# Intelligent Systems (AI-2)

## Computer Science cpsc422, Lecture 30

Nov, 23, 2015

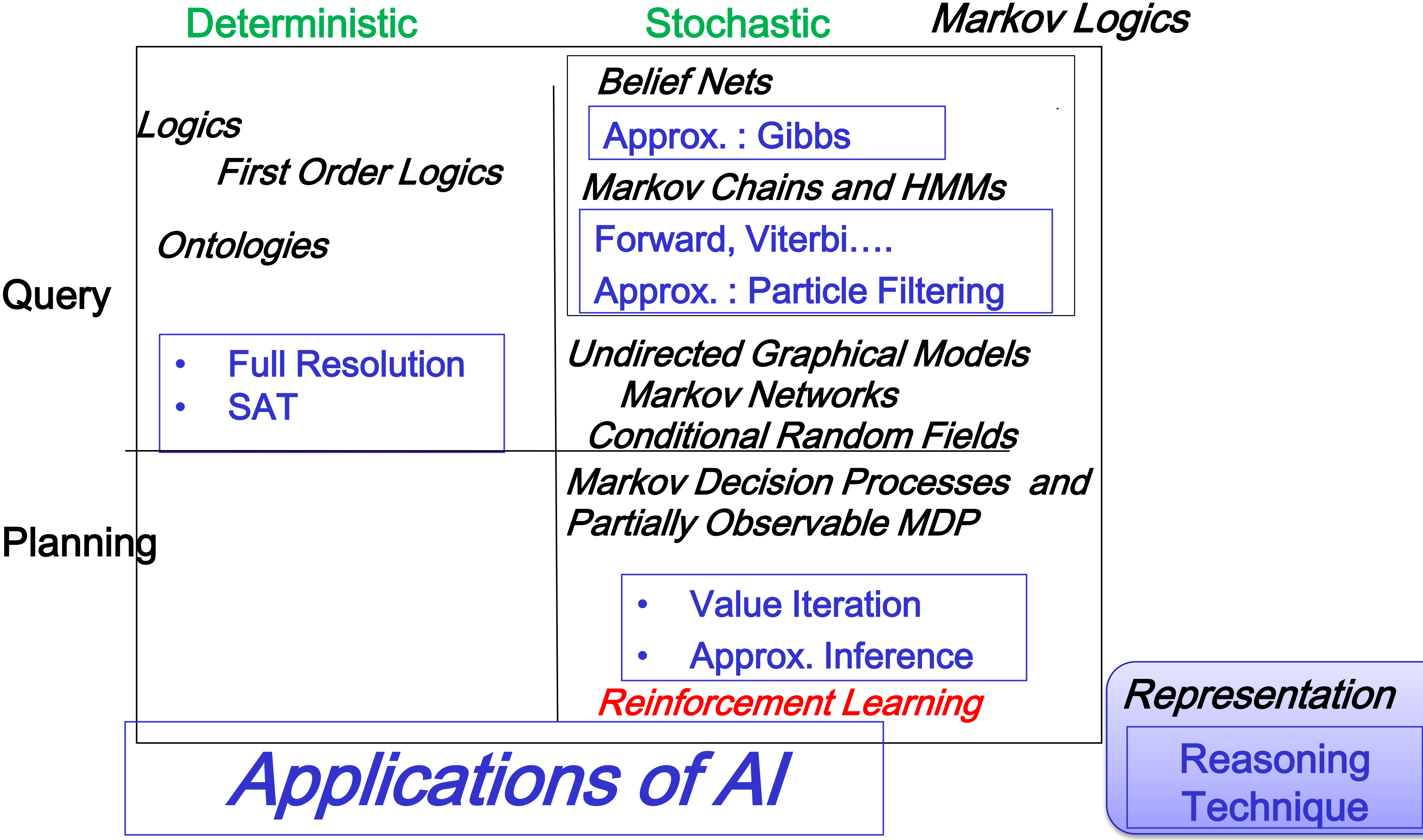
Slide source: from Pedro Domingos UW

# 422 big picture: Where are we?

**StarAI (statistical relational AI)**

**Hybrid: Det +Sto**

*Prob CFG*
*Prob Relational Models*
*Markov Logics*



Deterministic

Stochastic

Query

Planning

*Logics*
*First Order Logics*
*Ontologies*

- Full Resolution
- SAT

*Belief Nets*

Approx. : Gibbs

*Markov Chains and HMMs*

Forward, Viterbi….
Approx. : Particle Filtering

*Undirected Graphical Models*
*Markov Networks*
*Conditional Random Fields*

*Markov Decision Processes and Partially Observable MDP*

- Value Iteration
- Approx. Inference

*Reinforcement Learning*

*Applications of AI*

*Representation*

Reasoning Technique

# Lecture Overview

- Statistical Relational Models (*for us aka Hybrid*)
- Recap Markov Networks and log-linear models
- Markov Logic

# Statistical Relational Models

**Goals:**

- Combine **(subsets of) logic** and **probability** into a single language (**R&R** system)
- Develop efficient **inference** algorithms
- Develop efficient **learning** algorithms
- Apply to real-world problems

L. Getoor & B. Taskar (eds.), *Introduction to Statistical Relational Learning,* MIT Press, 2007.

# Plethora of Approaches

- Knowledge-based model construction [Wellman et al., 1992]
- Stochastic logic programs [Muggleton, 1996]
- Probabilistic relational models [Friedman et al., 1999]
- Relational Markov networks [Taskar et al., 2002]
- Bayesian logic [Milch et al., 2005]
- Markov logic [Richardson & Domingos, 2006]
- *And many others….!*

# Prob. Rel. Models vs. Markov Logic

PRM

- Relational Skeleton
- Dependency Graph
- Parameters (CPT)

⇒ BNET

ML

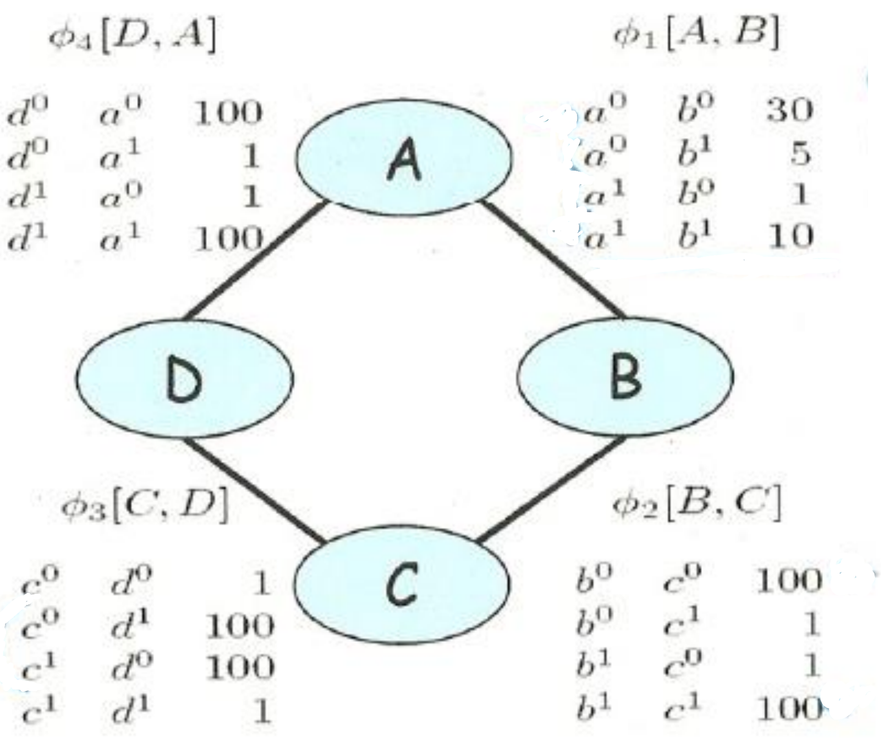
- weighted logical formulas
- set of constants

⇒ MARKOV LOGIC NETWORK

# Lecture Overview

- Statistical Relational Models (*for us aka Hybrid*)
- **Recap Markov Networks and log-linear models**
- Markov Logic
    - Markov Logic Network (MLN)

# Parameterization of Markov Networks



$\phi_4[D, A]$

| | | |
|---|---|---|
| $d^0$ | $a^0$ | 100 |
| $d^0$ | $a^1$ | 1 |
| $d^1$ | $a^0$ | 1 |
| $d^1$ | $a^1$ | 100 |

$\phi_1[A, B]$

| | | |
|---|---|---|
| $a^0$ | $b^0$ | 30 |
| $a^0$ | $b^1$ | 5 |
| $a^1$ | $b^0$ | 1 |
| $a^1$ | $b^1$ | 10 |

$\phi_3[C, D]$

| | | |
|---|---|---|
| $c^0$ | $d^0$ | 1 |
| $c^0$ | $d^1$ | 100 |
| $c^1$ | $d^0$ | 100 |
| $c^1$ | $d^1$ | 1 |

$\phi_2[B, C]$

| | | |
|---|---|---|
| $b^0$ | $c^0$ | 100 |
| $b^0$ | $c^1$ | 1 |
| $b^1$ | $c^0$ | 1 |
| $b^1$ | $c^1$ | 100 |

X set of random vars: A factor is $\Phi(Val(X)) \rightarrow \mathbb{R}$

Factors define the local interactions (like CPTs in Bnets)

What about the global model? What do you do with Bnets?

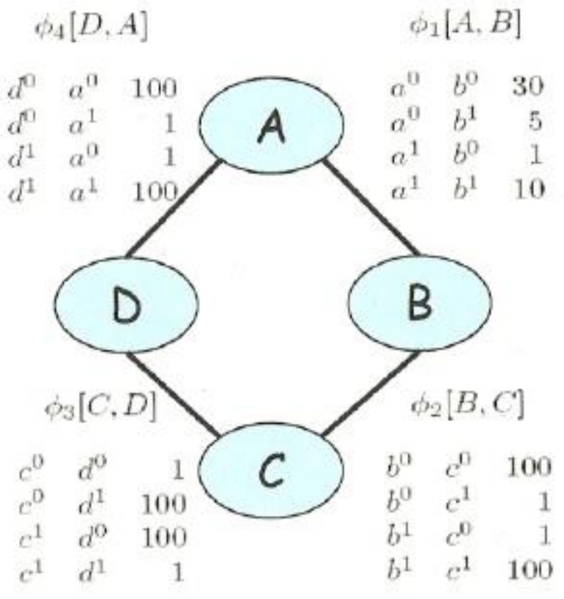
# How do we combine local models?

As in BNets by multiplying them!

$$\tilde{P}(A,B,C,D) = \phi_1(A,B) \times \phi_2(B,C) \times \phi_3(C,D) \times \phi_4(A,D)$$

$$P(A,B,C,D) = \frac{1}{Z}\tilde{P}(A,B,C,D)$$

| *Assignment* | | | | *Unnormalized* | Normalized |
|---|---|---|---|---|---|
| $a^0$ | $b^0$ | $c^0$ | $d^0$ | 300000 | .04 |
| $a^0$ | $b^0$ | $c^0$ | $d^1$ | 300000 | .04 |
| $a^0$ | $b^0$ | $c^1$ | $d^0$ | 300000 | .04 |
| $a^0$ | $b^0$ | $c^1$ | $d^1$ | 30 | $4.1 \times 10^{-6}$ |
| $a^0$ | $b^1$ | $c^0$ | $d^0$ | 500 | |
| $a^0$ | $b^1$ | $c^0$ | $d^1$ | 500 | |
| $a^0$ | $b^1$ | $c^1$ | $d^0$ | 5000000 | .69 |
| $a^0$ | $b^1$ | $c^1$ | $d^1$ | 500 | |
| $a^1$ | $b^0$ | $c^0$ | $d^0$ | 100 | |
| $a^1$ | $b^0$ | $c^0$ | $d^1$ | 1000000 | |
| $a^1$ | $b^0$ | $c^1$ | $d^0$ | 100 | |
| $a^1$ | $b^0$ | $c^1$ | $d^1$ | 100 | |
| $a^1$ | $b^1$ | $c^0$ | $d^0$ | 10 | |
| $a^1$ | $b^1$ | $c^0$ | $d^1$ | 100000 | |
| $a^1$ | $b^1$ | $c^1$ | $d^0$ | 100000 | |
| $a^1$ | $b^1$ | $c^1$ | $d^1$ | 100000 | |



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|---|---|---|
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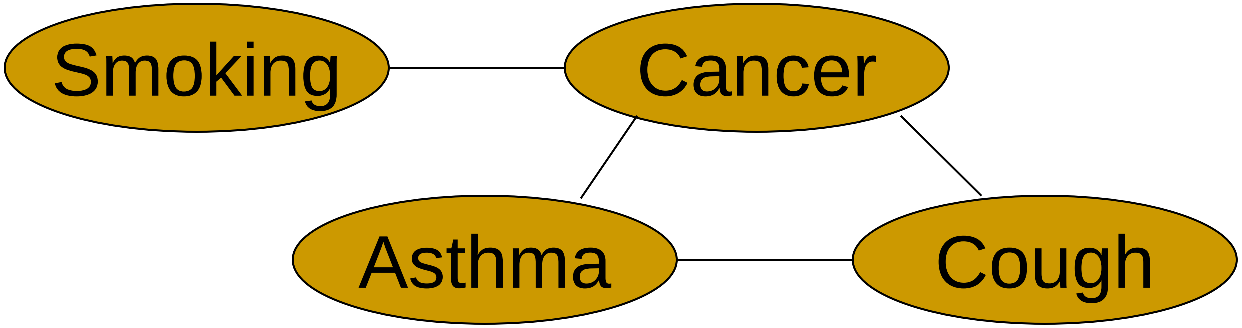
| $c^0$ | $d^1$ | 100 |
| $c^1$ | $d^0$ | 100 |
| $c^1$ | $d^1$ | 1 |

$\phi_2[B,C]$

| | | |
|---|---|---|
| $b^0$ | $c^0$ | 100 |
| $b^0$ | $c^1$ | 1 |
| $b^1$ | $c^0$ | 1 |
| $b^1$ | $c^1$ | 100 |

# Markov Networks

- **Undirected** graphical models



- Factors/Potential-functions defined over cliques

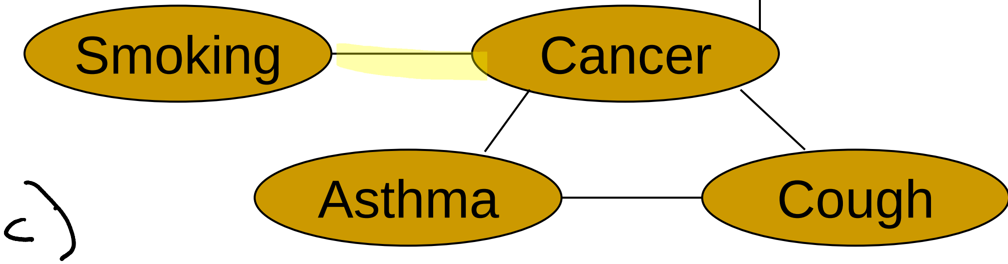
$$P(x) = \frac{1}{Z} \prod_c \Phi_c(x_c)$$

$$Z = \sum_x \prod_c \Phi_c(x_c)$$

| **Smoking** | **Cancer** | **Φ(S,C)** |
|---|---|---|
| F | F | 4.5 |
| F | T | 4.5 |
| T | F | 2.7 |
| T | T | 4.5 |

# Markov Networks :log-linear model

$$P(x) = \frac{1}{Z} \prod_c \Phi_c(x_c)$$



- Log-linear model:

each $\Phi_c(x_c) = e^{w_c f_c(x_c)}$

$$w_1 = 0.51$$

$$f_1(\text{Smoking, Cancer}) = \begin{cases} 1 & \text{if } \neg \text{Smoking} \vee \text{Cancer} \\ 0 & \text{otherwise} \end{cases}$$

| Smoking | Cancer | |
|---|---|---|
| 1 | 1 | $e^{.51}$ |
| 1 | 0 | $e^{0}$ |
| 0 | 1 | $e^{.51}$ |
| 0 | 0 | $e^{.51}$ |

$$P(x) = \frac{1}{Z} \exp\left( \sum_i \boxed{w_i} \boxed{f_i(x_i)} \right)$$

Weight of Feature *i*

Feature *i*

# Lecture Overview

- Statistical Relational Models (for us aka Hybrid)
- Recap Markov Networks
- **Markov Logic**

# Markov Logic: Intuition(1)

- A logical KB is a set of **hard constraints** on the set of possible worlds

$$\hat{f} \quad \boxed{\forall x \; Smokes(x) \Rightarrow Cancer(x)}$$

CONSTANT INDIVIDUALS = {a, b}

$$\left.\begin{array}{l} Smokes(a)=T \\ Cancer(a)=F \\ Smokes(b)=F \\ Cancer(b)=F \end{array}\right\} \hat{w}$$

in FOL $\hat{w}$ is...

A. possible    B. impossible

C. cannot tell

# Markov Logic: Intuition(1)

- A logical KB is a set of **hard constraints** on the set of possible worlds

$\hat{f}$ $\forall x\ Smokes(x) \Rightarrow Cancer(x)$

INDIVIDUALS = {a, b}

$\left.\begin{array}{l} Smokes(a) = T \\ Cancer(a) = F \\ Smokes(b) = F \\ Cancer(b) = F \end{array}\right\} \hat{w}$

in FOL $\hat{w}$ is impossible

- Let's make them **soft constraints**: When a world violates a formula, the world becomes less probable, not impossible

If $\hat{f}$ is True → $P(\hat{w})$ decreases

False in $\hat{w}$ → $P(\hat{w})$ increases

# Markov Logic: Intuition (2)

- The more formulas in the KB a possible world satisfies the more it should be likely
- Give each formula a **weight**
- By design, if a possible world satisfies a formula its **log probability** should go up proportionally to the formula weight.

$$\log(\mathrm{P(world)}) \propto \left(\sum \text{weights of formulas it satisfies}\right)$$

$$\mathrm{P(world)} \propto \exp\left(\sum \text{weights of formulas it satisfies}\right)$$

# Markov Logic: Definition

- A Markov Logic Network (MLN) is
  - a set of pairs (F, w) where
    - F is a **formula** in first-order logic
    - w is a **real number**
  - Together with a set C of **constants**,
- It defines a **Markov network** with
  - One *binary node* for each **grounding** of each **predicate** in the MLN
  - One *feature/factor* for each **grounding** of each **formula** F in the MLN, with the corresponding weight w

**Grounding**: substituting vars with constants

# (not required)consider Existential and functions

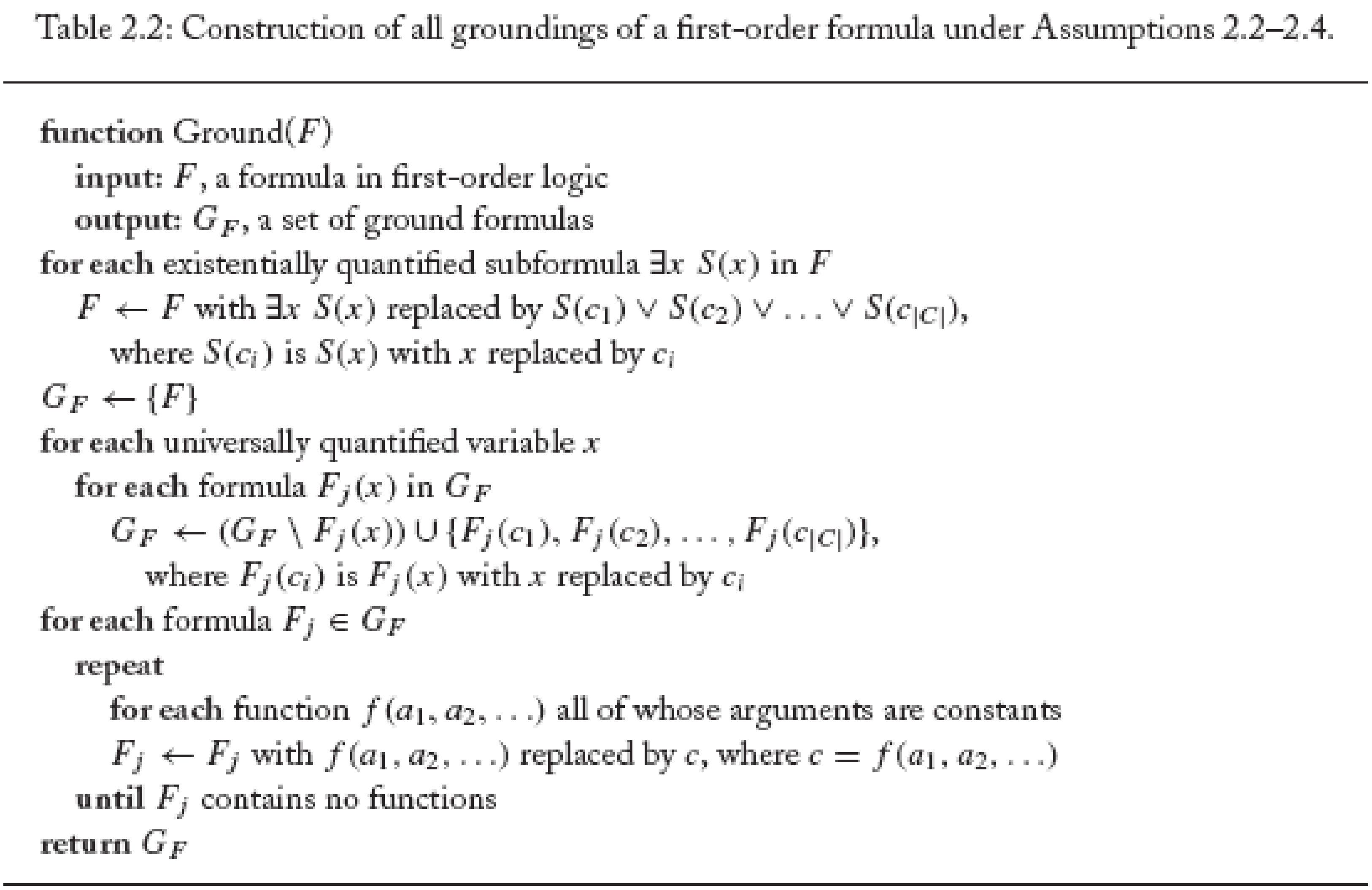


Table 2.2: Construction of all groundings of a first-order formula under Assumptions 2.2–2.4.

```
function Ground($F$)
   input: $F$, a formula in first-order logic
   output: $G_F$, a set of ground formulas
for each existentially quantified subformula $\exists x\ S(x)$ in $F$
   $F \leftarrow F$ with $\exists x\ S(x)$ replaced by $S(c_1) \vee S(c_2) \vee \ldots \vee S(c_{|C|})$,
      where $S(c_i)$ is $S(x)$ with $x$ replaced by $c_i$
$G_F \leftarrow \{F\}$
for each universally quantified variable $x$
   for each formula $F_j(x)$ in $G_F$
      $G_F \leftarrow (G_F \setminus F_j(x)) \cup \{F_j(c_1), F_j(c_2), \ldots, F_j(c_{|C|})\}$,
         where $F_j(c_i)$ is $F_j(x)$ with $x$ replaced by $c_i$
for each formula $F_j \in G_F$
   repeat
      for each function $f(a_1, a_2, \ldots)$ all of whose arguments are constants
      $F_j \leftarrow F_j$ with $f(a_1, a_2, \ldots)$ replaced by $c$, where $c = f(a_1, a_2, \ldots)$
   until $F_j$ contains no functions
return $G_F$
```

# Example: Friends & Smokers

Smoking causes cancer.

Friends have similar smoking habits.

# Example: Friends & Smokers

$$\forall x \; Smokes(x) \Rightarrow Cancer(x)$$

$$\forall x, y \; Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$$

# Example: Friends & Smokers

|  |  |
|---|---|
| 1.5 | $\forall x\ Smokes(x) \Rightarrow Cancer(x)$ |
| 1.1 | $\forall x, y\ Friends(x, y) \Rightarrow \left(Smokes(x) \Leftrightarrow Smokes(y)\right)$ |

# Example: Friends & Smokers

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Two constants: **Anna** (A) and **Bob** (B)

# MLN nodes

| | |
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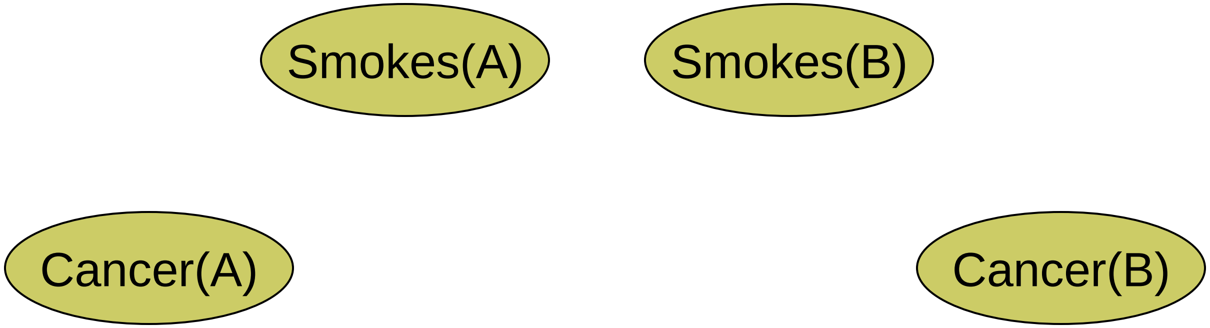
- One *binary node* for each grounding of each predicate in the MLN

**Grounding**: substituting vars with constants



Smokes(A)

Smokes(B)

Cancer(A)

Cancer(B)

- Any nodes missing?

# MLN nodes (complete)

| | |
|---|---|
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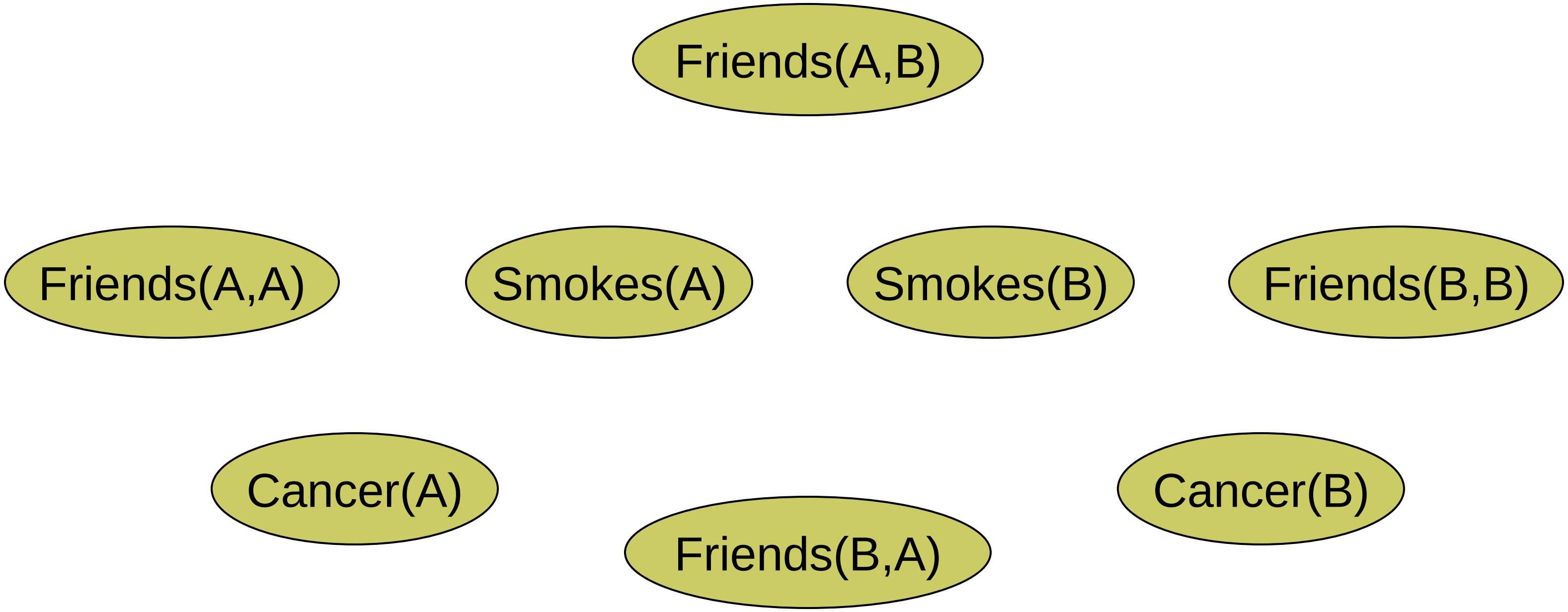

# MLN features

| | |
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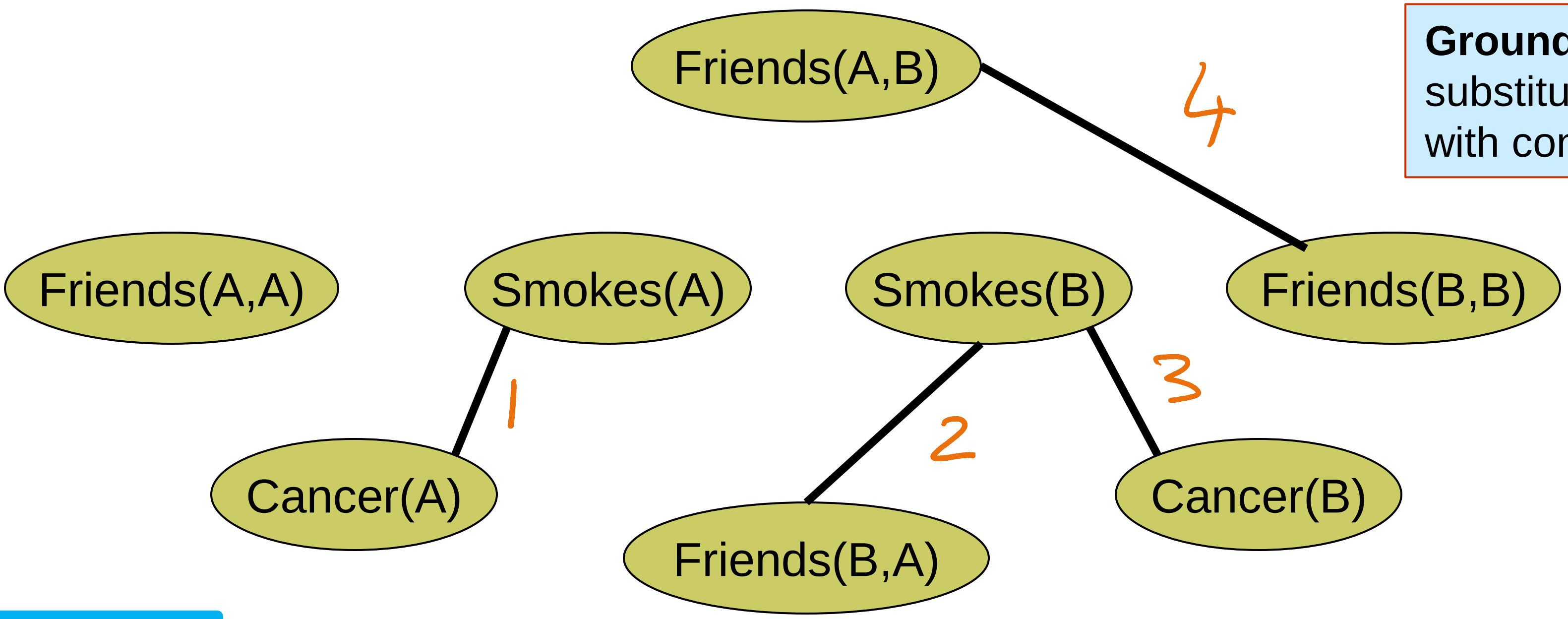
Edge between two nodes iff the corresponding ground predicates appear together in at least one grounding of one formula



**Grounding**: substituting vars with constants

Which edge should not be there?

A. 1 B. 2 C. 3 D. 4

# MLN features

|  |  |
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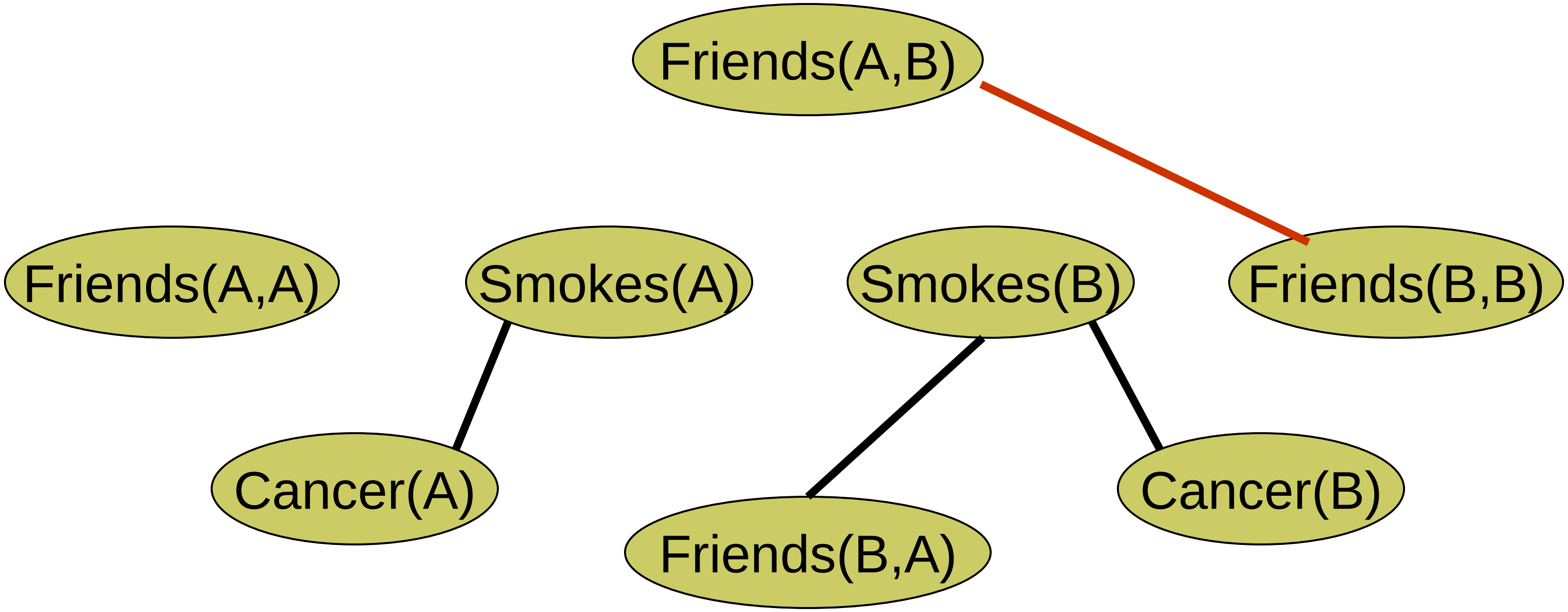

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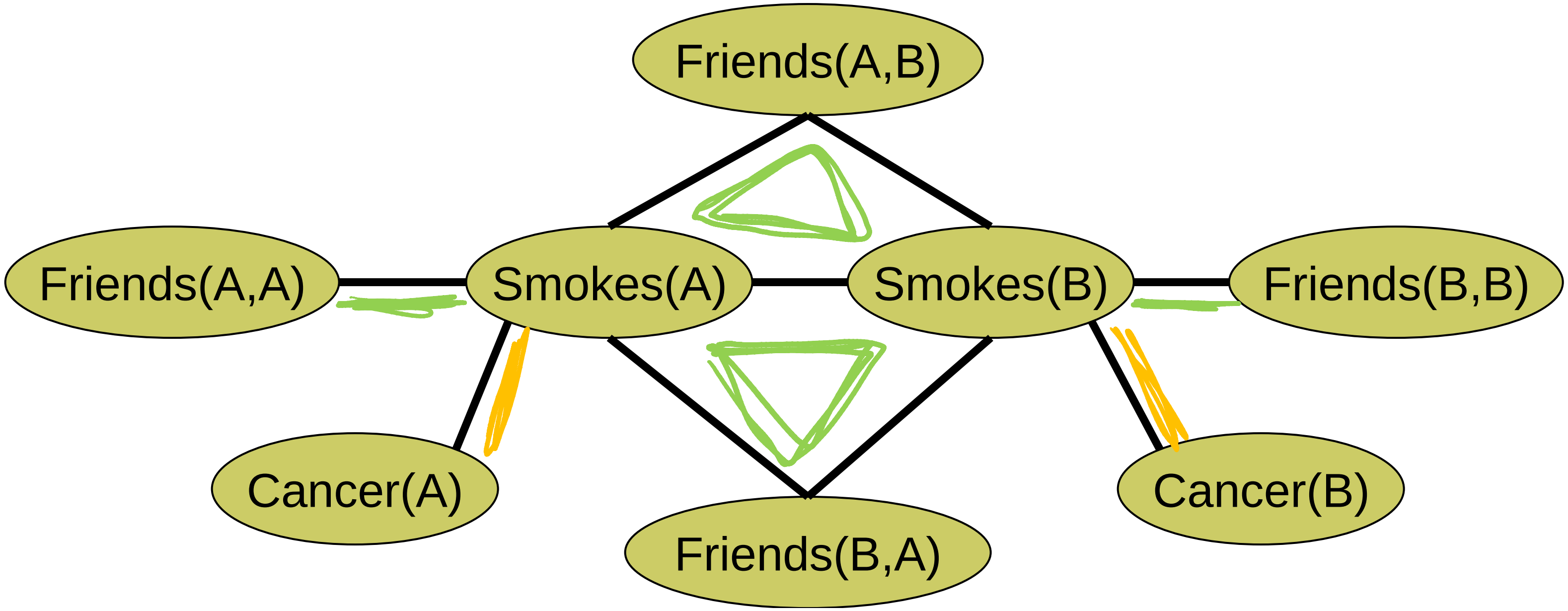

One *feature/factor* for each **grounding** of each **formula** F in the MLN

# MLN: parameters

- For each formula *i* we have a factor

$$\Phi_i(pw) = e^{w_i f_i(pw)} \leftarrow \text{possible world}$$

$w_i$ weight of formula

$$f_i(pw) = \begin{cases} 1 & \text{when formula is true in } pw \\ 0 & \text{otherwise} \end{cases}$$

$$1.5 \quad \forall x\ Smokes(x) \Rightarrow Cancer(x)$$

$$f(\text{Smokes}(x),\ \text{Cancer}(x)) = \begin{cases} 1 & \text{if } \text{Smokes}(x) \Rightarrow \text{Cancer}(x) \\ 0 & \text{otherwise} \end{cases}$$

$pw_1$ . . . . . .

Smokes(A) T

Cancer(A) F $e^0 = 1$

$pw_2$ . . . . .

Smokes(A) T

Cancer(A) T $e^{1.5}$

# MLN: prob. of possible world

| | |
|---|---|
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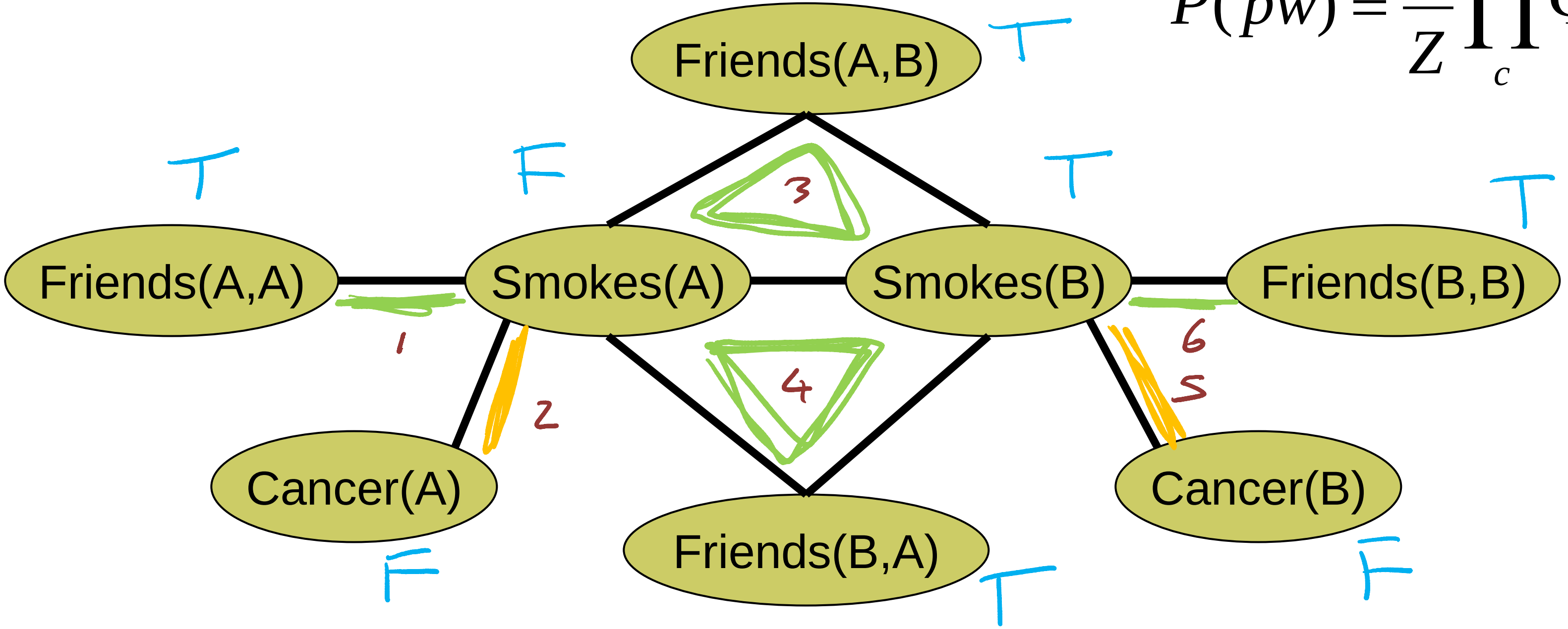
$$P(pw) = \frac{1}{Z}\prod_c \Phi_c(pw_c)$$



$$P(pw) = \left(\underset{1}{e^{1.1}} * \underset{6}{e^{1.1}} * \underset{3}{e^{0}} * \underset{4}{e^{0}} * \underset{2}{e^{1.5}} * \underset{5}{e^{0}}\right) / Z$$

# MLN: prob. of possible world

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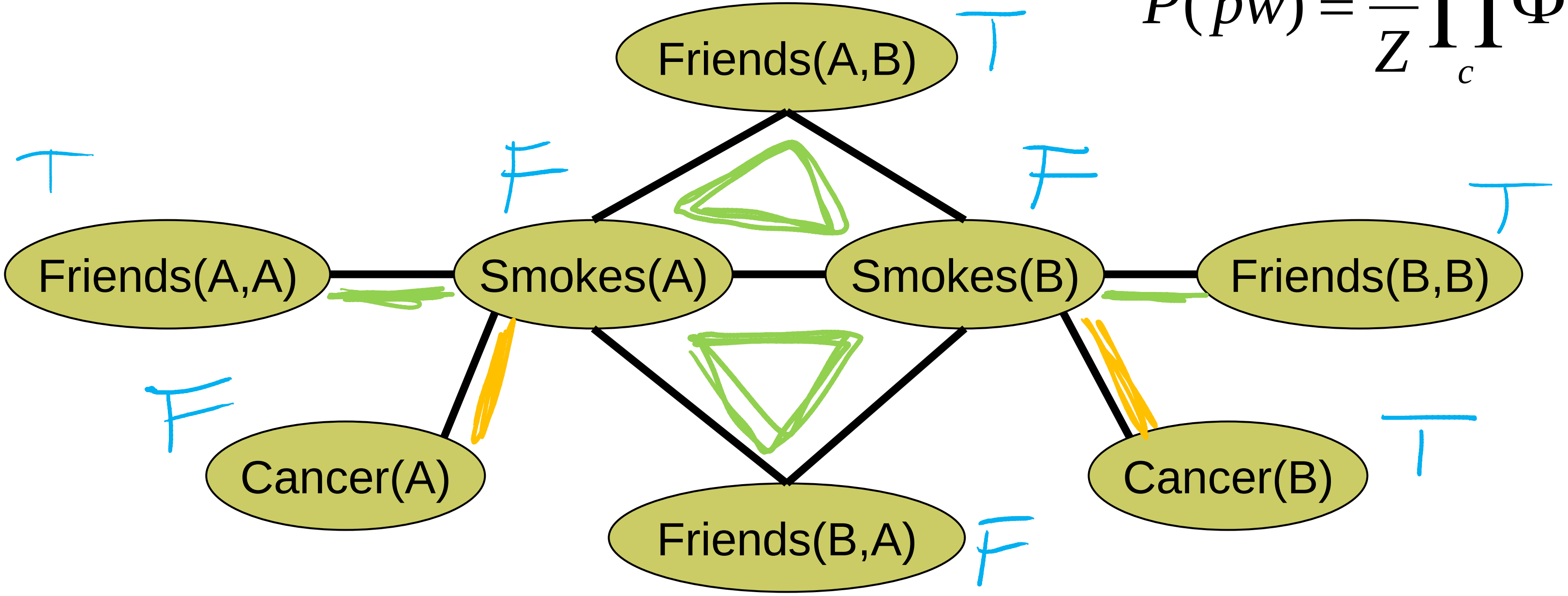


$$\left(e^{1,1} * e^{1.1} * e^{1.1} * e^{1.1} * e^{1.5} * e^{1.5}\right) / Z$$

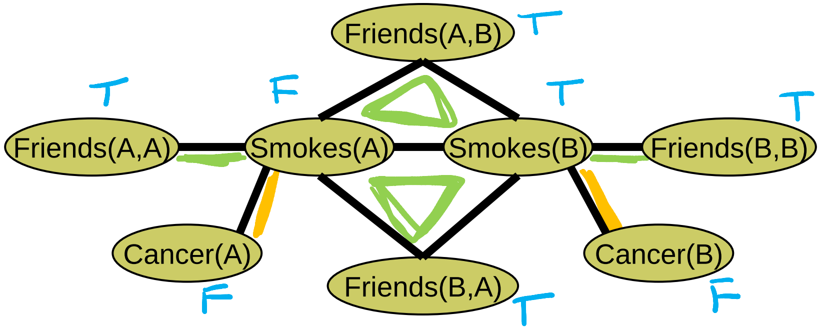
# MLN: prob. Of possible world

- Probability of a world $pw$:

$$P(pw) = \frac{1}{Z}\exp\left(\sum_i \boxed{w_i}\,\boxed{n_i(pw)}\right)$$

Weight of formula *i*

No. of true groundings of formula *i* in *pw*



$$P(pw) = (\underbrace{e^{1.1} * e^{1.1}}_{n_2(pw) = 2} * e^{0} * e^{0} * \underbrace{e^{1.5}}_{n_1(pw) = 1} * e^{0})/Z$$

# Learning Goals for today's class

**You can:**

- Describe the intuitions behind the design of a Markov Logic
- Define and Build a Markov Logic Network
- Justify and apply the formula for computing the probability of a possible world

# Next class on Wed

**Markov Logic**

**-relation to FOL**

**- Inference (MAP and Cond. Prob)**

Assignment-4 posted, due on Dec 2

# Relation to First-Order Logic

- Example pag 17
- Infinite weights ⇒ First-order logic
- Satisfiable KB, positive weights ⇒ Satisfying assignments = Modes of distribution
- Markov logic allows contradictions between formulas

# Relation to Statistical Models

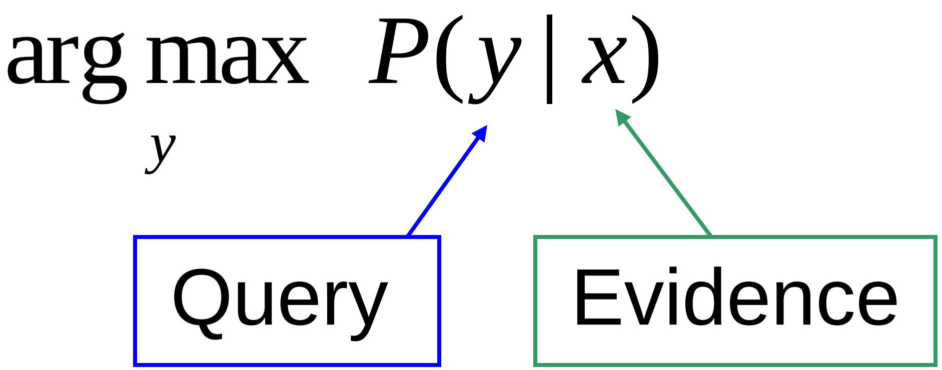
- Special cases:
  - Markov networks
  - Markov random fields
  - Bayesian networks
  - Log-linear models
  - Exponential models
  - Max. entropy models
  - Gibbs distributions
  - Boltzmann machines
  - Logistic regression
  - Hidden Markov models
  - Conditional random fields

- Obtained by making all predicates zero-arity

- Markov logic allows objects to be interdependent (non-i.i.d.)

# MAP Inference

- **Problem:** Find most likely state of world given evidence



$$\arg\max_{y} \; P(y \mid x)$$

# MAP Inference

- **Problem:** Find most likely state of world given evidence

$$\arg\max_{y} \frac{1}{Z_x} \exp\left( \sum_i w_i n_i(x, y) \right)$$

# MAP Inference

- **Problem:** Find most likely state of world given evidence

$$\arg\max_{y} \sum_{i} w_i n_i(x, y)$$

# MAP Inference

- **Problem:** Find most likely state of world given evidence

$$\arg\max_{y} \sum_{i} w_i n_i(x, y)$$

- This is just the weighted MaxSAT problem
- Use weighted SAT solver
(e.g., MaxWalkSAT [Kautz et al., 1997]

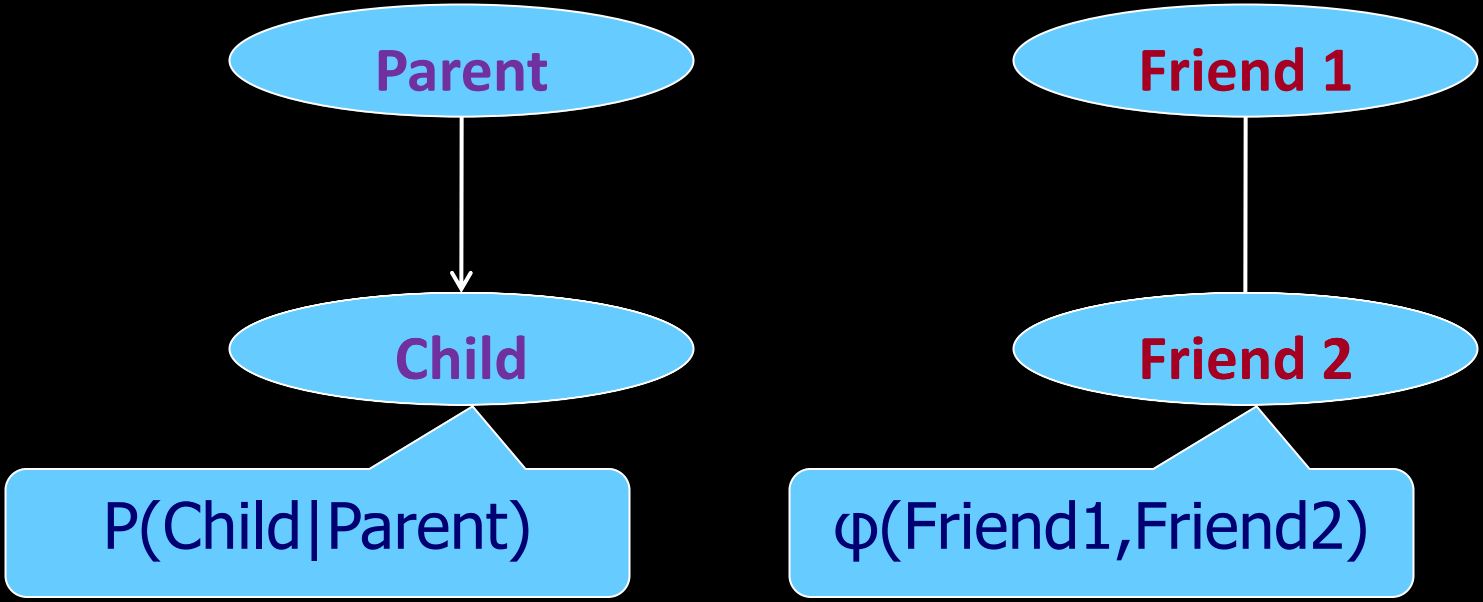
# The MaxWalkSAT Algorithm

```
for i ← 1 to max-tries do
    solution = random truth assignment
    for j ← 1 to max-flips do
        if ∑ weights(sat. clauses) > threshold then
            return solution
        c ← random unsatisfied clause
        with probability p
            flip a random variable in c
        else
            flip variable in c that maximizes
                ∑ weights(sat. clauses)
return failure, best solution found
```

# Computing Probabilities

- P(Formula|MLN,C) = ?
- Brute force: Sum probs. of worlds where formula holds
- MCMC: Sample worlds, check formula holds
- P(Formula1|Formula2,MLN,C) = ?
- Discard worlds where Formula 2 does not hold
- In practice: More efficient alternatives

# Directed Models vs. Undirected Models



# Undirected Probabilistic Logic Models

- Upgrade undirected propositional models to relational setting
  - Markov Nets → Markov Logic Networks
  - Markov Random Fields → Relational Markov Nets
  - Conditional Random Fields → Relational CRFs

# Markov Logic Networks (Richardson & Domingos)

- Soften logical clauses
  - A first-order clause is a **hard** constraint on the world

$$\forall x, person(x) \rightarrow \exists y, person(y), father(x,y)$$

  - Soften the constraints so that when a constraint is violated, the world is less probably, not impossible

$$w : friends(x,y) \wedge smokes(x) \rightarrow smokes(y)$$

  - Higher weight ⇒ Stronger constraint
  - Weight of $\infty$ ⇒ first-order logic

Probability( World S ) = ( 1 / Z ) × exp { Σ weight $_i$ x **numberTimesTrue(f $_i$, S)** }

# Example: Friends & Smokers

| | |
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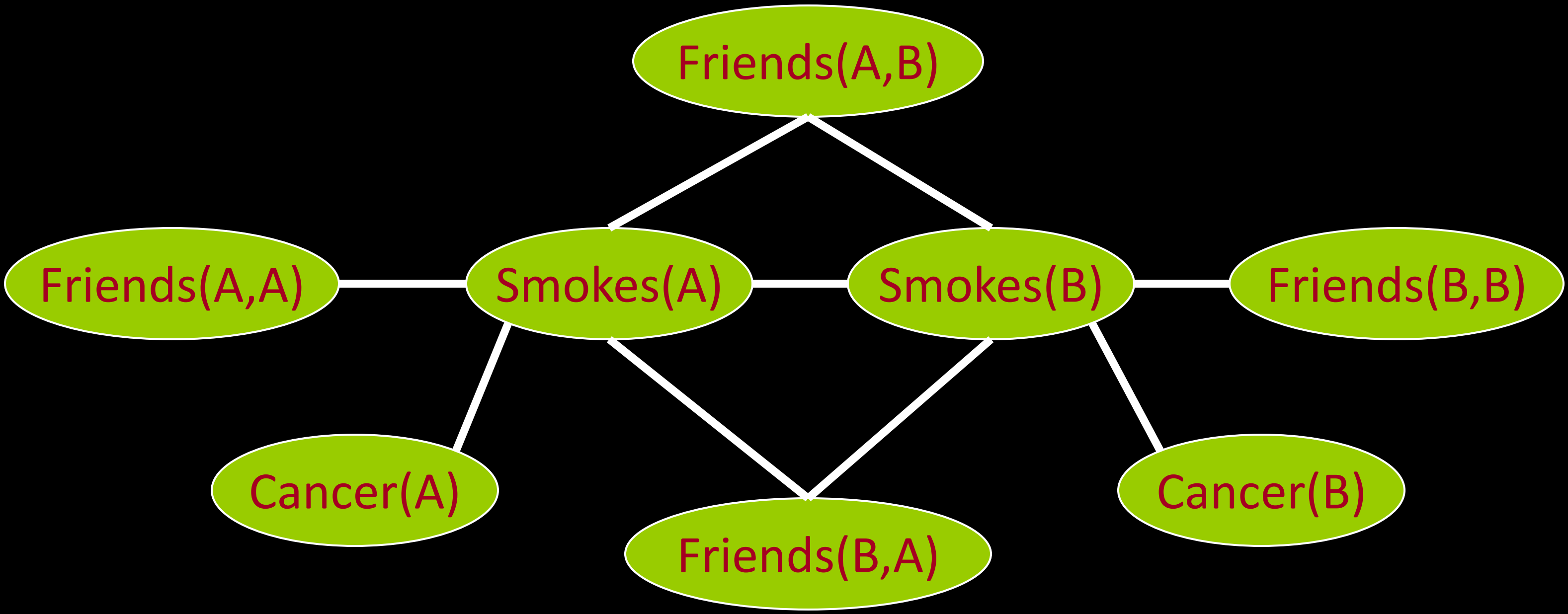

# Alphabetic Soup => Endless Possibilities

- Probabilistic Relational Models (PRM)
- Bayesian Logic Programs (BLP)
- PRISM
- Stochastic Logic Programs (SLP)
- Independent Choice Logic (ICL)
- Markov Logic Networks (MLN)
- Relational Markov Nets (RMN)
- CLP-BN
- Relational Bayes Nets (RBN)
- Probabilistic Logic Progam (PLP)
- ProbLog

….

- Web data (**web**)
- Biological data (**bio**)
- Social Network Analysis (**soc**)
- Bibliographic data (**cite**)
- Epidimiological data (**epi**)
- Communication data (**comm**)
- Customer networks (**cust**)
- Collaborative filtering problems (**cf**)
- Trust networks (**trust**)

…

Fall 2003– Dietterich @ OSU, Spring 2004 –Page @ UW, Spring 2007-Neville @ Purdue, Fall 2008 – Pedro @ CMU

# Recent Advances in SRL Inference

- Preprocessing for Inference
  - FROG – Shavlik & Natarajan (2009)
- Lifted Exact Inference
  - Lifted Variable Elimination – Poole (2003), Braz et al(2005) Milch et al (2008)
  - Lifted VE + Aggregation – Kisynski & Poole (2009)
- Sampling Methods
  - MCMC techniques – Milch & Russell (2006)
  - Logical Particle Filter – Natarajan et al (2008), ZettleMoyer et al (2007)
  - Lazy Inference – Poon et al (2008)
- Approximate Methods
  - Lifted First-Order Belief Propagation – Singla & Domingos (2008)
  - Counting Belief Propagation – Kersting et al (2009)
  - MAP Inference – Riedel (2008)
- Bounds Propagation
  - Anytime Belief Propagation – Braz et al (2009)

# Conclusion

- Inference is the key issue in several SRL formalisms
- FROG - Keeps the count of unsatisfied groundings
  - ☐ Order of Magnitude reduction in number of groundings
  - ☐ Compares favorably to Alchemy in different domains
- Counting BP - BP + grouping nodes sending and receiving identical messages
  - ☐ Conceptually easy, scaleable BP algorithm
  - ☐ Applications to challenging AI tasks
- Anytime BP – Incremental Shattering + Box Propagation
  - ☐ Only the most necessary fraction of model considered and shattered
  - ☐ Status – Implementation and evaluation