# Heuristic Search

## Computer Science cpsc322, Lecture 7

*(Textbook Chpt 3.6)*

Sept, 20, 2013

# Course Announcements

Marks for Assignment0: will be posted on Connect next week

Assignment1: will also be posted on Mon/Tue

If you are confused on basic search algorithm, different search strategies….. Check learning goals at the end of lectures. Work on the Practice Exercises and Please do come to office hours

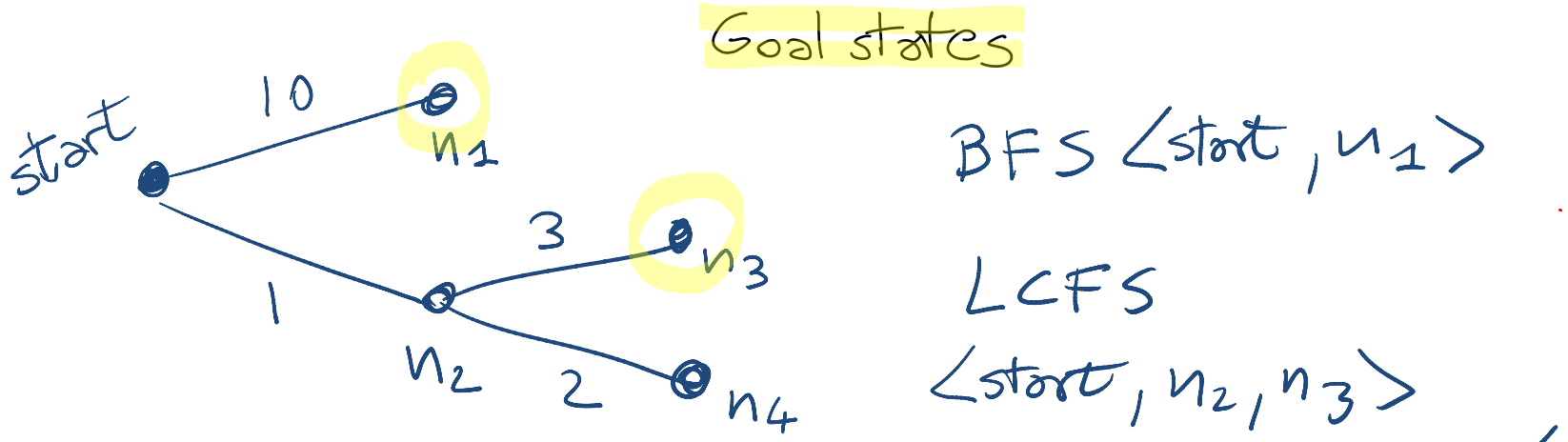
*Giuseppe* : Fri 2-3, my office CICSR 105
*Kamyar Ardekani* Mon 2-3, X150 (Learning Center)
*Tatsuro Oya* Thur 11-12, X150 (Learning Center)
*Xin Ru (Nancy) Wang* Tue 2-3, X150 (Learning Center)

# Lecture Overview

- Recap
  - Search with Costs
  - Summary Uninformed Search
- Heuristic Search

# Recap: Search with Costs

- Sometimes there are costs associated with arcs.
  - The cost of a path is the sum of the costs of its arcs.




- Optimal solution: not the one that minimizes the *number of links*, but the one that minimizes *cost*
- Lowest-Cost-First Search: expand paths from the frontier in order of their costs.

# Recap Uninformed Search

| | Complete | Optimal | Time | Space |
|---|---|---|---|---|
| DFS | N<br>Y if no cycles and finite search space | N | $O(b^m)$ | $O(mb)$ |
| BFS | Y | Y | $O(b^m)$ | $O(b^m)$ |
| IDS | Y | Y | $O(b^m)$ | $O(mb)$ |
| LCFS | Y<br>Costs > 0 | Y<br>Costs >=0 | $O(b^m)$ | $O(b^m)$ |

# Recap Uninformed Search

- Why are all these strategies called uninformed?

Because they **do not consider any information about the states (end nodes)** to decide which path to expand first on the frontier

eg

(⟨n0, n2, n3⟩ 12), (⟨n0, n3⟩ 8) , (⟨n0, n1, n4⟩ 13)

In other words, they are general they do not take into account the **specific nature of the problem**.

# Lecture Overview

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# Beyond uninformed search….

i-clicker

What information we could use to better select paths from the frontier?

A. an estimate of the distance from the last node on the path to the goal
B. an estimate of the distance from the start state to the goal
C. an estimate of the cost of the path
D. None of the above

# Heuristic Search

Uninformed/Blind search algorithms do not take into account the goal until they are at a goal node.

Often there is extra knowledge that can be used to guide the search: **an *estimate* of the distance from node *n* to a goal node.**

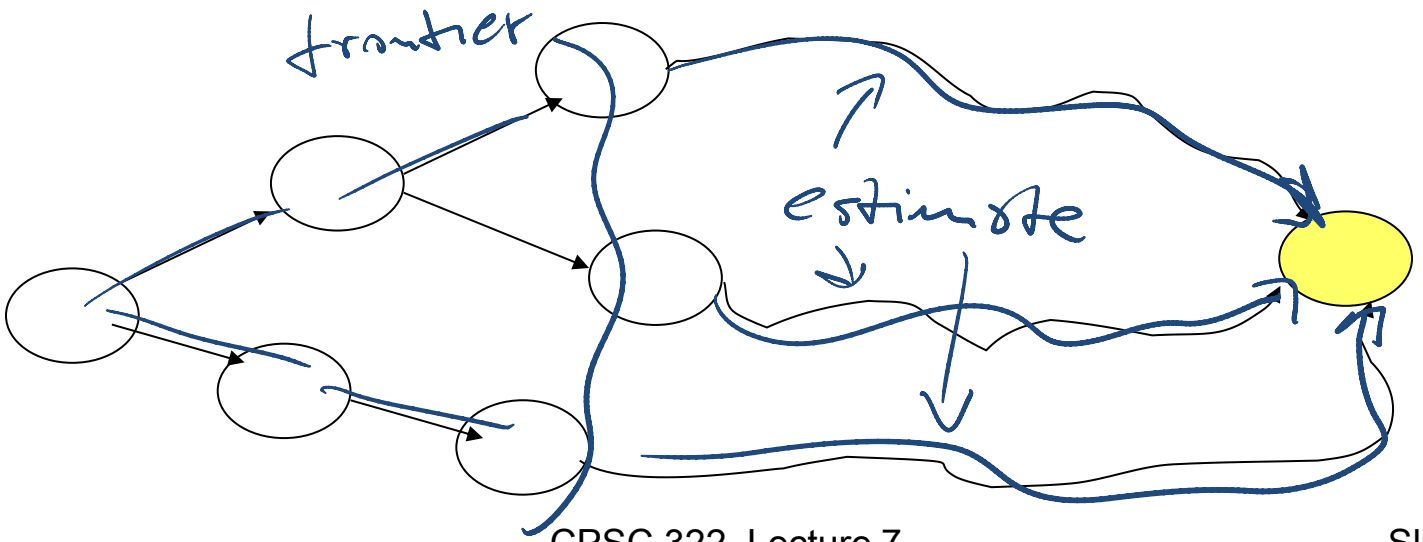
This is called a ***heuristic***

# More formally

Definition (search heuristic)

A search heuristic *h(n)* is an estimate of the cost of the shortest path from node *n* to a goal node.

- *h* can be extended to paths: $h(\langle n_0,\ldots,n_k\rangle)=h(n_k)$
- For now think of *h(n)* as only using readily obtainable information (that is easy to compute) about a node.



# More formally (cont.)

Definition (**admissible heuristic**)

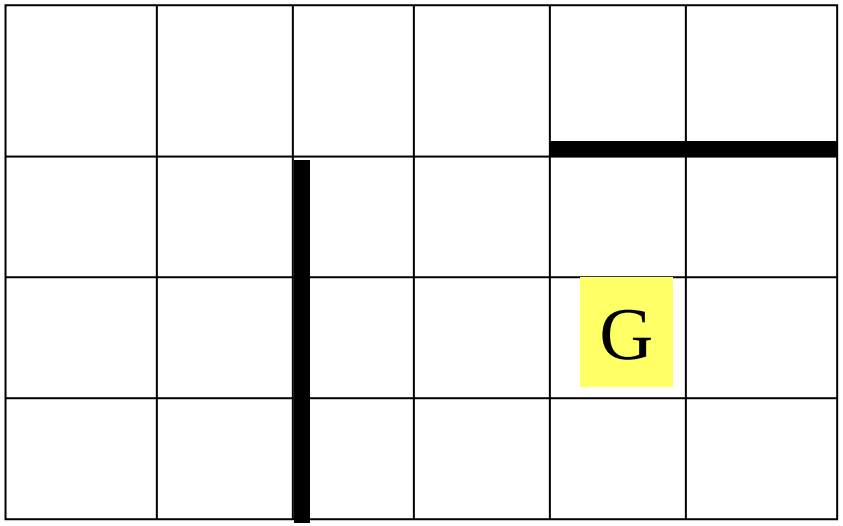
A search heuristic *h(n)* is admissible if it is never an overestimate of the cost from *n* to a goal.

- There is never a path from *n* to a goal that has path cost less than *h(n)*.
- another way of saying this: *h(n)* is a lower bound on the cost of getting from *n* to the nearest goal.

# Example Admissible Heuristic Functions

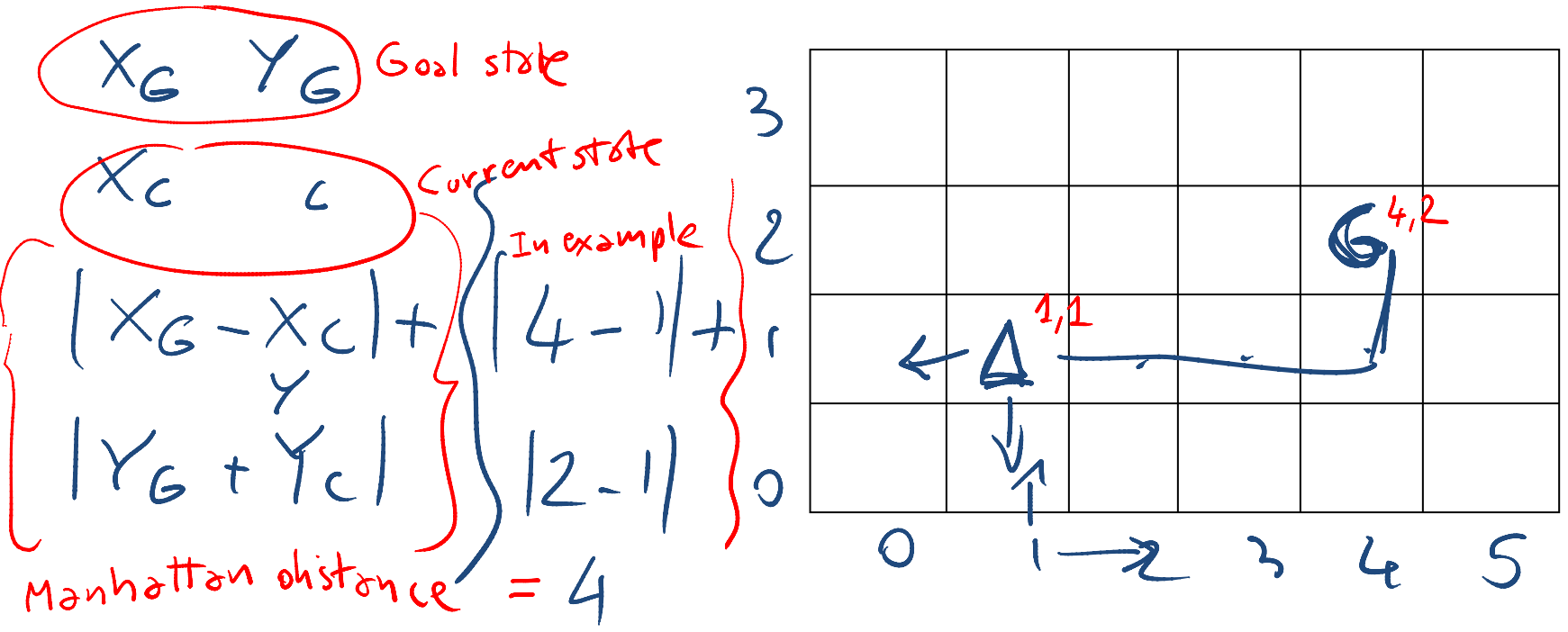
**Search problem:** robot has to find a route from start location to goal location on a grid (discrete space with obstacles)

**Final cost** (quality of the solution) is the number of steps



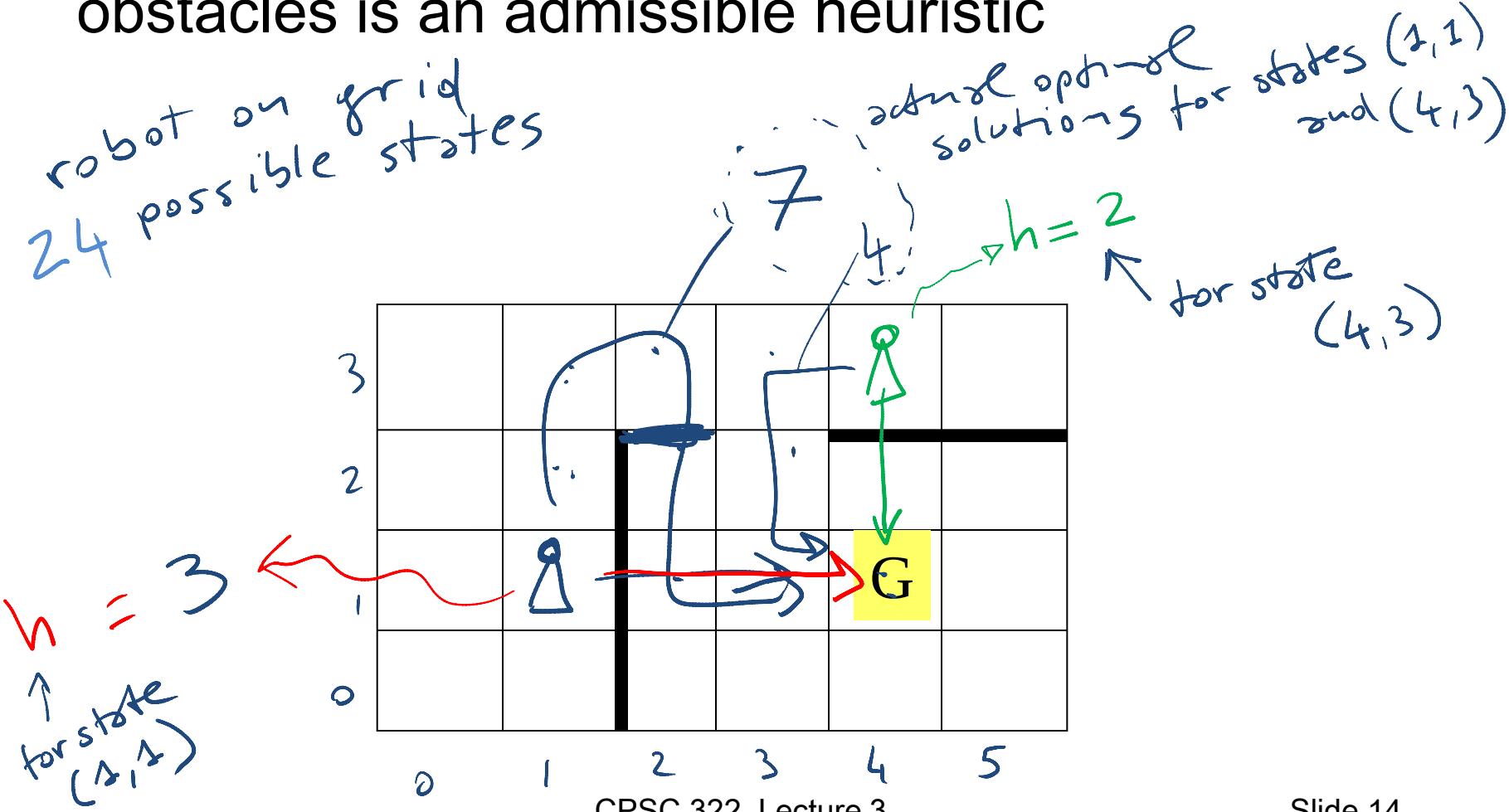
# Example Admissible Heuristic Functions

If no obstacles, cost of optimal solution is…



# Example Admissible Heuristic Functions

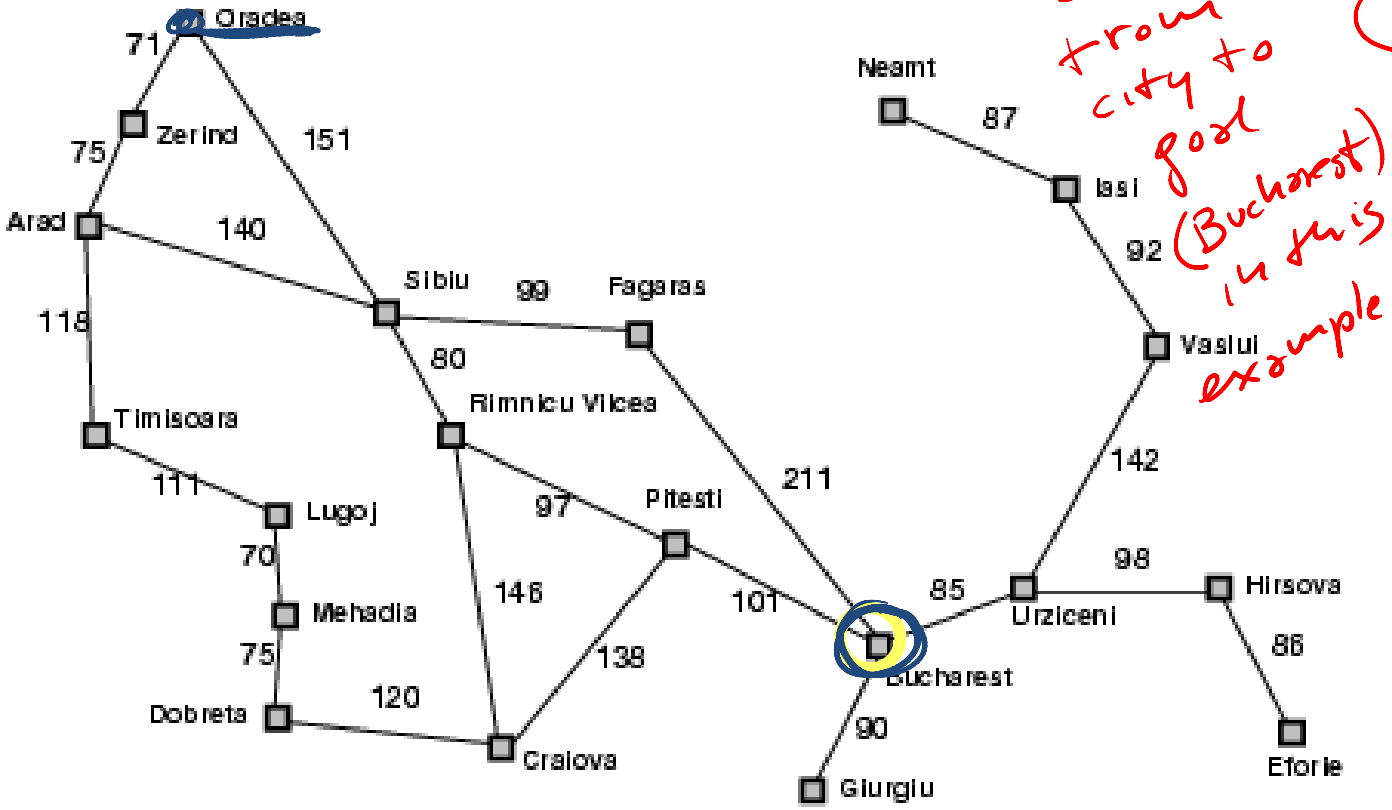
If there are obstacle, the optimal solution without obstacles is an admissible heuristic



# Example Admissible Heuristic Functions

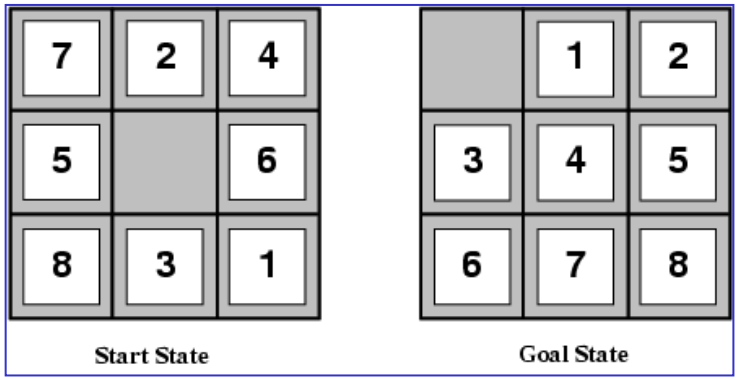
- Similarly, If the nodes are points on a Euclidean plane and the cost is the distance, we can use the straight-line distance from *n* to the closest goal as the value of *h(n)*.

always <= than any road from city to goal (Bucharest) in this example




| Straight–line distance to Bucharest | |
|---|---|
| Arad | 366 |
| Bucharest | 0 |
| Craiova | 160 |
| Dobreta | 242 |
| Eforie | 161 |
| Fagaras | 176 |
| Giurgiu | 77 |
| Hirsova | 151 |
| Iasi | 226 |
| Lugoj | 244 |
| Mehadia | 241 |
| Neamt | 234 |
| Oradea | 380 |
| Pitesti | 10 |
| Rimnicu Vilcea | 193 |
| Sibiu | 253 |
| Timisoara | 329 |
| Urziceni | 80 |
| Vaslui | 199 |
| Zerind | 374 |

# Heuristic Function for 8-puzzle

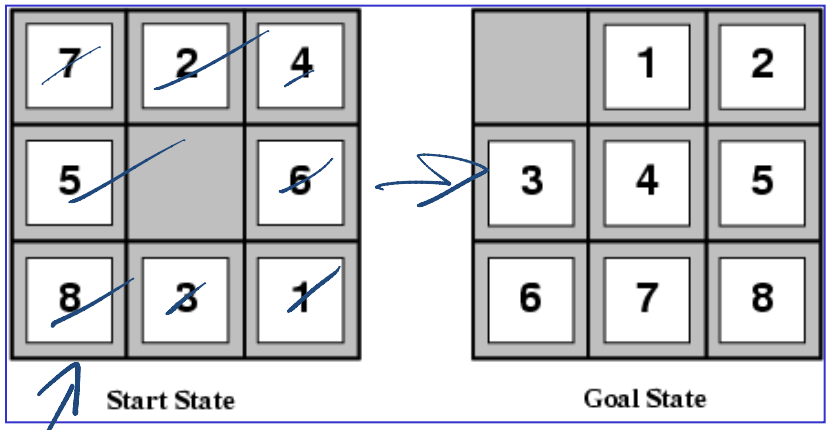



A reasonable heuristics for the 8-puzzle is?

A. Number of misplaced tiles plus number of correctly place tiles
B. Number of misplaced tiles
C. Number of correctly placed tiles
D. None of the above

# Example Heuristic Functions

- In the 8-puzzle, we can use the number of misplaced tiles

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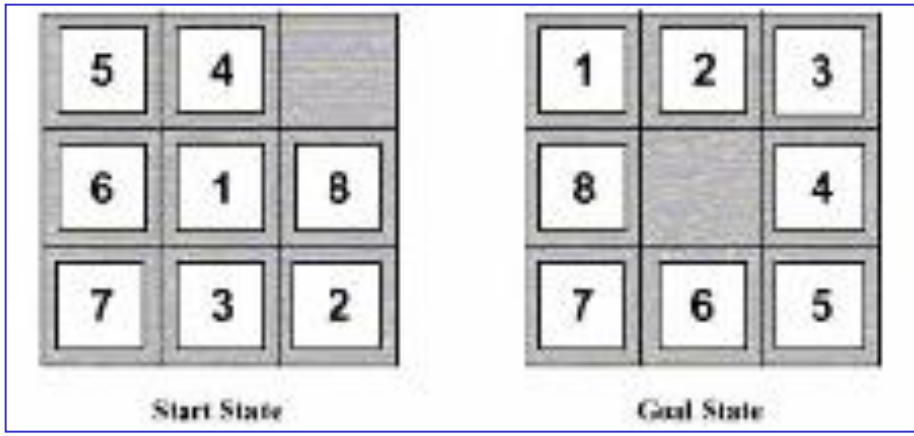


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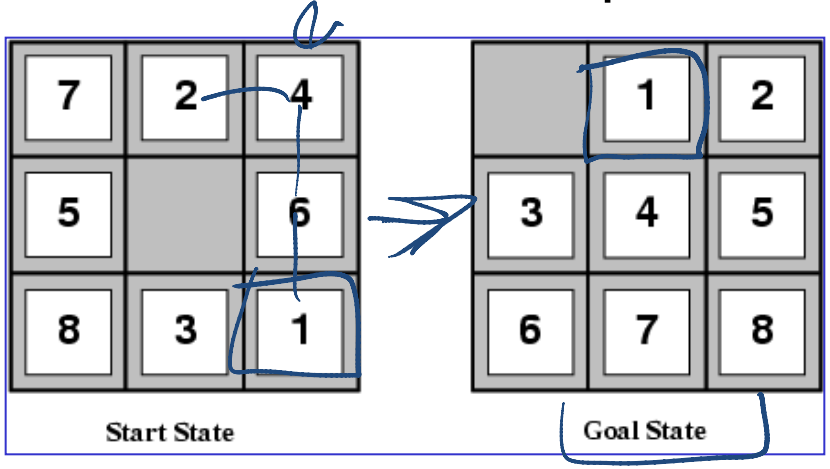
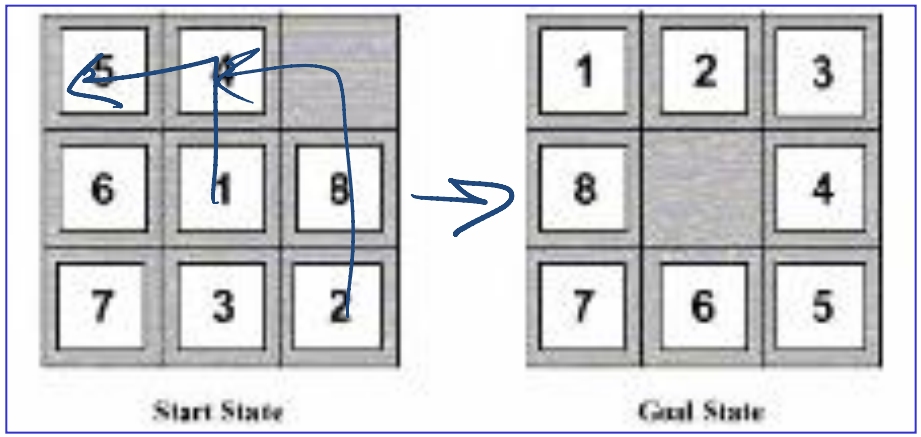
# Example Heuristic Functions (2)

- Another one we can use the number of moves between each tile's current position and its position in the solution




tiles

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|

3 1 2 2 2 3 3 2 2 3 . . .

= 18

# How to Construct a Heuristic

You identify relaxed version of the problem:

- where one or more constraints have been dropped
- problem with fewer restrictions on the actions

**Robot**: the agent can move through walls

**Driver**: the agent can move straight

**8puzzle**: (1) tiles can move anywhere

(2) tiles can move to any adjacent square

**Result:** The cost of an optimal solution to the relaxed problem is an admissible heuristic for the original problem (because it is always weakly less costly to solve a less constrained problem!)

# How to Construct a Heuristic (cont.)

You should identify constraints which, when dropped, make the problem extremely easy to solve

- this is important because heuristics are not useful if they're as hard to solve as the original problem!

This was the case in our examples

Robot: ***allowing*** the agent to move through walls. Optimal solution to this relaxed problem is Manhattan distance

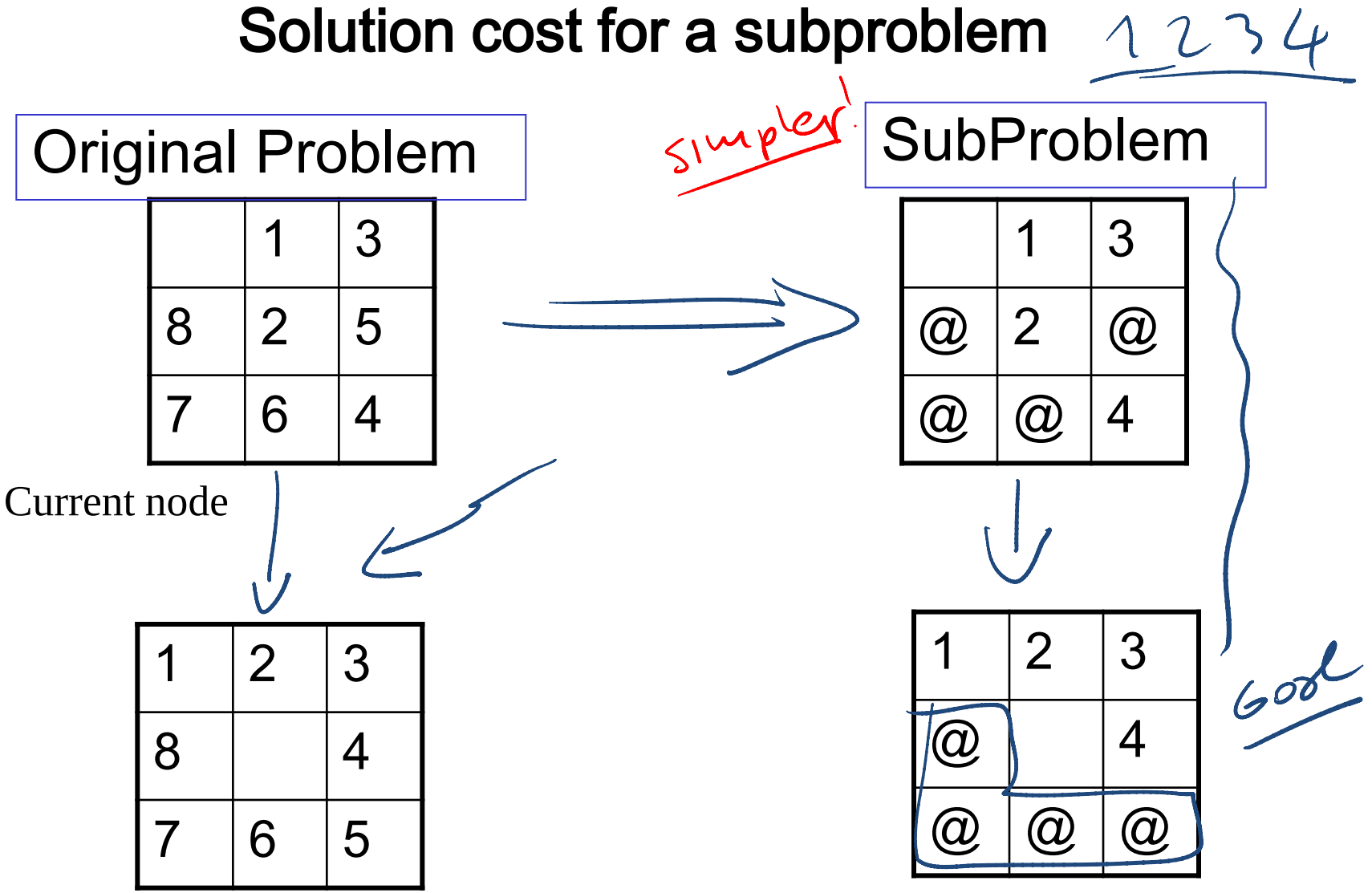
Driver: ***allowing*** the agent to move straight. Optimal solution to this relaxed problem is straight-line distance

8puzzle: (1) tiles **can move anywhere** Optimal solution to this relaxed problem is number of misplaced tiles

(2) tiles can move to **any adjacent square**….

# Another approach to construct heuristics

## Solution cost for a subproblem



# Heuristics: Dominance

If $h_2(n) \geq h_1(n)$ for every state $n$ (both admissible) then $h_2$ dominates $h_1$

*Which one* is better for search ?

i>clicker

*A.* $h_1$

*B.* $h_2$

*C.* *It depends*

# Heuristics: Dominance


**8puzzle**: (1) tiles can move anywhere

(2) tiles can move to any adjacent square

(Original problem: tiles can move to an adjacent square if it is empty)

Iterative deepening (not using any heuristic)

search costs for the 8-puzzle (average number of paths expanded):

→ depth of solution

| | | |
|---|---|---|
| *d=12* | IDS | = 3,644,035 paths |
| | $A^*(h_1)$ | = 227 paths |
| | $A^*(h_2)$ | = 73 paths |
| *d=24* | IDS | = too many paths |
| | $A^*(h_1)$ | = 39,135 paths |
| | $A^*(h_2)$ | = 1,641 paths |

why

| | $h_1$ | $h_2$ |
|---|---|---|
| If tile in correct position | 0 | 0 |
| If tile 1 moves from correct position | 1 | 1 |
| otherwise | 1 | >1 |

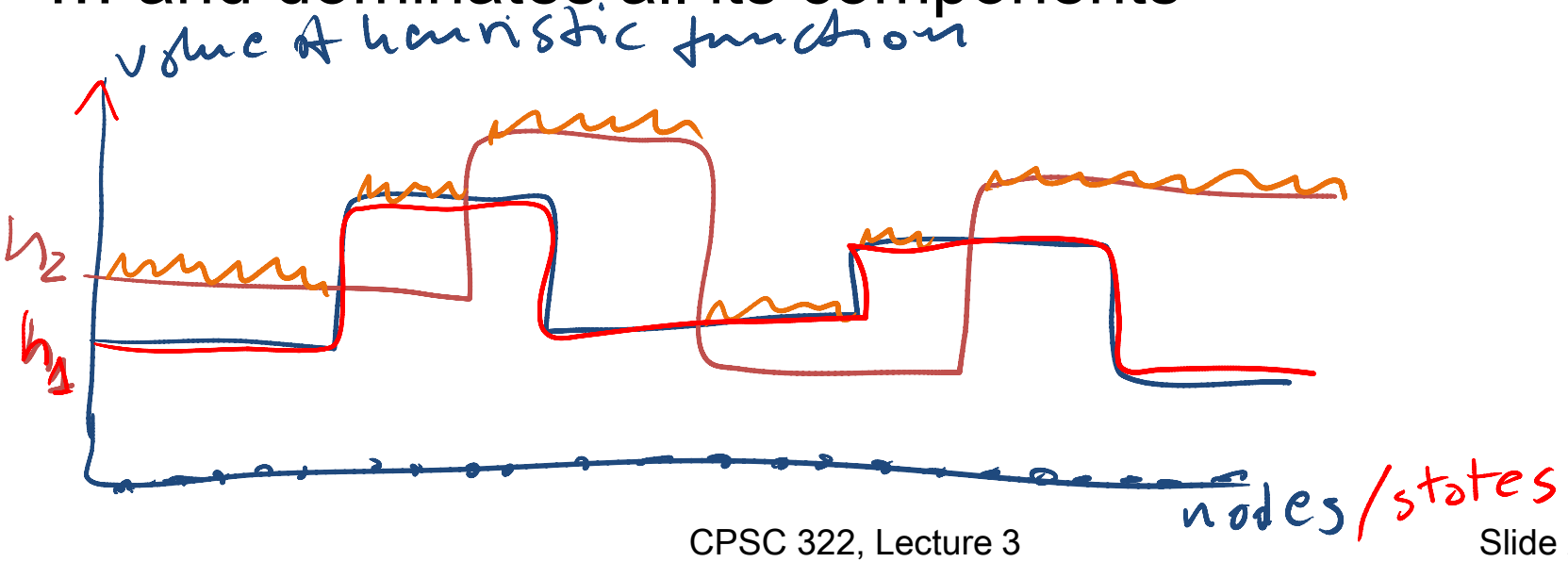
# Combining Heuristics

**How to combine heuristics when there is no dominance?**

If $h_1(n)$ is admissible and $h_2(n)$ is also admissible then

h(n)= max($h_1$, $h_2$) is also admissible

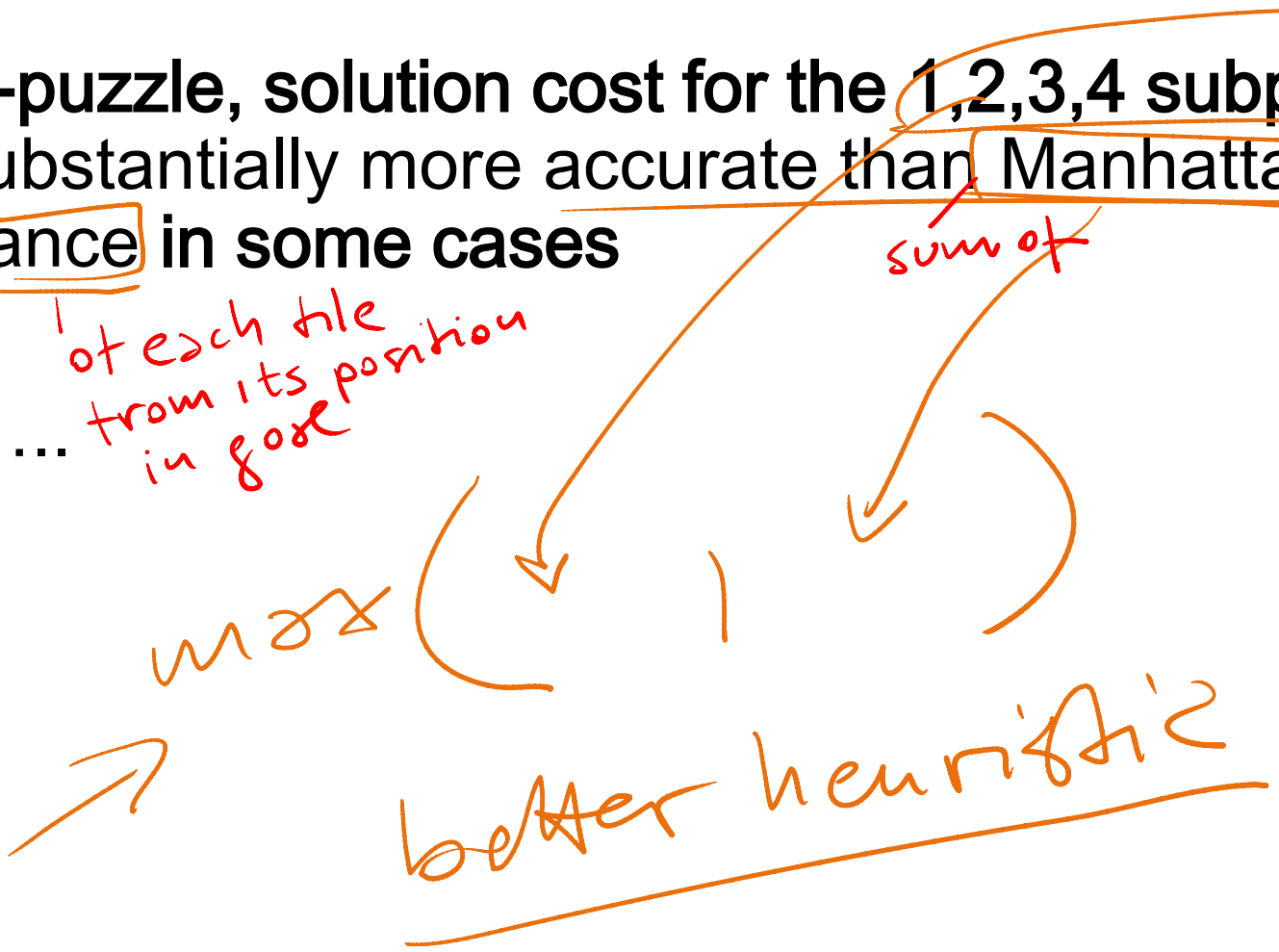
… and dominates all its components



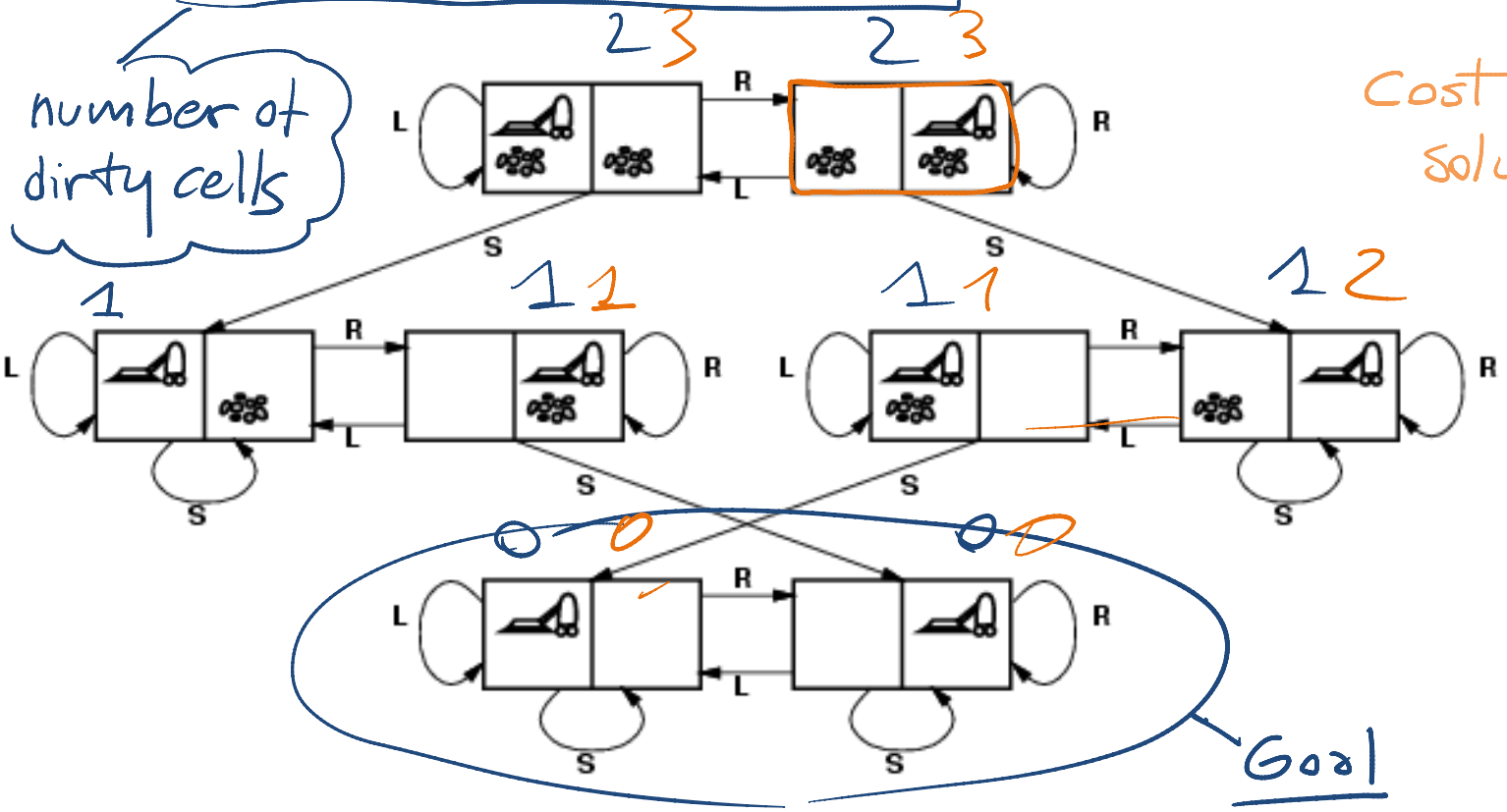
# Combining Heuristics: Example

**In 8-puzzle, solution cost for the 1,2,3,4 subproblem** is substantially more accurate than Manhattan distance **in some cases**

So…..



# Admissible heuristic for Vacuum world?




states? Where it is dirty and robot location

actions? *Left*, *Right*, *Suck*

Possible goal test? no dirt at all locations

# Learning Goals for today's class

- **Construct admissible heuristics** for a given problem.
- **Verify Heuristic Dominance**.
- **Combine admissible heuristics**

• From previous classes
Define/read/write/trace/debug different search algorithms
    - •With / Without cost
    - •Informed / Uninformed

# Next Class

- Best-First Search
- Combining LCFS and BFS: A*  (finish 3.6)
- A*  Optimality