

# Intelligent Systems (AI-2)

## Computer Science cpsc422, Lecture 33

Nov, 25, 2015

**Slide source:** from Pedro Domingos UW & Markov Logic: An Interface Layer for Artificial Intelligence Pedro Domingos and Daniel Lowd University of Washington, Seattle

# Lecture Overview

- Recap Markov Logic (Networks)
- Relation to First-Order Logics
- Inference in MLN
  - MAP Inference (most likely  $p_w$ )
  - Probability of a formula, Conditional Probability

# Prob. Rel. Models vs. Markov Logic



PRM

- Relational Skeleton
  - Dependency Graph
  - Parameters (CPT)
- }  $\Rightarrow$  BNENET

ML

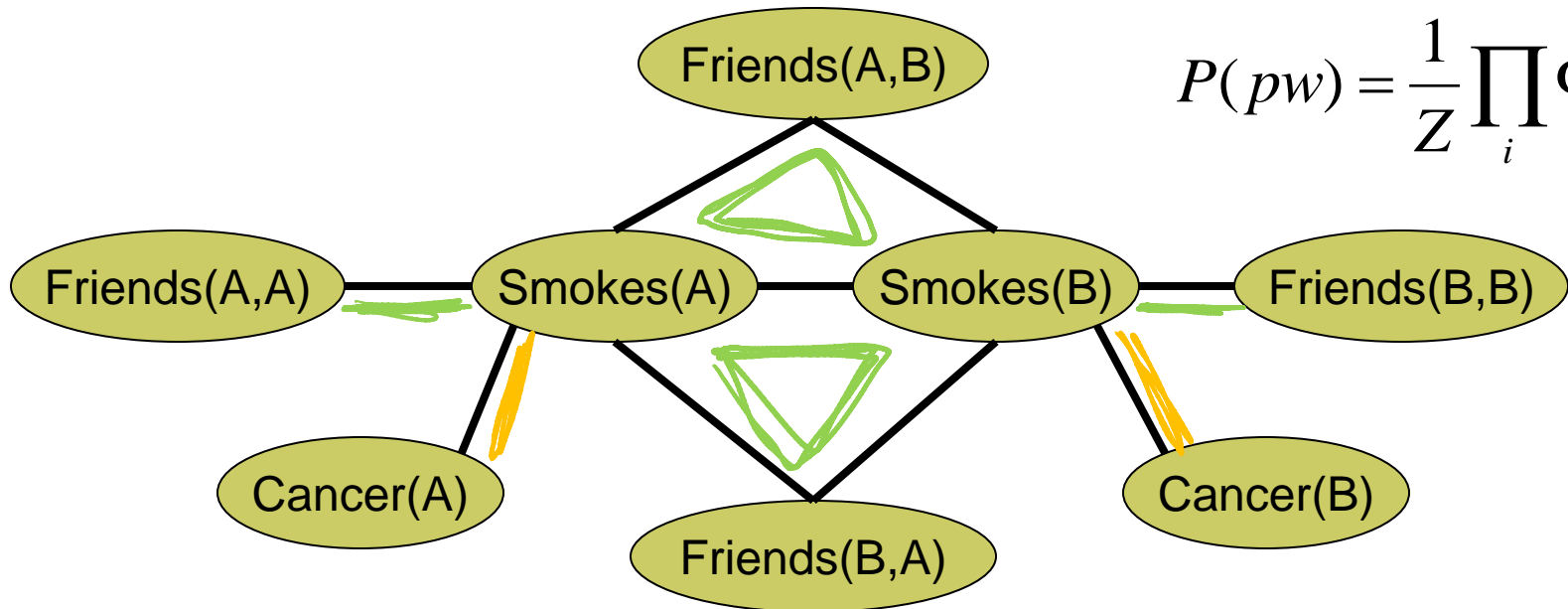
- weighted logical formulas
  - set of constants
- }  $\Rightarrow$  MARKOV LOGIC NETWORK

# MLN features



- 1.5  $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$
- 1.1  $\forall x, y \text{ Friends}(x, y) \Rightarrow (\text{Smokes}(x) \Leftrightarrow \text{Smokes}(y))$

Two constants: **Anna** (A) and **Bob** (B)



$$P(pw) = \frac{1}{Z} \prod_i \Phi_i(pw)$$

# MLN: parameters



- For each grounded formula  $i$  we have a **factor**

$$\Phi_i(pw) = e^{w_i f_i(pw)}$$

← possible world

$w_i$  weight of formula

- Same for all the groundings of the same formula

$$f_i(pw) = \begin{cases} 1 & \text{when formula is true in } pw \\ 0 & \text{otherwise} \end{cases}$$

1.5  $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$

$$f(\text{Smokes}(x), \text{Cancer}(x)) = \begin{cases} 1 & \text{if } \text{Smokes}(x) \Rightarrow \text{Cancer}(x) \\ 0 & \text{otherwise} \end{cases}$$

$pw_1$  ...  
 $\text{Smokes}(A) \quad T$   
 $\text{Cancer}(A) \quad F \quad e^0 = 1$

$pw_2$  ...  $e^{1.5}$   
 $\text{Smokes}(A) \quad T$   
 $\text{Cancer}(A) \quad T$

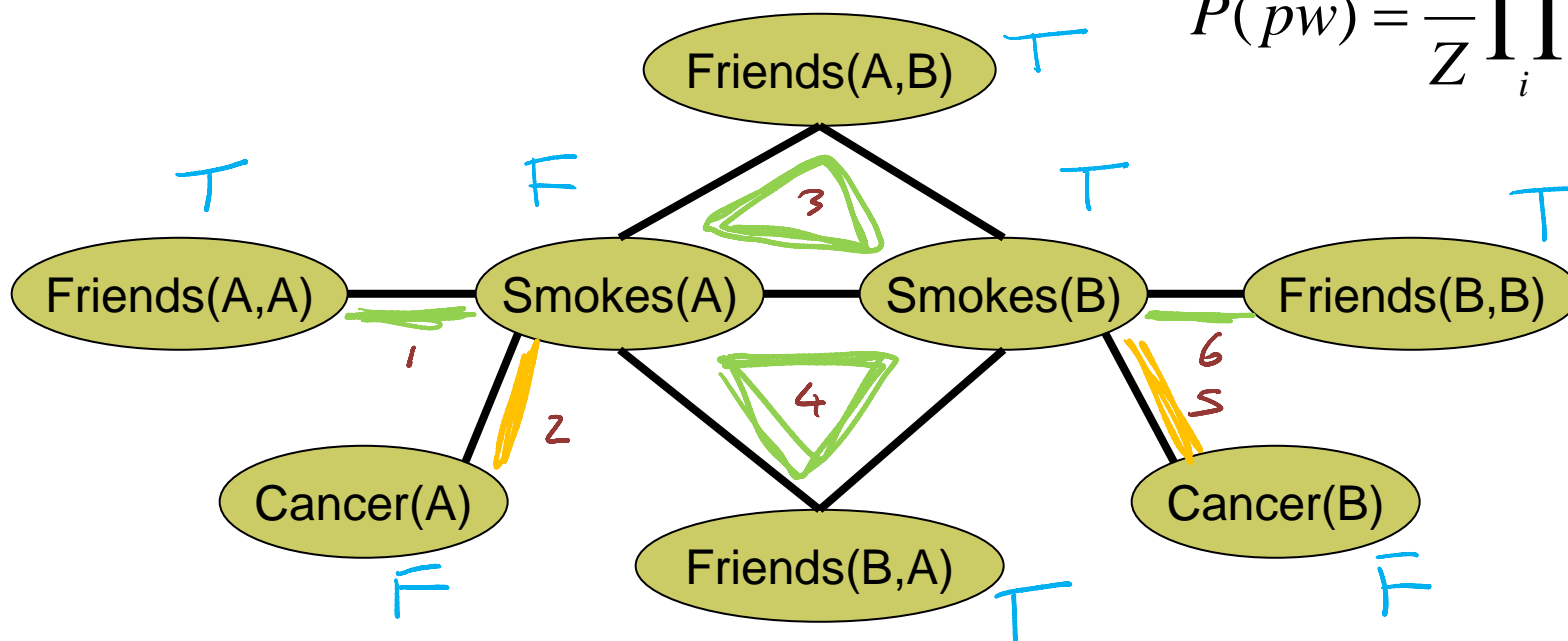
# MLN: prob. of possible world



- 1.5  $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$
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Two constants: **Anna** (A) and **Bob** (B)

$$P(pw) = \frac{1}{Z} \prod_i \Phi_i(pw)$$



$$P(pw) = \left( e^{1.1} * e^{1.1} * e^0 * e^0 * e^{1.5} * e^0 \right) / Z$$

# MLN: prob. Of possible world



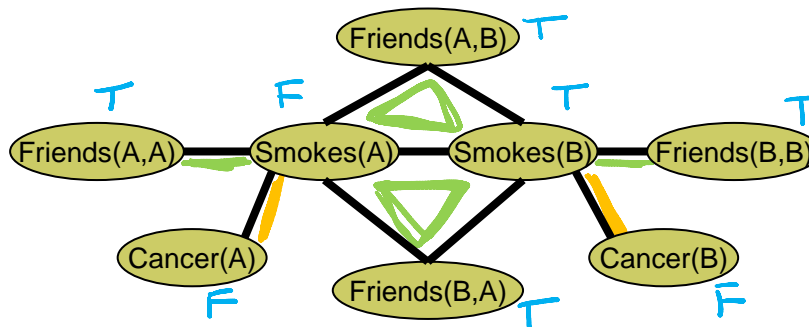
- ① 1.5  $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$
- ② 1.1  $\forall x, y \text{ Friends}(x, y) \Rightarrow (\text{Smokes}(x) \Leftrightarrow \text{Smokes}(y))$

- Probability of a world  $pw$ :

$$P(pw) = \frac{1}{Z} \exp \left( \sum_i w_i n_i(pw) \right)$$

Weight of formula  $i$

No. of true groundings of formula  $i$  in  $pw$



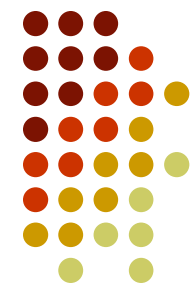
$$P(pw) = \left( \underbrace{e^{1.1} * e^{1.1}}_{n_2(pw)=2} * e^0 * e^0 * \underbrace{e^{1.5}}_{n_1(pw)=1} * e^0 \right)^{\frac{1}{Z}}$$

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# How MLNs generalize FOL



- Consider MLN containing only one formula

$$w \quad \forall x R(x) \Rightarrow S(x) \quad C = \{A\}$$



$$\Phi_1(pw) = e^{w f_1(pw)}$$

$$z = 1 + 3e^w$$

4 pws

R(A)	S(A)	$f_1(pw)$	$\Phi_1(pw)$
T	T	1	$e^w$
F	T	1	$e^w$
T	F	0	$1$
F	F	1	$e^w$

P(pw)
$e^w / (1 + 3e^w)$
$e^w / (1 + 3e^w)$
$1 / (1 + 3e^w)$
$e^w / (1 + 3e^w)$

$$P(S(A) | R(A)) = \frac{P(S(A) \wedge R(A))}{P(R(A))} = \frac{e^w / z}{\frac{1}{z} + \frac{e^w}{z}} = \frac{e^w}{1 + e^w} = \frac{1}{e^{-w} + 1}$$

$$w \rightarrow \infty$$



$w \rightarrow \infty, P(S(A) | R(A)) \rightarrow 1$  “recovering logical entailment”

# How MLNs generalize FOL



First order logic (with some mild assumptions) is a special Markov Logics obtained when

- all the weight are equal
- and tend to infinity

# Lecture Overview

- Recap Markov Logic (Networks)
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- **Inference in MLN**
  - MAP Inference (most likely  $pw$ )
  - Probability of a formula, Conditional Probability

# Inference in MLN



- MLN acts as a template for a Markov Network
- We can always answer prob. queries using standard Markov network inference methods on the instantiated network
- **However**, due to the size and complexity of the resulting network, this is often infeasible.
- Instead, we combine **probabilistic methods** with ideas from **logical inference**, including **satisfiability** and **resolution**.
- This leads to efficient methods that take full advantage of the logical structure.

# MAP Inference



- **Problem:** Find most likely state of world

$$\arg \max_{pw} P(pw)$$

- Probability of a world  $pw$ :

$$P(pw) = \frac{1}{Z} \exp \left( \sum_i w_i n_i(pw) \right)$$

Weight of formula  $i$

No. of true groundings of formula  $i$  in  $pw$

$$\arg \max_{pw} \frac{1}{Z} \exp \left( \sum_i w_i n_i(pw) \right)$$



# MAP Inference

$$\arg \max_{pw} \frac{1}{Z} \exp \left( \sum_i w_i n_i(pw) \right)$$

$$\arg \max_{pw} \sum_i w_i n_i(pw)$$

- Are these two equivalent?



A. Yes

B. No

C. It depends . . . .

# MAP Inference



- Therefore, the MAP problem in Markov logic reduces to finding the truth assignment that maximizes the sum of weights of satisfied formulas (let's assume clauses)

$$\arg \max_{pw} \sum_i w_i n_i(pw)$$

- This is just the weighted MaxSAT problem
- Use weighted SAT solver (e.g., MaxWalkSAT [Kautz et al., 1997])

# WalkSAT algorithm (in essence) (from lecture 21 – one change)

**(Stochastic) Local Search Algorithms** can be used for this task!

**Evaluation Function:** number of **satisfied clauses**

**WalkSat:** One of the simplest and most effective algorithms:

Start from a randomly generated interpretation ( $\rho$ )

- Pick randomly an unsatisfied clause ←
- Pick a proposition/atom to flip (randomly 1 or 2)
  1. Randomly
  2. To maximize **# of satisfied clauses**

if all clauses satisfied DONE 😊  
else



# MaxWalkSAT algorithm (in essence)

**Evaluation Function**  $f(pw)$  :  $\sum$  weights(sat. clauses in  $pw$ )

*current pw*  $\leftarrow$  randomly generated interpretation

Generate *new pw* by doing the following

- Pick randomly an unsatisfied clause
- Pick a proposition/atom to flip (randomly 1 or 2)
  1. Randomly
  2. To maximize  $\sum$  weights(sat. clauses in resulting  $pw$ )

If  $f(\text{new } pw) > f(\text{current } pw)$

$\text{current } pw \leftarrow \text{new } pw$

# Computing Probabilities



$$P(\text{Formula} | M_{L,C}) = ?$$

- **Brute force:** Sum probs. of possible worlds where formula holds

$M_{L,C}$  Markov Logic Network

$PW_F$  possible worlds in which  $F$  is true

$$P(F, M_{L,C}) = \sum_{pw \in PW_F} P(pw, M_{L,C})$$

- **MCMC:** Sample worlds, check formula holds

$S$  all samples

$S_F$  samples (i.e. possible worlds) in which  $F$  is true

$$P(F, M_{L,C}) = \frac{|S_F|}{|S|}$$

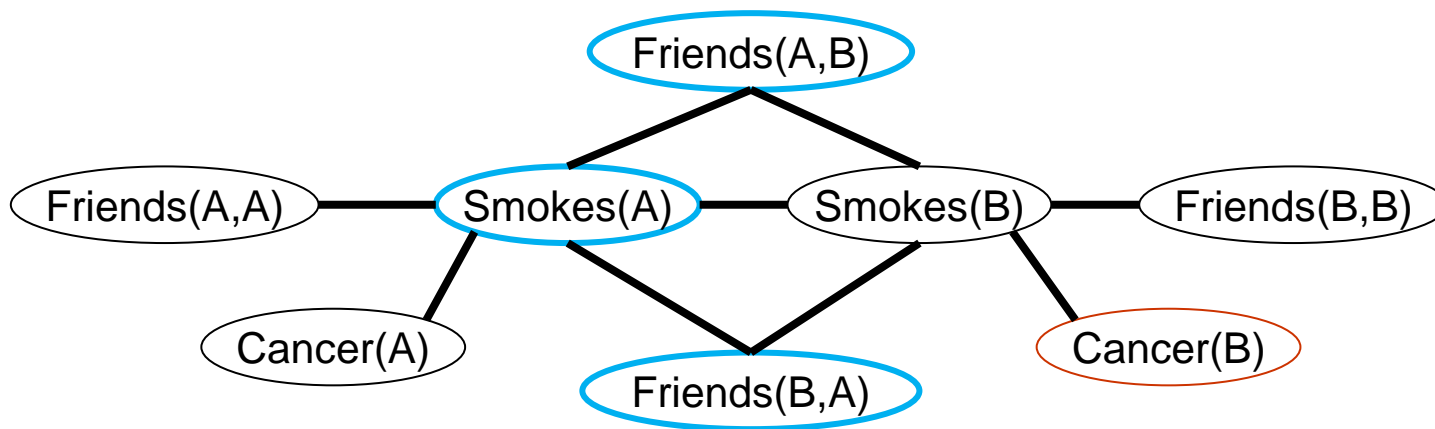
# Computing Cond. Probabilities



Let's look at the simplest case

$P(\text{ground literal} \mid \text{conjunction of ground literals}, M_{L,C})$

$P(\text{Cancer}(B) \mid \text{Smokes}(A), \text{Friends}(A, B), \text{Friends}(B, A))$



To answer this query do you need to create (ground) the whole network?



A. Yes

B. No

C. It depends...

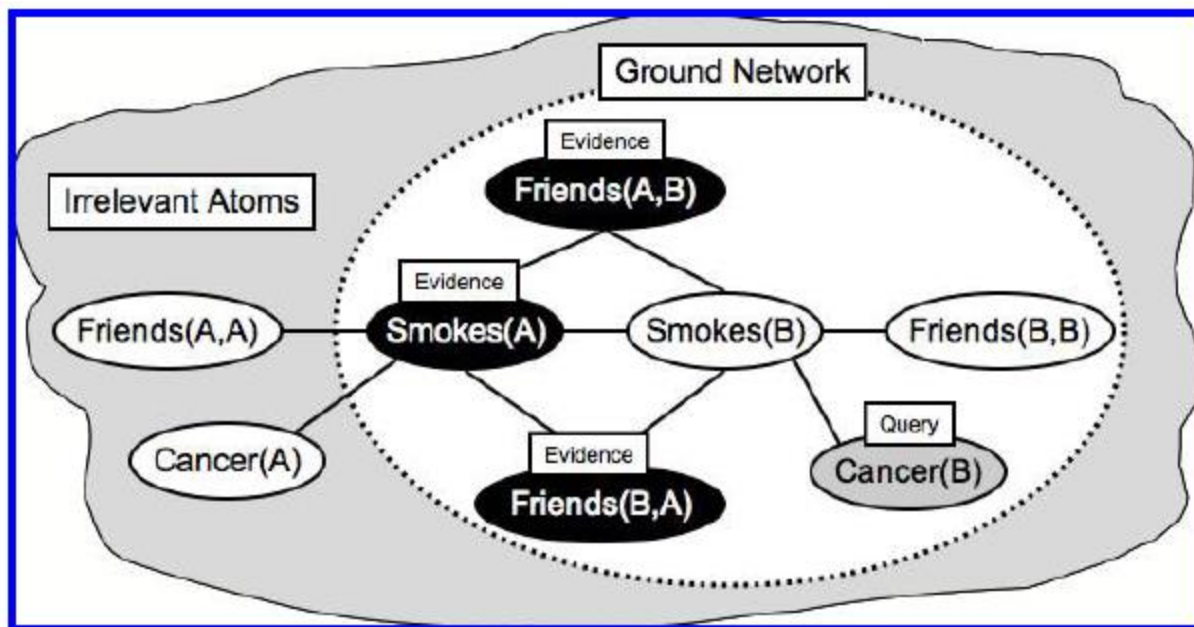
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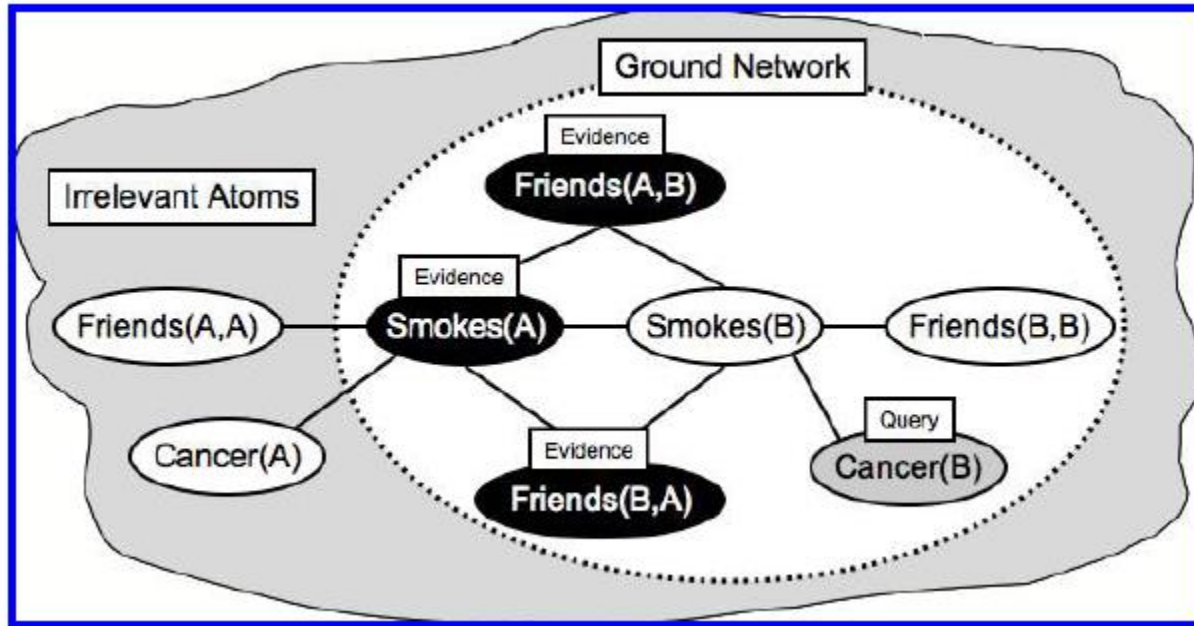


You do not need to create (ground) the part of the Markov Network from which the query is independent given the evidence

# Computing Cond. Probabilities



$$P(\text{Cancer}(B) \mid \text{Smokes}(A), \text{Friends}(A, B), \text{Friends}(B, A))$$



Then you can perform Gibbs Sampling in this Sub Network

# Learning Goals for today's class

## You can:

- Show on an example how MLNs generalize FOL
- Compute the most likely  $pw$  (given some evidence)
- Probability of a formula, Conditional Probability

# Next class on Fri

- Markov Logic: applications
- Start. Prob Relational Models

# Inference in MLN



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- **However**, due to the size and complexity of the resulting network, this is often infeasible.
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# MAP Inference



- Find most likely state of world

$$\arg \max_{pw} P(pw)$$

- Reduces to finding the  $pw$  that maximizes the sum of weights of satisfied clauses

$$\arg \max_{pw} \sum_i w_i n_i(pw)$$

- Use weighted SAT solver  
(e.g., MaxWalkSAT [Kautz et al., 1997])

Probabilistic problem solved by logical inference method

# Computing Probabilities



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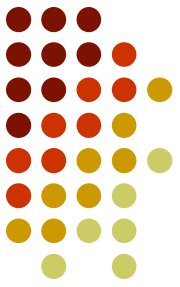
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# Computing Cond. Probabilities



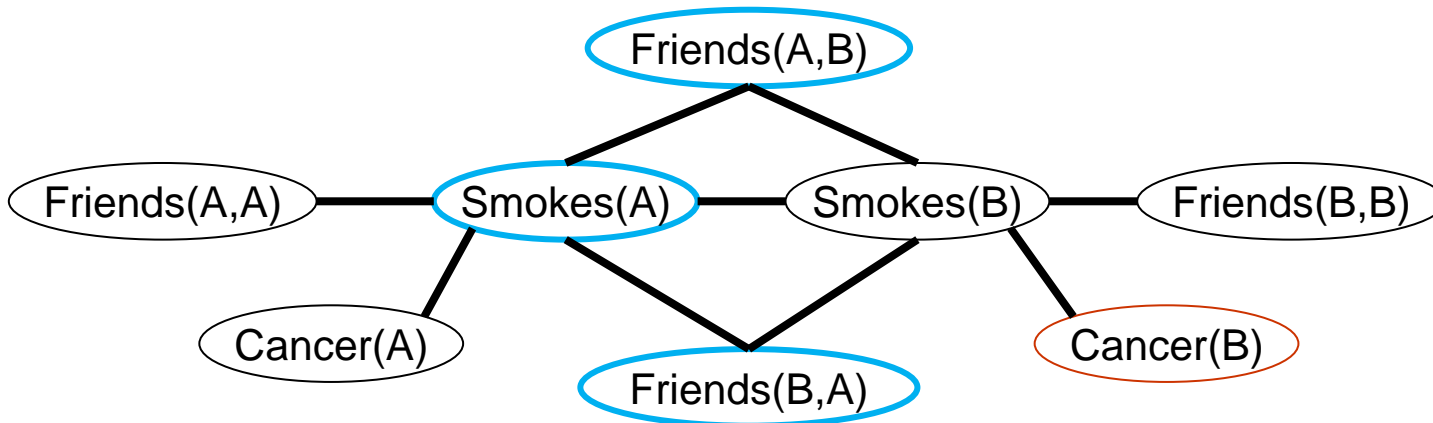
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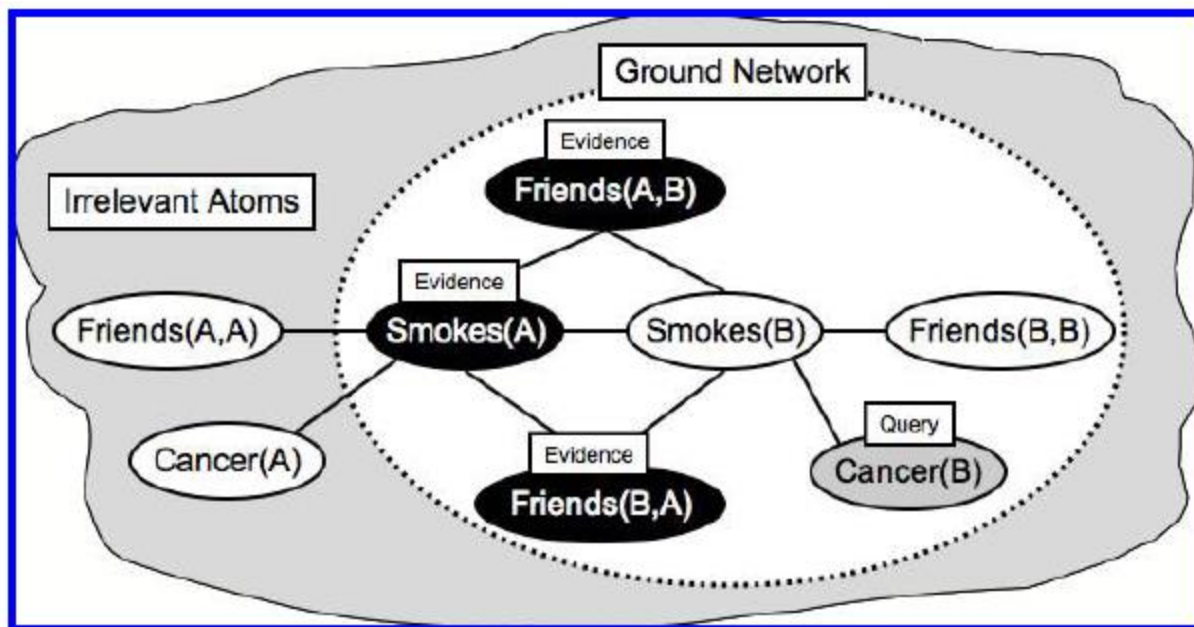
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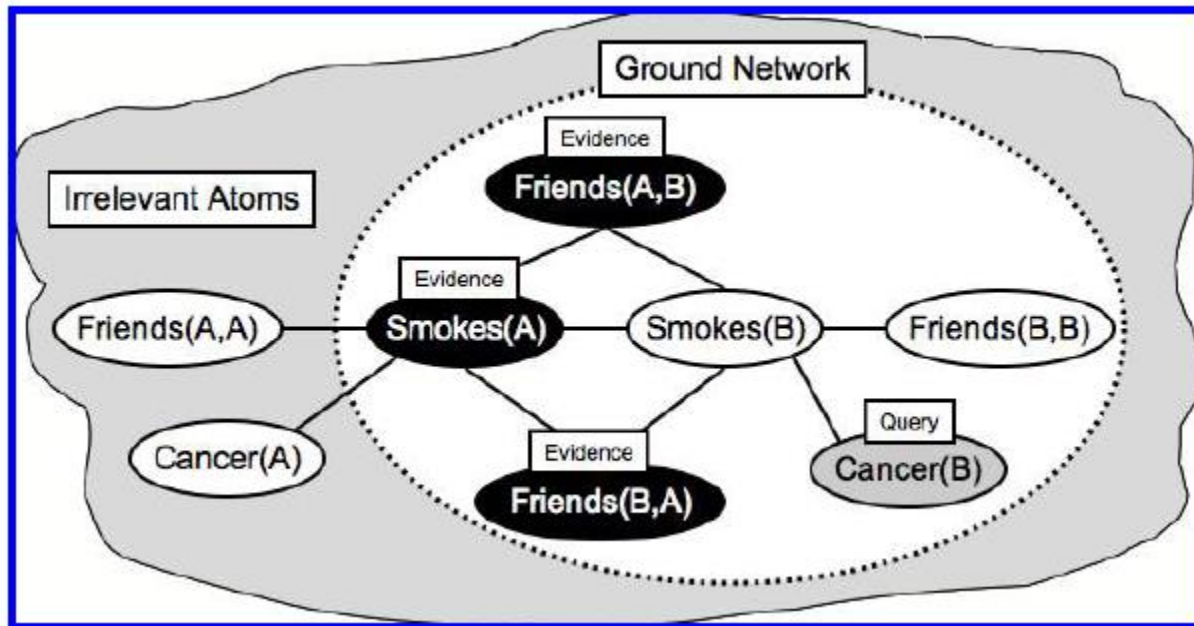
# Computing Cond. Probabilities

$$P(\text{Cancer}(B) \mid \text{Smokes}(A), \text{Friends}(A, B), \text{Friends}(B, A))$$

The sub network is determined by the formulas  
(the logical structure of the problem)

1.5  $\forall x \text{Smokes}(x) \Rightarrow \text{Cancer}(x)$

1.1  $\forall x, y \text{Friends}(x, y) \Rightarrow (\text{Smokes}(x) \Leftrightarrow \text{Smokes}(y))$



You can then perform Gibbs Sampling in  
this Sub Network



# Lecture Overview

- Finish Inference in MLN
  - Probability of a formula, Conditional Probability
- **Markov Logic: applications**
- **Beyond 322/422 (ML + grad courses)**
- **AI conf. and journals**
- **Watson.....**
- **Final Exam (office hours, samples)**
- **TA evaluation**



# Entity Resolution

- Determining which observations correspond to the same real-world objects
- (e.g., database records, noun phrases, video regions, etc)
- Crucial importance in many areas (e.g., data cleaning, NLP, Vision)

# Entity Resolution: Example



**AUTHOR:** *H. POON & P. DOMINGOS*  
**TITLE:** *UNSUPERVISED SEMANTIC PARSING*  
**VENUE:** *EMNLP-09*

**AUTHOR:** *Hoifung Poon and Pedro Domingos*  
**TITLE:** *Unsupervised semantic parsing*  
**VENUE:** *Proceedings of the 2009 Conference on Empirical Methods in Natural Language Processing*

**AUTHOR:** *Poon, Hoifung and Domingos, Pedro*  
**TITLE:** *Unsupervised ontology induction from text*  
**VENUE:** *Proceedings of the Forty-Eighth Annual Meeting of the Association for Computational Linguistics*

**AUTHOR:** *H. Poon, P. Domingos*  
**TITLE:** *Unsupervised ontology induction*  
**VENUE:** *ACL-10*

SAME?

SAME?



# Entity Resolution (relations)



**Problem:** Given citation database, find duplicate records  
Each citation has author, title, and venue fields  
We have 10 relations

**Author**(bib, author)

**Title**(bib, title)

**Venue**(bib, venue)

relate citations to their fields

**HasWord**(author, word)

**HasWord**(title, word)

**HasWord**(venue, word)

indicate which words are present  
in each field;

**SameAuthor**(author, author) represent field equality;

**SameTitle**(title, title)

**SameVenue**(venue, venue)

**SameBib**(bib, bib) represents citation equality;

provided as evidence

To be  
inferred

# Entity Resolution (formulas)



Predict citation equality based on words in the fields

$\text{Title}(b1, t1) \wedge \text{Title}(b2, t2) \wedge$   
 $\text{HasWord}(t1, +\text{word}) \wedge \text{HasWord}(t2, +\text{word}) \Rightarrow$   
 $\text{SameBib}(b1, b2)$

*1000s  
of rules  
one for  
each word*

(NOTE: +word is a shortcut notation, you actually have a rule for each word e.g.,  
 $\text{Title}(b1, t1) \wedge \text{Title}(b2, t2) \wedge$   
 $\text{HasWord}(t1, \text{"bayesian"}) \wedge$   
 $\text{HasWord}(t2, \text{"bayesian"}) \Rightarrow \text{SameBib}(b1, b2)$  )

Same 1000s of rules for **author**

Same 1000s of rules for **venue**

# Entity Resolution (formulas)



## Transitive closure

$\text{SameBib}(b1, b2) \wedge \text{SameBib}(b2, b3) \Rightarrow \text{SameBib}(b1, b3)$

$\text{SameAuthor}(a1, a2) \wedge \text{SameAuthor}(a2, a3) \Rightarrow \text{SameAuthor}(a1, a3)$

*Same rule for title*

*Same rule for venue*

**Link fields equivalence to citation equivalence** – *e.g., if two citations are the same, their authors should be the same*

$\text{Author}(b1, a1) \wedge \text{Author}(b2, a2) \wedge \text{SameBib}(b1, b2) \Rightarrow \text{SameAuthor}(a1, a2)$

*...and that citations with the same author are more likely to be the same*

$\text{Author}(b1, a1) \wedge \text{Author}(b2, a2) \wedge \text{SameAuthor}(a1, a2) \Rightarrow \text{SameBib}(b1, b2)$

*Same rules for title*

*Same rules for venue*

# Benefits of MLN model



**Standard approach:** build a classifier that given two citations tells you if they are the same or not, and then apply transitive closure

**New MLN approach:**

- performs *collective* entity resolution, where resolving one pair of entities helps to resolve pairs of related entities

e.g., inferring that a pair of citations are equivalent can provide evidence that the names *AAAI-06* and *21st Natl. Conf. on AI* refer to the same venue, even though they are superficially very different. This equivalence can then aid in resolving other entities.

# Other MLN applications



- **Information Extraction**
- **Co-reference Resolution (see lecture 1!)**
- **Robot Mapping** (infer the map of an indoor environment from laser range data)
- **Link-based Clustering** (uses relationships among the objects in determining similarity)
- **Ontologies extraction from Text**
- .....

# Summary of tutorial on MLN for NLP at NA-ACL (2010)



- We need to unify logical and statistical NLP
- **Markov logic** provides a language for this
  - **Syntax:** Weighted first-order formulas
  - **Semantics:** Feature templates of Markov nets
  - **Inference:** Satisfiability, MCMC, lifted BP, etc.
  - **Learning:** Pseudo-likelihood, VP, PSCG, ILP, etc.
- Growing set of NLP applications
- Open-source software: Alchemy

[alchemy.cs.washington.edu](http://alchemy.cs.washington.edu)

- Book: Domingos & Lowd, *Markov Logic*, Morgan & Claypool, 2009.

# Learning Goals for today's class

## You can:

- Show on an example how MLNs generalize FOL
- Compute the most likely  $pw$  given some evidence
- Probability of a formula, Conditional Probability

# Next class on Mon (last class)

- Markov Logic: applications
- Watson....
- Beyond 322/422 (ML + grad courses)
- AI conf. and journals
- Final Exam (office hours, samples)

Assignment-4 due on Mon (last class)

Marked Summaries for last paper discussion



# The MaxWalkSAT Algorithm



```
for  $i \leftarrow 1$  to max-tries do
  solution = random truth assignment
  for  $j \leftarrow 1$  to max-flips do
    if  $\sum \text{weights}(\text{sat. clauses}) > \text{threshold}$  then
      return solution
     $c \leftarrow$  random unsatisfied clause
    with probability  $p$ 
      flip a random variable in  $c$ 
    else
      flip variable in  $c$  that maximizes
         $\sum \text{weights}(\text{sat. clauses})$ 
  return failure, best solution found
```



# Markov Logic Network

What is the probability that a formula  $F_1$  holds given that formula  $F_2$  does?

$$\begin{aligned} & P(F_1 \mid F_2, L, C) \\ &= P(F_1 \mid F_2, M_{L,C}) \\ &= \frac{P(F_1 \wedge F_2, M_{L,C})}{P(F_2, M_{L,C})} \\ &= \frac{\sum_{x \in \chi_{F_1} \cap x \in \chi_{F_2}} P(X = x, M_{L,C})}{\sum_{x \in \chi_{F_2}} P(X = x, M_{L,C})} \end{aligned}$$

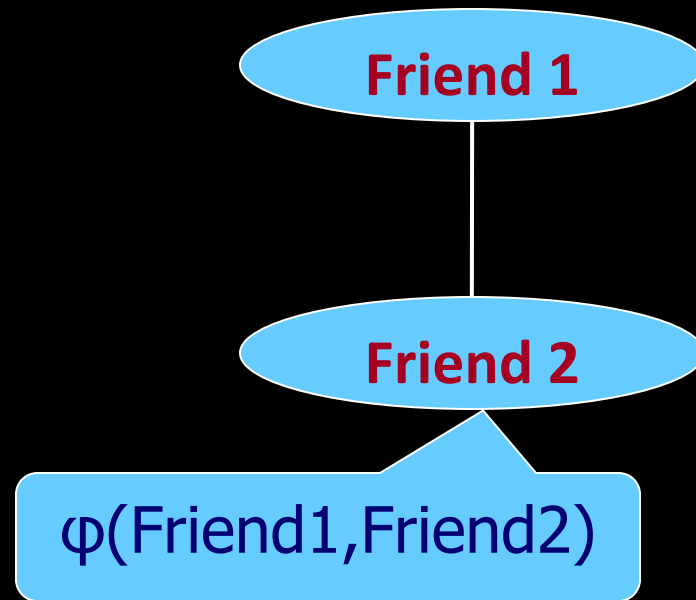
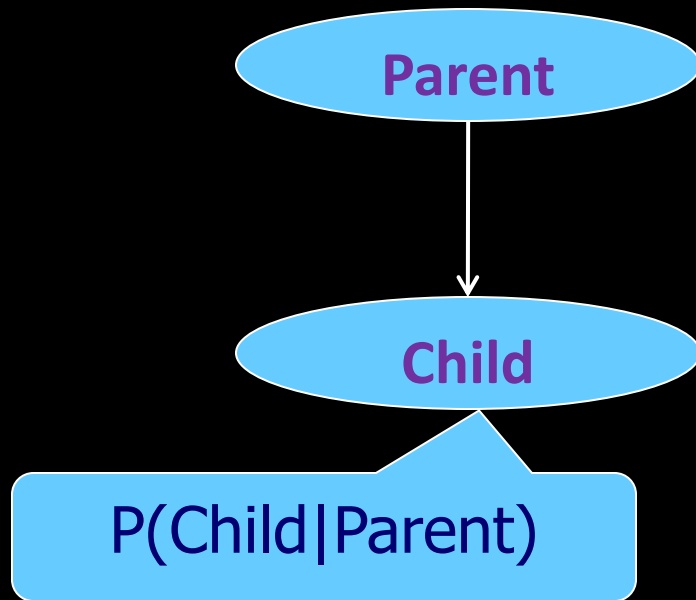
# Computing Probabilities



$$P(\text{Formula} | \text{Formula2}, \text{MLN}, \text{C}) = ?$$

- Discard worlds where Formula 2 does not hold
- In practice: More efficient alternatives

# Directed Models vs. Undirected Models



# Undirected Probabilistic Logic Models

- Upgrade undirected propositional models to relational setting
  - Markov Nets → **Markov Logic Networks**
  - Markov Random Fields → Relational Markov Nets
  - Conditional Random Fields → Relational CRFs

# Markov Logic Networks (Richardson & Domingos)

## ■ Soften logical clauses

- A first-order clause is a **hard** constraint on the world

$$\forall x, \text{person}(x) \rightarrow \exists y, \text{person}(y), \text{father}(x, y)$$

- **Soften** the constraints so that when a constraint is violated, the world is less probably, not impossible

$$w : \text{friends}(x, y) \wedge \text{smokes}(x) \rightarrow \text{smokes}(y)$$

- **Higher** weight  $\Rightarrow$  **Stronger** constraint
- Weight of  $\infty \Rightarrow$  first-order logic

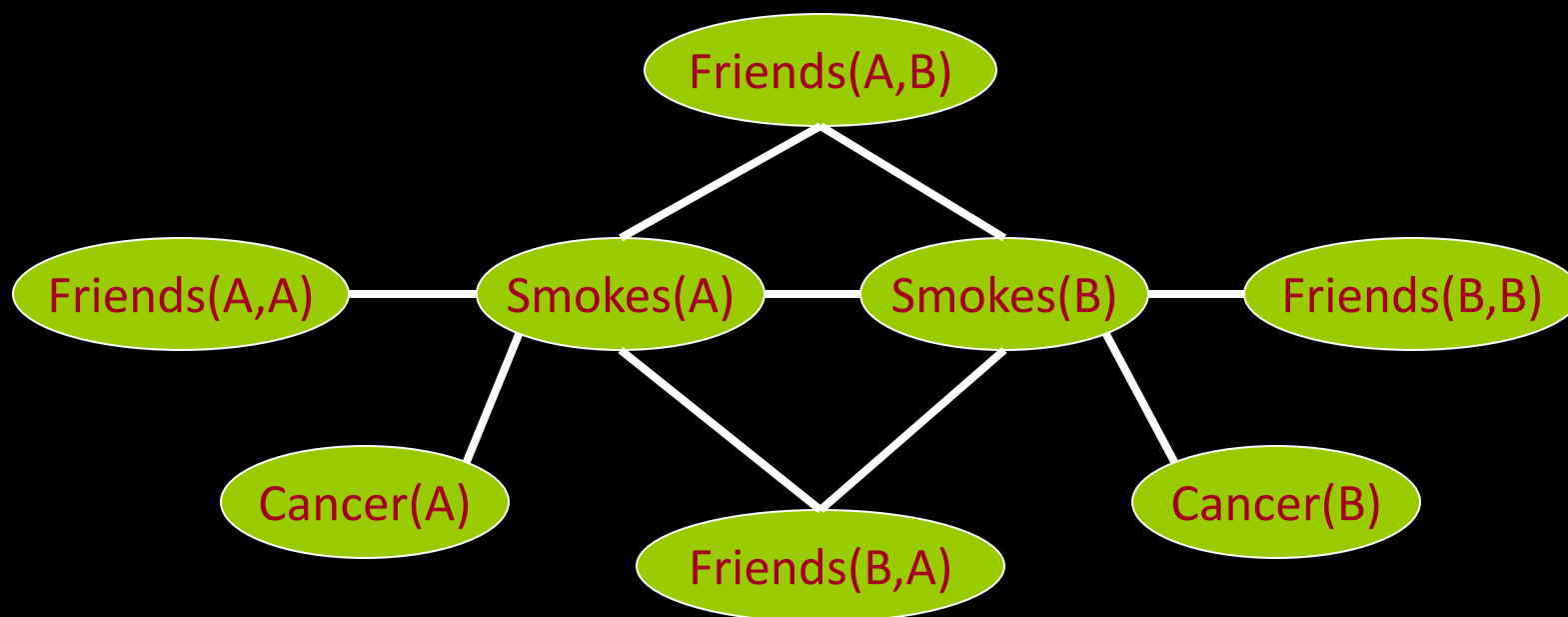
$$\text{Probability}(\text{World } S) = (1 / Z) \times \exp \{ \sum \text{weight}_i \times \text{numberTimesTrue}(f_i, S) \}$$

## Example: Friends & Smokers

1.5  $\forall x \text{ Smokes}(x) \Rightarrow \text{Cancer}(x)$

1.1  $\forall x, y \text{ Friends}(x, y) \Rightarrow (\text{Smokes}(x) \Leftrightarrow \text{Smokes}(y))$

Two constants: Anna (A) and Bob (B)



# Alphabetic Soup => Endless Possibilities

- Probabilistic Relational Models (PRM)
  - Bayesian Logic Programs (BLP)
  - PRISM
  - Stochastic Logic Programs (SLP)
  - Independent Choice Logic (ICL)
  - Markov Logic Networks (MLN)
  - Relational Markov Nets (RMN)
  - CLP-BN
  - Relational Bayes Nets (RBN)
  - Probabilistic Logic Program (PLP)
  - ProbLog
  - ....
- Web data (**web**)
  - Biological data (**bio**)
  - Social Network Analysis (**soc**)
  - Bibliographic data (**cite**)
  - Epidemiological data (**epi**)
  - Communication data (**comm**)
  - Customer networks (**cust**)
  - Collaborative filtering problems (**cf**)
  - Trust networks (**trust**)
  - ...



# Recent Advances in SRL Inference

- Preprocessing for Inference
  - ❑ FROG – Shavlik & Natarajan (2009)
- Lifted Exact Inference
  - ❑ Lifted Variable Elimination – Poole (2003), Braz et al(2005) Milch et al (2008)
  - ❑ Lifted VE + Aggregation – Kisynski & Poole (2009)
- Sampling Methods
  - ❑ MCMC techniques – Milch & Russell (2006)
  - ❑ Logical Particle Filter – Natarajan et al (2008), ZettleMoyer et al (2007)
  - ❑ Lazy Inference – Poon et al (2008)
- Approximate Methods
  - ❑ Lifted First-Order Belief Propagation – Singla & Domingos (2008)
  - ❑ Counting Belief Propagation – Kersting et al (2009)
  - ❑ MAP Inference – Riedel (2008)
- Bounds Propagation
  - ❑ Anytime Belief Propagation – Braz et al (2009)

# Conclusion

- Inference is the key issue in several SRL formalisms
- **FROG** - Keeps the count of unsatisfied groundings
  - ❑ **Order of Magnitude** reduction in number of groundings
  - ❑ Compares favorably to **Alchemy** in different domains
- **Counting BP** - BP + grouping nodes sending and receiving identical messages
  - ❑ Conceptually **easy, scalable** BP algorithm
  - ❑ Applications to **challenging AI tasks**
- **Anytime BP** – Incremental Shattering + Box Propagation
  - ❑ Only the most **necessary** fraction of model considered and **shattered**
  - ❑ **Status** – Implementation and evaluation

# Relation to Statistical Models



- Special cases:
  - Markov networks
  - Markov random fields
  - Bayesian networks
  - Log-linear models
  - Exponential models
  - Max. entropy models
  - Gibbs distributions
  - Boltzmann machines
  - Logistic regression
  - Hidden Markov models
  - Conditional random fields
- Obtained by making all predicates zero-arity
- Markov logic allows objects to be interdependent (non-i.i.d.)