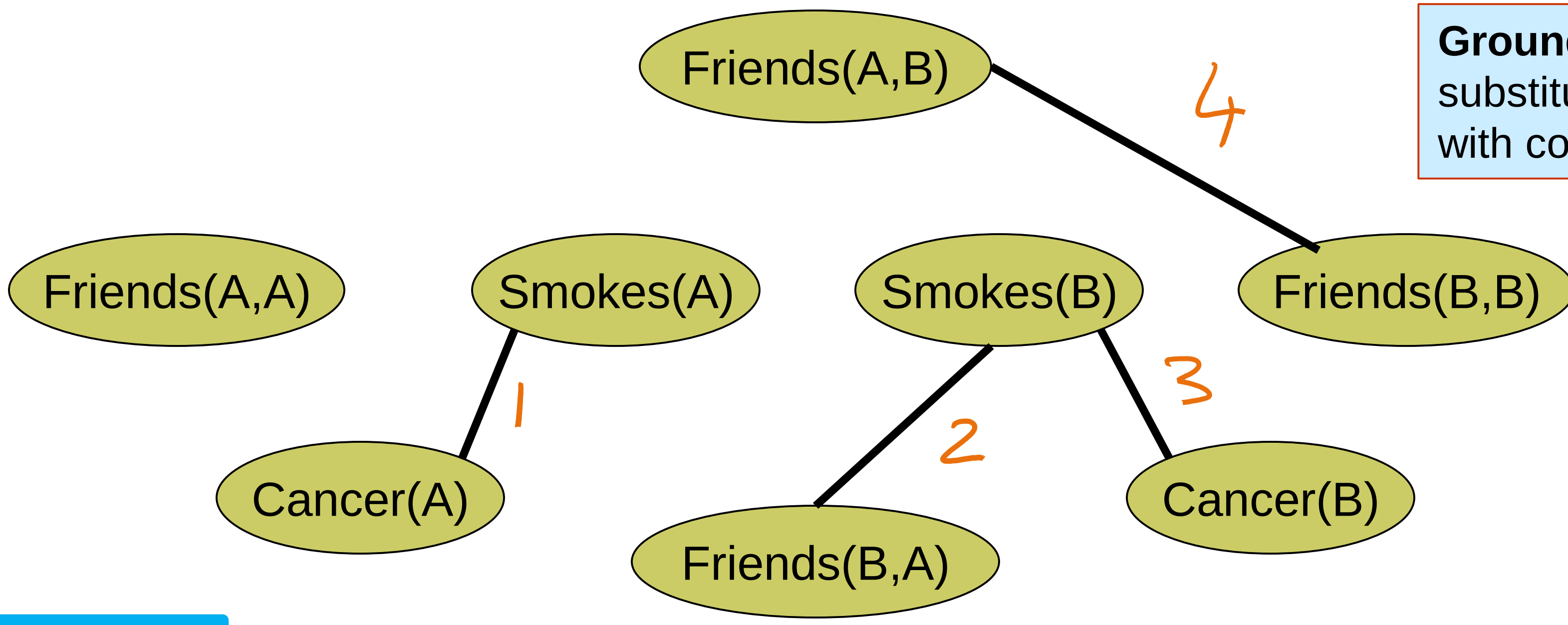
MLN features

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x,y) \Rightarrow \big(Smokes(x) \Leftrightarrow Smokes(y)\big)$

Two constants: **Anna** (A) and **Bob** (B)

Edge between two nodes iff the corresponding ground predicates appear together in at least one grounding of one formula

Grounding: substituting vars with constants



Which edge should not be there?

A. 1 B. 2 C. 3 D. 4

MLN features

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

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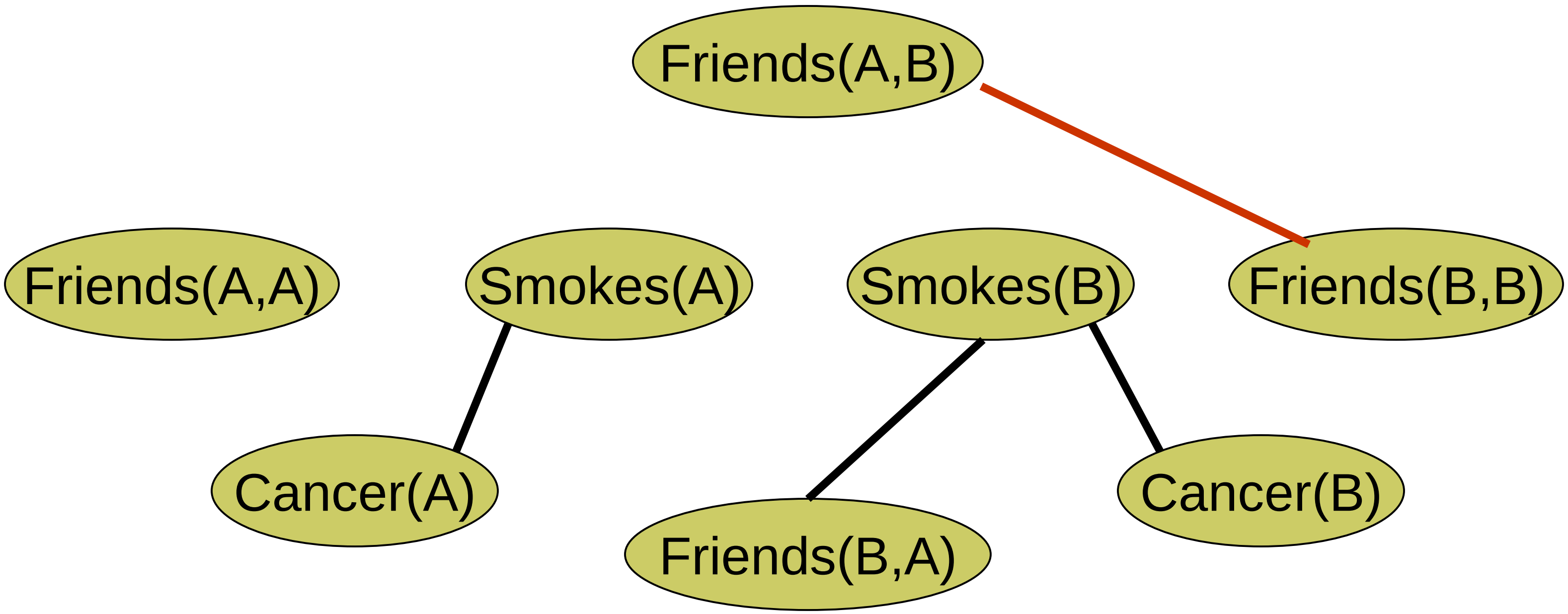

MLN features

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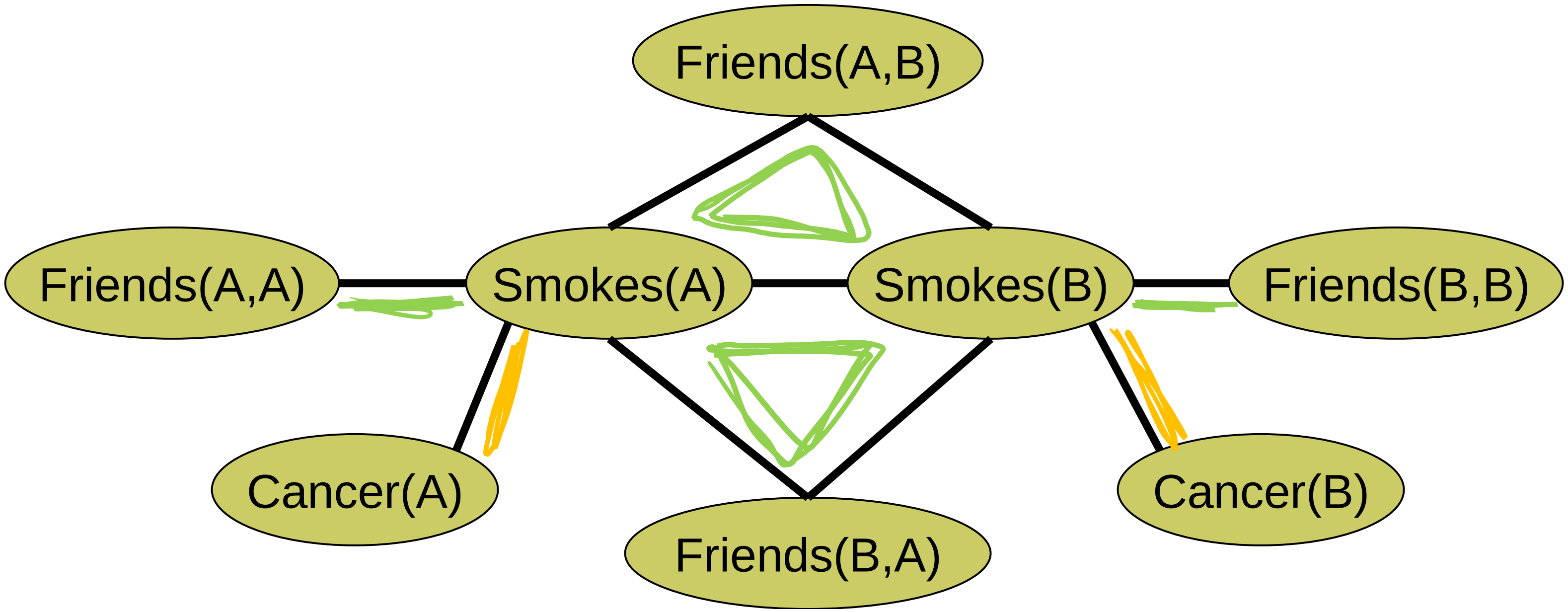


One *feature/factor* for each **grounding** of each **formula** F in the MLN

MLN: parameters

- For each formula *i* we have a factor

$\Phi_i(pw) = e^{w_i f_i(pw)}$ ← possible world

w_i weight of formula

$$f_i(pw) = \begin{cases} 1 & \text{when formula is true in } pw \\ 0 & \text{otherwise} \end{cases}$$

$1.5 \quad \forall x\ Smokes(x) \Rightarrow Cancer(x)$

$$f(\text{Smokes(x), Cancer(x)}) = \begin{cases} 1 & \text{if Smokes(x)} \Rightarrow \text{Cancer(x)} \\ 0 & \text{otherwise} \end{cases}$$

pw_1 ....

Smokes(A) T
Cancer(A) F $e^0 = 1$

pw_2 ....

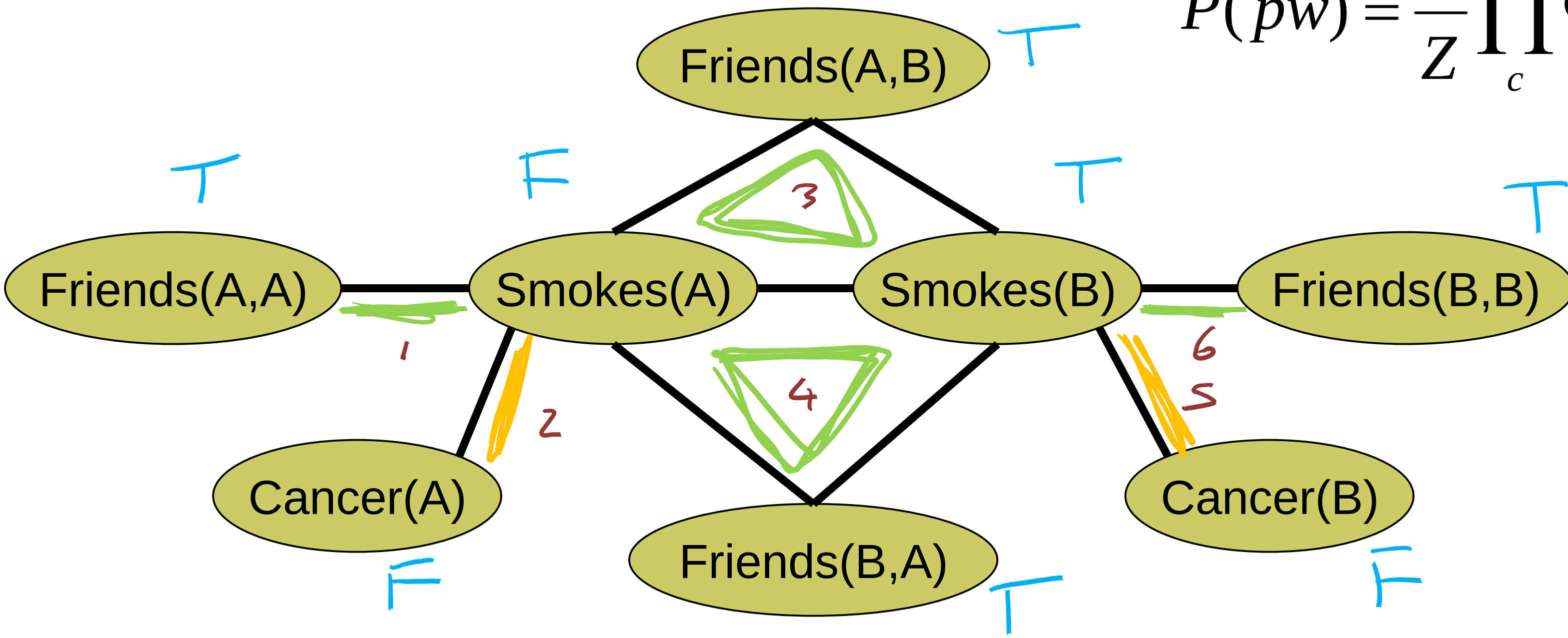
Smokes(A) T
Cancer(A) T $e^{1.5}$

MLN: prob. of possible world

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x,y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Two constants: **Anna** (A) and **Bob** (B)

$$P(pw) = \frac{1}{Z}\prod_c \Phi_c(pw_c)$$



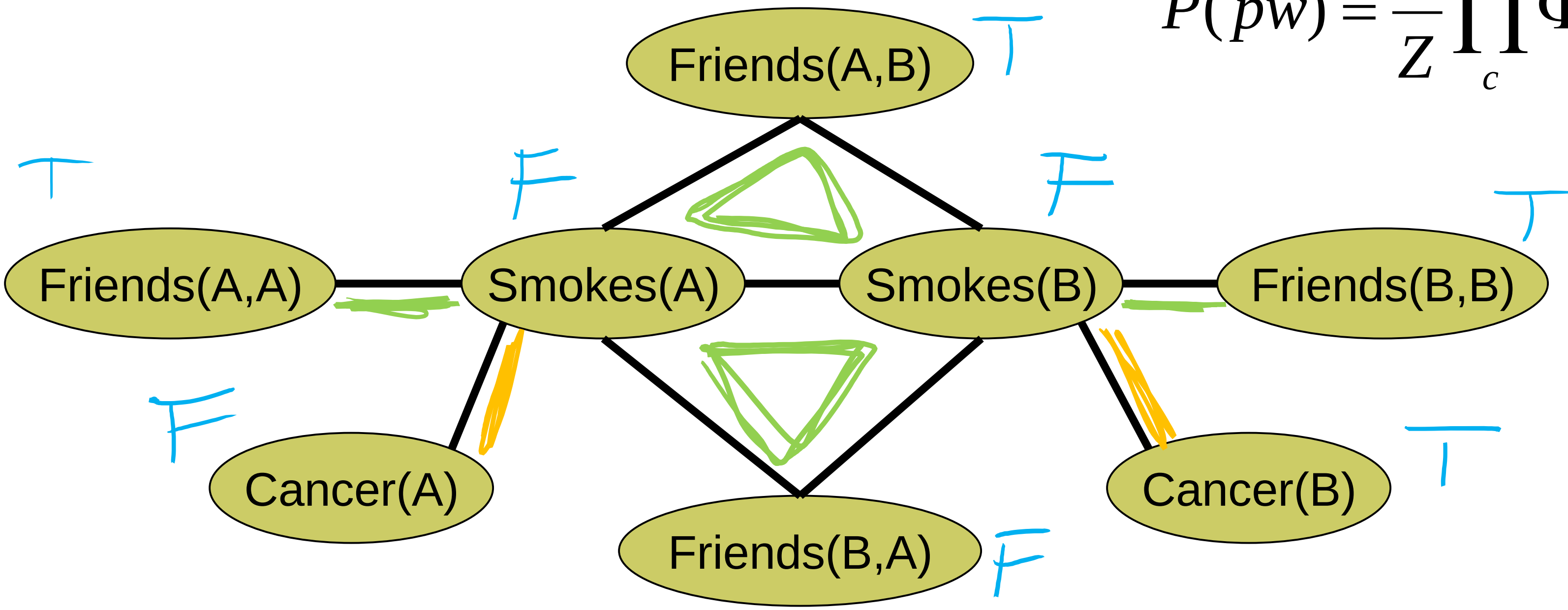
$$P(pw) = \left(\underset{1}{e^{1.1}} * \underset{6}{e^{1.1}} * \underset{3}{e^{0}} * \underset{4}{e^{0}} * \underset{2}{e^{1.5}} * \underset{5}{e^{0}}\right) / Z$$

MLN: prob. of possible world

1.5	$\forall x\ Smokes(x) \Rightarrow Cancer(x)$
1.1	$\forall x, y\ Friends(x,y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$

Two constants: **Anna** (A) and **Bob** (B)

$$P(pw) = \frac{1}{Z} \prod_c \Phi_c(pw_c)$$



$$\left(e^{1.1} * e^{1.1} * e^{1.1} * e^{1.1} * e^{1.5} * e^{1.5}\right) / Z$$

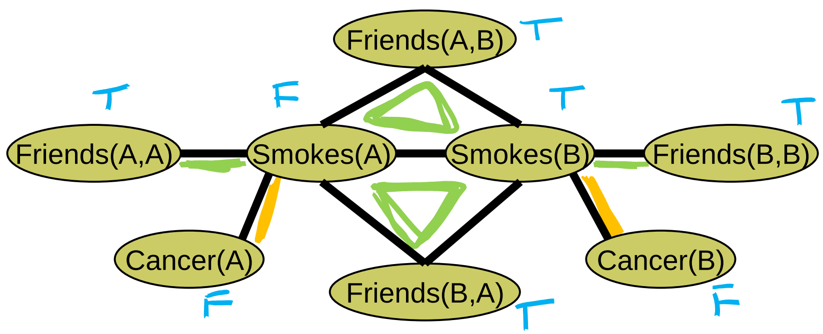
MLN: prob. Of possible world

- Probability of a world *pw*:

$$P(pw) = \frac{1}{Z}\exp\left(\sum_i \boxed{w_i}\,\boxed{n_i(pw)}\right)$$

Weight of formula *i*

No. of true groundings of formula *i* in *pw*



$$P(pw) = \left(\underbrace{e^{1.1} * e^{1.1}}_{n_2(pw)=2} * e^{0} * e^{0} * \underbrace{e^{1.5}}_{n_1(pw)=1} * e^{0}\right) / Z \rightarrow e^{(1.1*2)+(1.5*1)}$$

$$P(\text{world}) \propto \exp\left(\sum \text{weights of grounded formulas it satisfies}\right)$$

Learning Goals for today's class

You can:

- Describe the intuitions behind the design of a Markov Logic
- Define and Build a Markov Logic Network
- Justify and apply the formula for computing the probability of a possible world

Next class on Wed

Markov Logic

-relation to FOL

- Inference (MAP and Cond. Prob)

Assignment-4 posted, due on Dec 2