# Search: Advanced Topics

## Computer Science cpsc322, Lecture 9

*(Textbook Chpt 3.6)*

Sept, 25, 2013

# Lecture Overview

$f = c + h$

- **Recap A***
- Branch & Bound
- A* tricks
- Other Pruning

# A* advantages

i-clicker

What is a key advantage of A* ?

A. Does not need to consider the cost of the paths
B. Has a linear space complexity
C. It is often optimal
D. None of the above

both accepted

# Branch-and-Bound Search

- Biggest advantages of A*….

uses heuristics + optimal

- What is the biggest problem with A*?

space

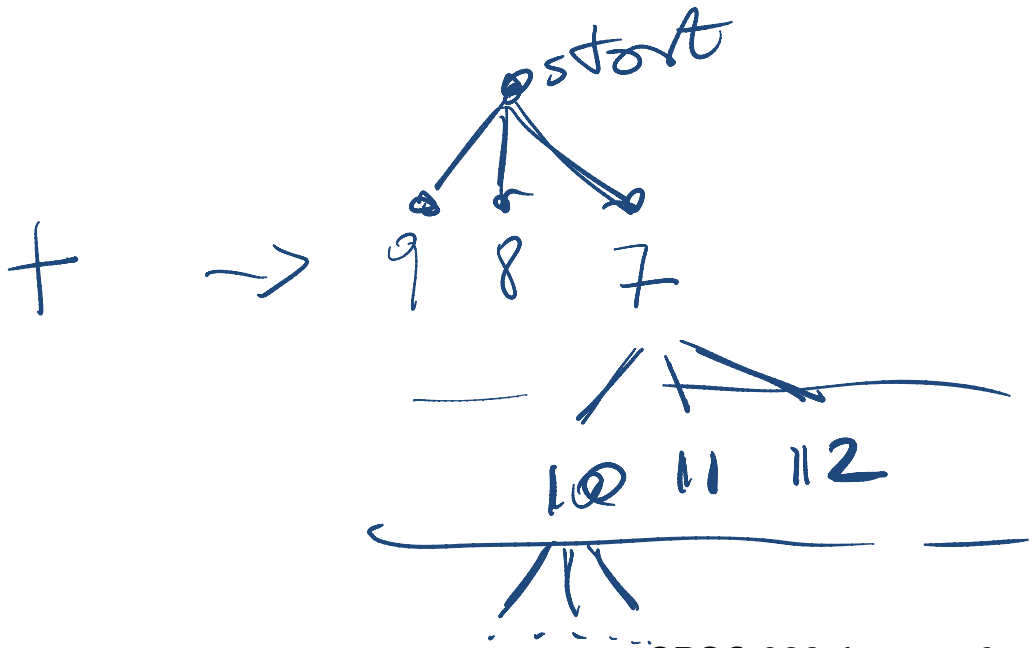
- Possible, preliminary Solution:

DFS + h

# Branch-and-Bound Search Algorithm

- Follow exactly the same search path as **depth-first search**
  - treat the frontier as a stack: expand the most-recently added path first
  - the order in which neighbors are expanded can be governed by some arbitrary node-ordering heuristic

we can use $f = c + h$



# Once this strategy has found a solution….

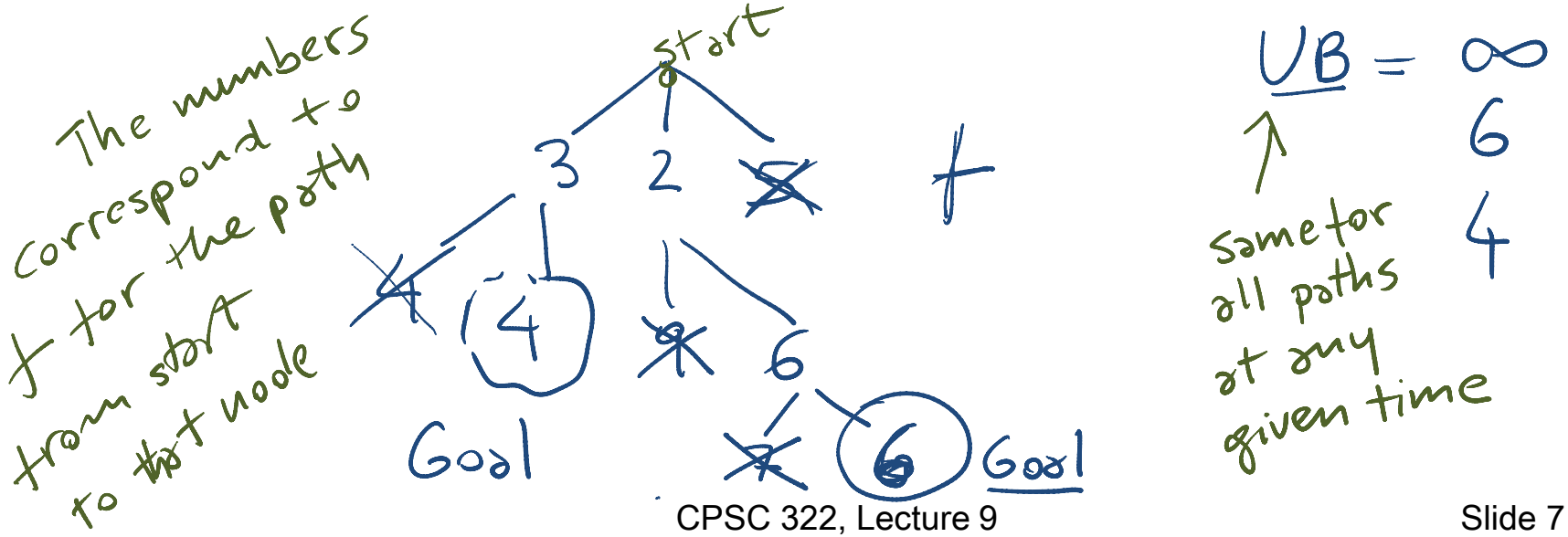
i-clicker

What should it do next ?

A. Keep running DFS, looking for deeper solutions?
B. Stop and return that solution
C. Keep searching, but only for shorter solutions
D. None of the above

# Branch-and-Bound Search Algorithm

- Keep track of a lower bound and upper bound on solution cost at each path
  - lower bound: $LB(p) = f(p) = cost(p) + h(p)$
  - upper bound: $UB =$ cost of the best solution found so far.
    - ✓ if no solution has been found yet, set the upper bound to $\infty$.
- When a path $p$ is selected for expansion:
  - if $LB(p) \geq UB$, remove $p$ from frontier without expanding it (pruning)
  - else expand $p$, adding all of its neighbors to the frontier



# Branch-and-Bound Analysis

- Complete ?

  yes  no  It depends

- Optimal ?

  yes  no  It depends

- Space complexity?

  $O(b^m)$  $O(m^b)$  $O(bm)$  $O(b+m)$

- Time complexity?

# Branch-and-Bound Analysis

- Completeness: no, for the same reasons that DFS isn't complete
  - however, for many problems of interest there are no infinite paths and no cycles
  - hence, for many problems B&B is complete
- Time complexity: $O(b^m)$
- Space complexity: $O(bm)$
  - Branch & Bound has the same space complexity as DFS
  - this is a big improvement over A*!
- Optimality: yes

# Lecture Overview

- Recap A*
- Branch & Bound
- A* tricks
- Pruning Cycles and Repeated States

# Other $A^*$ Enhancements

The main problem with $A^*$ is that it uses exponential space. Branch and bound was one way around this problem. Are there others?

- Iterative Deepening $A^*$ (IDA*)
- .....
- Memory-bounded $A^*$

# (Heuristic) Iterative Deepening – IDA*

**B & B** can still get stuck in infinite (extremely long) paths

- Search depth-first, but to a fixed depth / bound
  - if you don't find a solution, increase the depth tolerance and try again
  - depth is measured in ..... f .....

    start node $f(start) = h(start)$

  - Then update with the lowest f ..... that passed the previous bound

# Analysis of Iterative Deepening A* (IDA*)

- Complete and optimal:

yes | no | It depends

- Space complexity:

$O(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

- Time complexity:

$O(b^m)$ | $O(m^b)$ | $O(bm)$ | $O(b+m)$

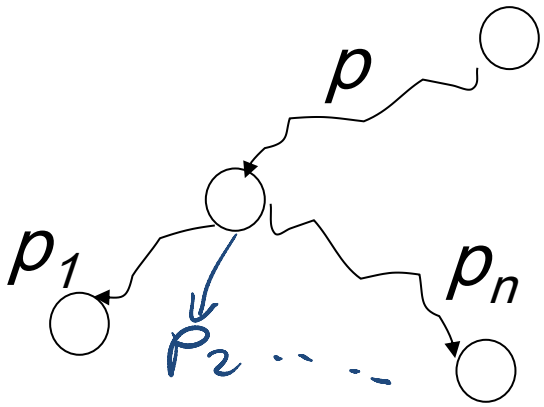
# (Heuristic) Iterative Deepening – IDA*

- Counter-intuitively, the asymptotic complexity is not changed, even though we visit paths multiple times (*go back to slides on uninformed ID*)

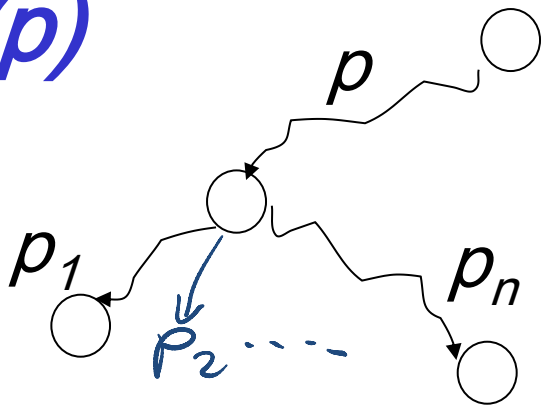
$$\left(\frac{b}{b-1}\right)^2$$

# Memory-bounded $A^*$

- Iterative deepening A* and B & B use a tiny amount of memory
- **what if we've got more memory to use?**
- keep as much of the fringe in memory as we can
- if we have to delete something:
  - delete the worst paths (with highest f)
  - ``back them up'' to a common ancestor



# MBA*: Compute New *h(p)*




A $$\text{New } h(p) = \min\left( \max_i [(\text{cost}(p_i) - \text{cost}(p)) + h(p_i)], \text{Old } h(p) \right)$$

B $$\text{New } h(p) = \max\left( \min_i [(\text{cost}(p_i) - \text{cost}(p)) + h(p_i)], \text{Old } h(p) \right)$$

C $$\text{New } h(p) = \max\left( \max_i [(\text{cost}(p_i) - \text{cost}(p)) + h(p_i)], \text{Old } h(p) \right)$$

# Memory-bounded $A^*$

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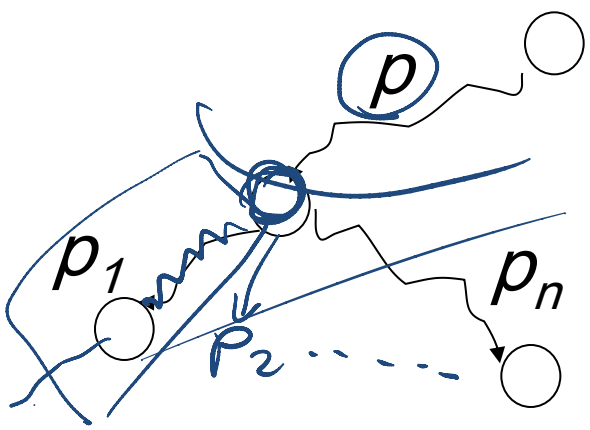
min max

max min




$$h(p) = \max\left(\min\left[\text{cost}(p, p_i) - \text{cost}(p)\right] + h(p_i)\right)$$

original $h(p)$

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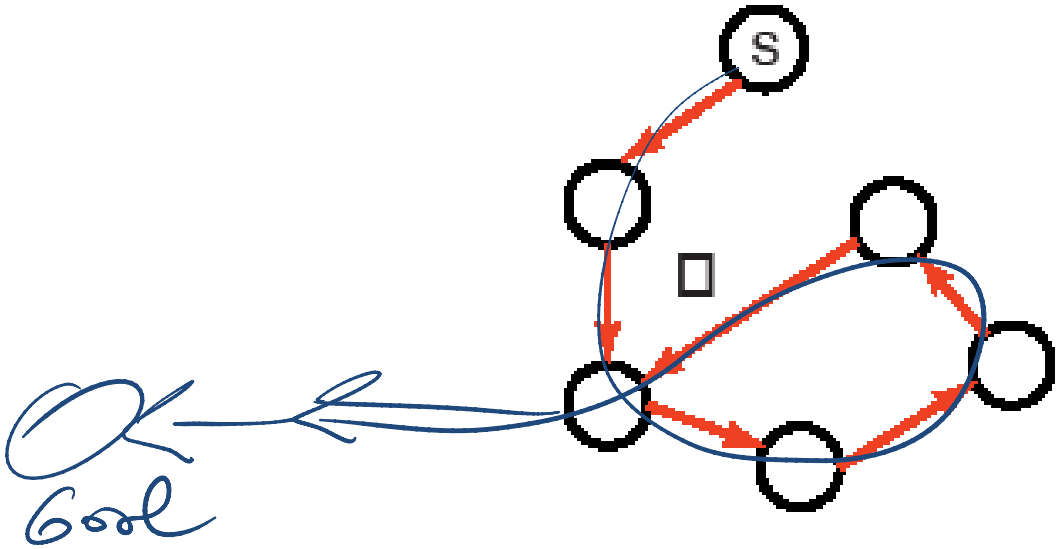
# Cycle Checking

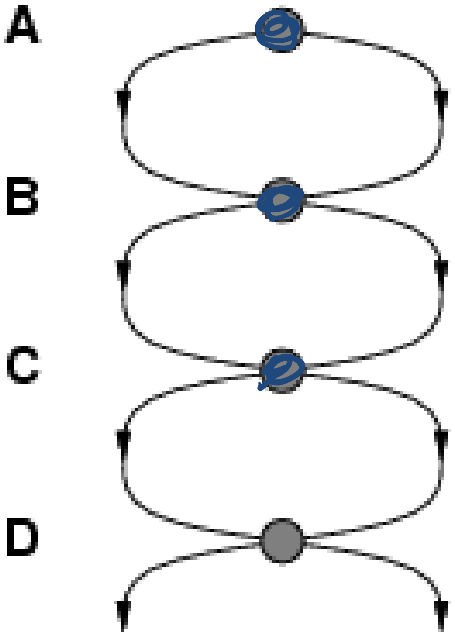


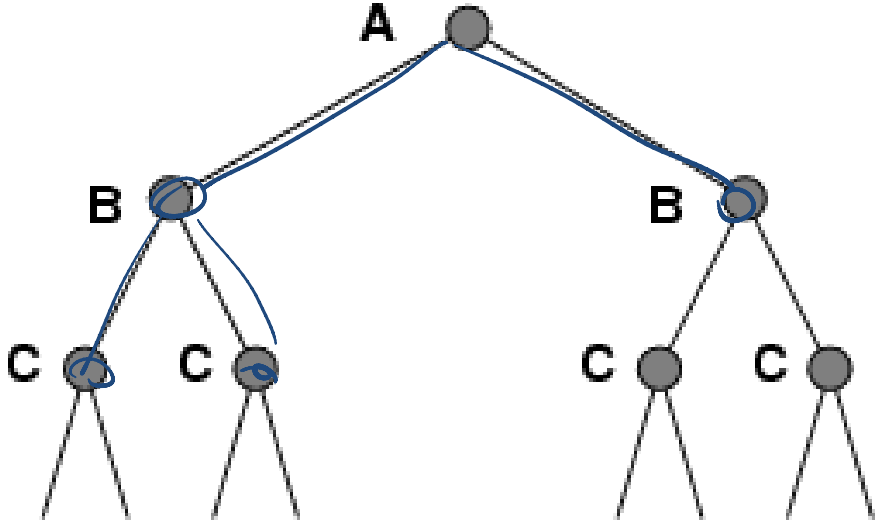

You can prune a path that ends in a node already on the path. This pruning cannot remove an optimal solution.

- The time is ..... in path length.

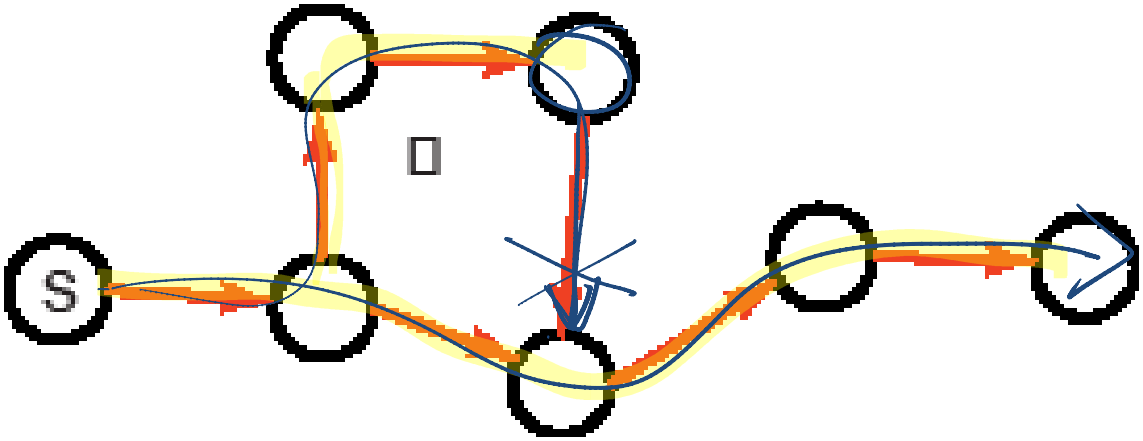
# Repeated States / Multiple Paths

Failure to detect repeated states can turn a linear problem into an exponential one!



# Multiple-Path Pruning

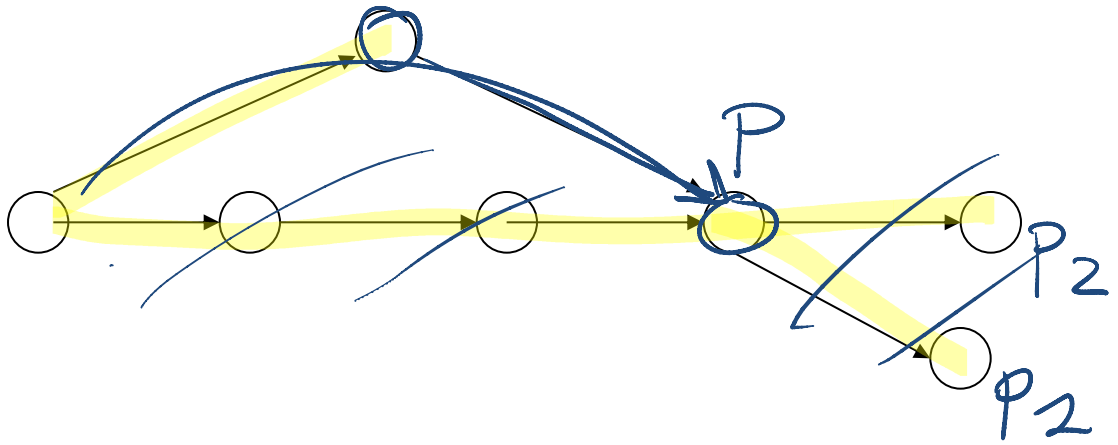



- You can prune a path to node ***n*** that you have already found a path to
- (if the new path is longer – more costly).

# Multiple-Path Pruning & Optimal Solutions

Problem: what if a subsequent path to *n* is shorter than the first path to *n* ?

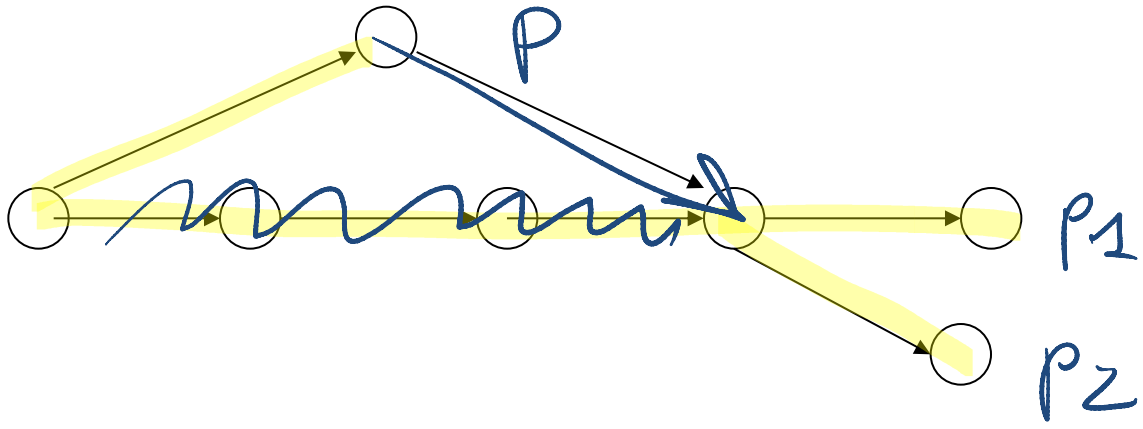
- You can remove all paths from the frontier that use the longer path. (as these can't be optimal)



# Multiple-Path Pruning & Optimal Solutions

Problem: what if a subsequent path to *n* is shorter than the first path to *n* ?

- You can change the initial segment of the paths on the frontier to use the shorter path.



# Learning Goals for today's class

- Define/read/write/trace/debug different search algorithms
  - With / Without cost
  - Informed / Uninformed

- Pruning cycles and Repeated States

# Next class: Fri

- Dynamic Programming
- Recap Search
- Start Constraint Satisfaction Problems (CSP)
- Chp 4.

- Start working on assignment-1 !