Probability and Time: Markov Models

Computer Science cpsc322, Lecture 31

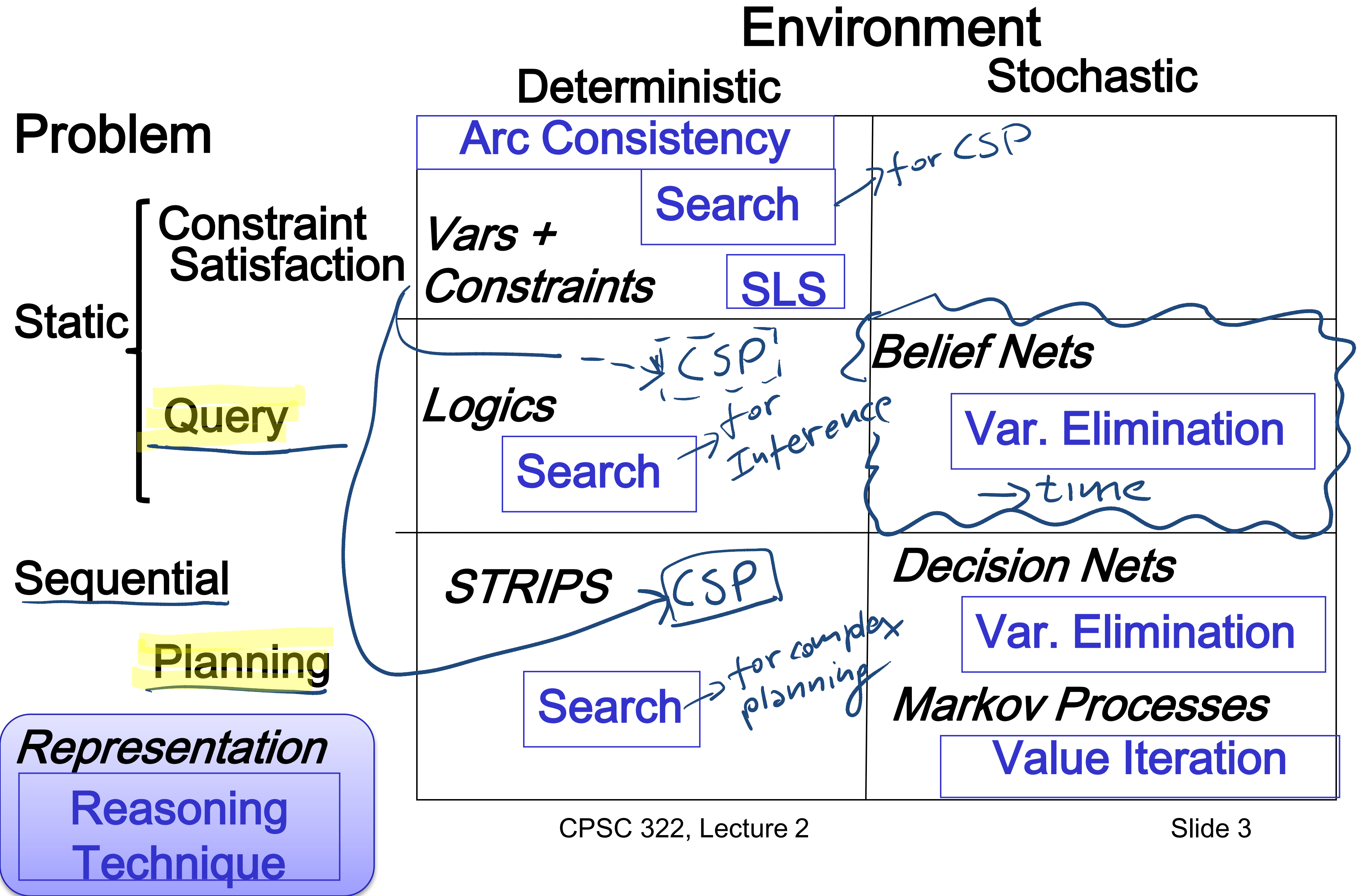
(Textbook Chpt 6.5.1)

Nov, 22, 2013

Lecture Overview

- Recap
- Temporal Probabilistic Models
- Start Markov Models
  - Markov Chain
  - Markov Chains in Natural Language Processing

Big Picture: R&R systems



Environment

Problem		Deterministic	Stochastic
Static	Constraint Satisfaction	*Vars + Constraints* Arc Consistency Search SLS	
	Query	*Logics* Search	*Belief Nets* Var. Elimination
Sequential	Planning	*STRIPS* Search	*Decision Nets* Var. Elimination *Markov Processes* Value Iteration

Representation

Reasoning Technique

→ for CSP

CSP

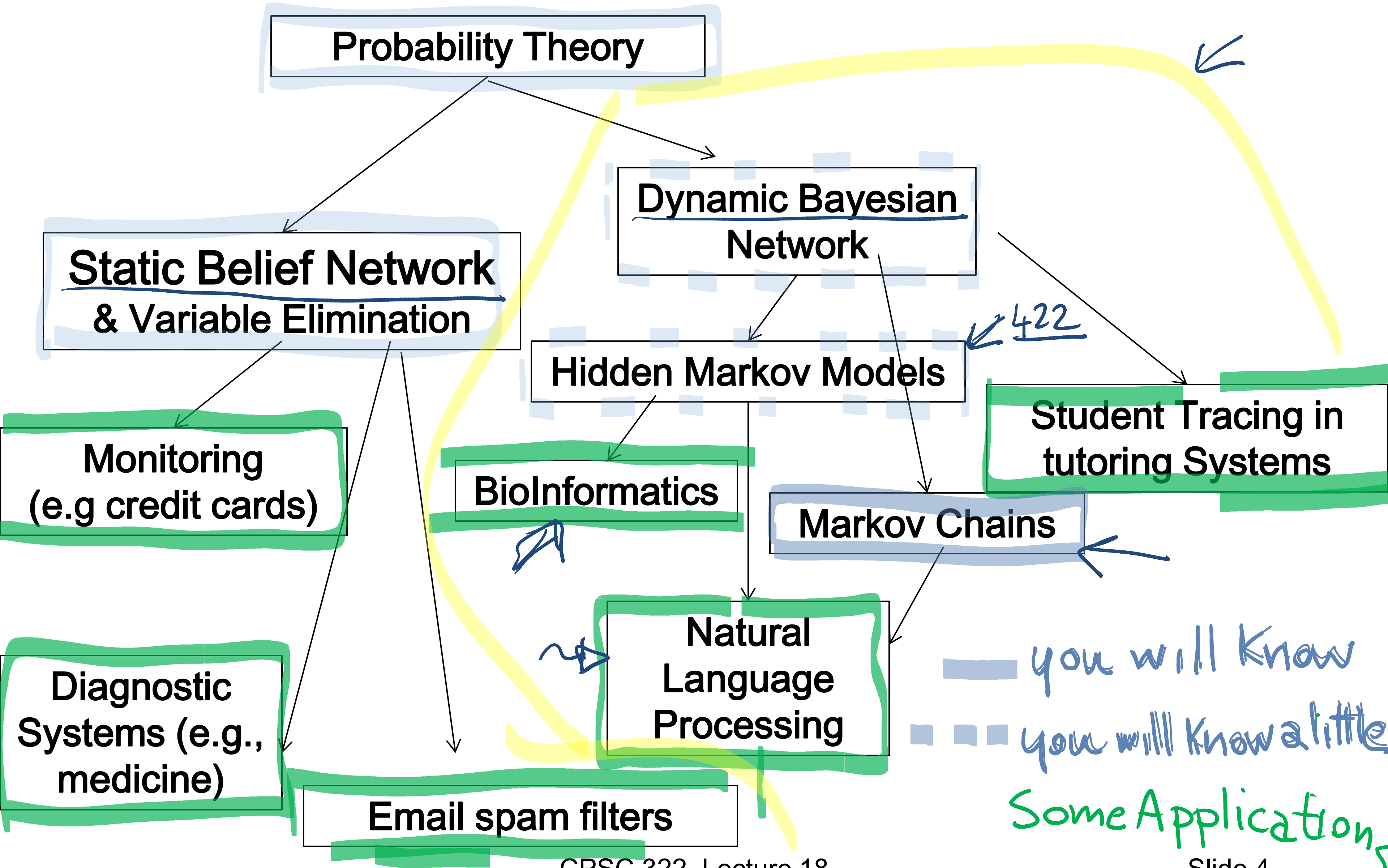
→ for Inference

→ time

CSP

→ for complex planning

Answering Query under Uncertainty



Probability Theory

Static Belief Network & Variable Elimination

Dynamic Bayesian Network

Hidden Markov Models

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Monitoring (e.g credit cards)

BioInformatics

Markov Chains

Student Tracing in tutoring Systems

Diagnostic Systems (e.g., medicine)

Email spam filters

Natural Language Processing

▬ you will know

▪ ▪ ▪ you will know a little

Some Applications

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Modelling static Environments

So far we have used Bnets to perform inference in **static environments**

- For instance, the system keeps collecting evidence to diagnose the cause of a fault in a system (e.g., a car).

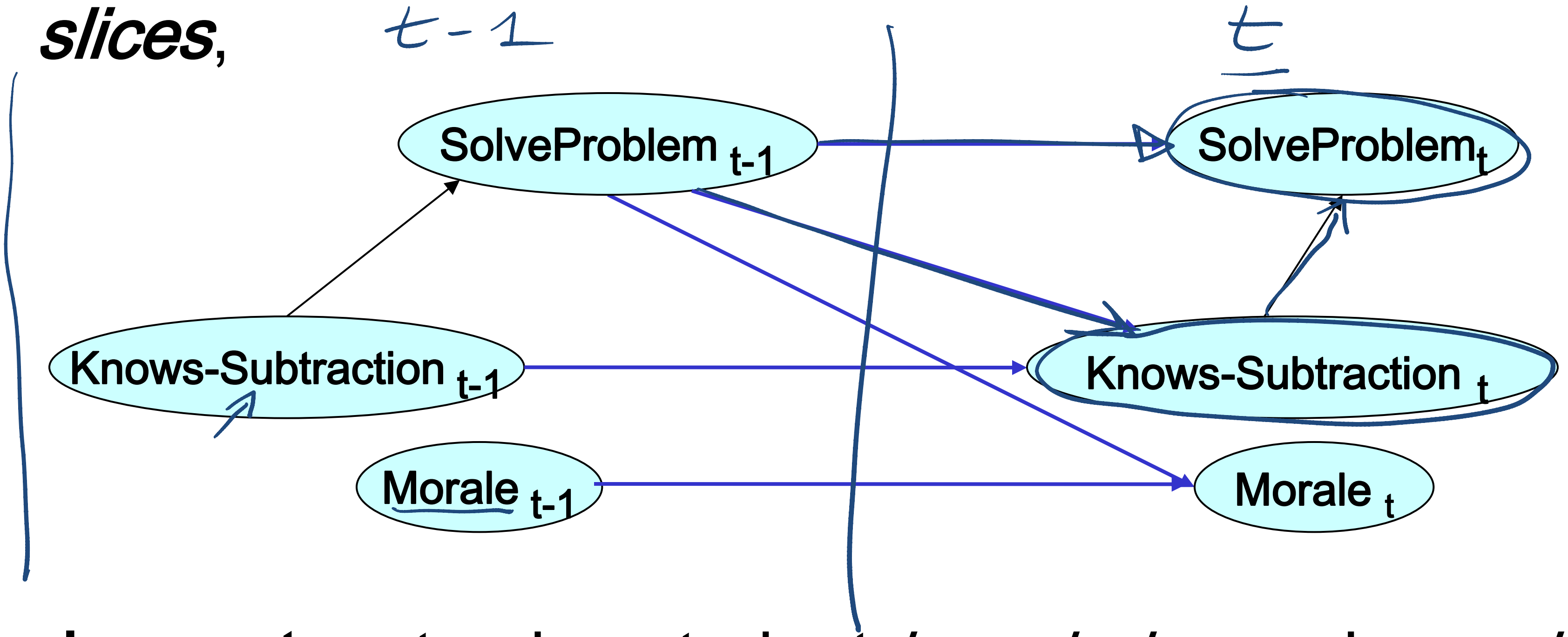
- The environment (values of the evidence, the true cause) does not change as I gather new evidence

- What does change?

The system's beliefs over possible causes

Modeling Evolving Environments

- Often we need to make inferences about evolving environments.
- Represent the state of the world at each specific point in time via a series of snapshots, or ***time slices***,



Tutoring system tracing student *knowledge* and *morale*

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  - **Markov Chain**
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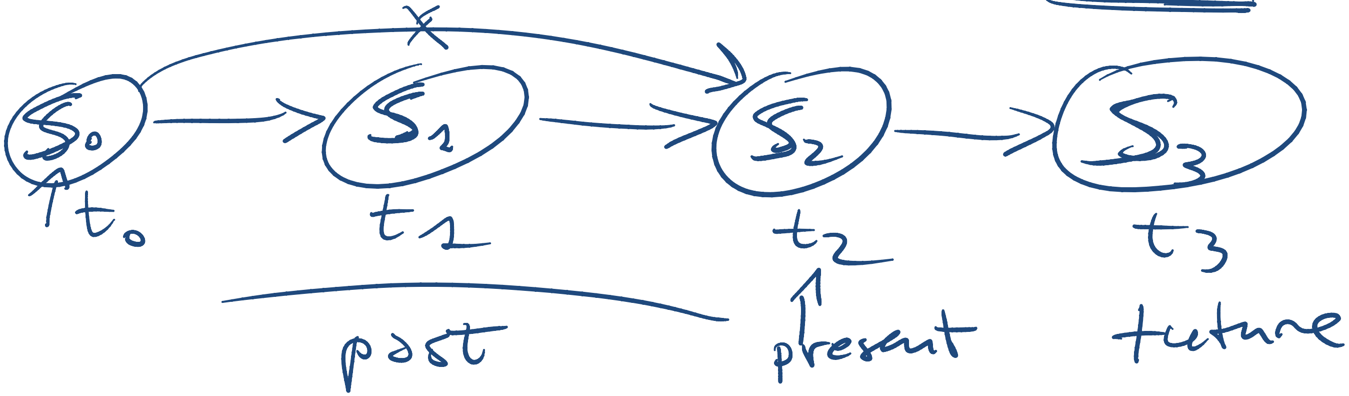
Simplest Possible DBN

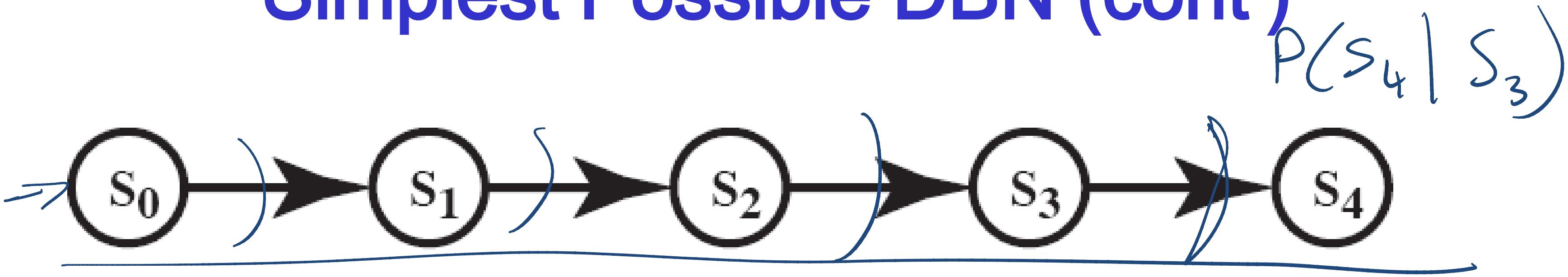
- **One random variable** for each time slice: let's assume S_t represents the state at time *t*. with domain $\{\boxed{\underline{v_1 \ldots v_n}}\}$



- **Each random variable depends only on the previous one**
- Thus $P(S_{t+1} | S_0 \cdots S_t) = P(\underline{S_{t+1}} | \underline{S_t})$

- Intuitively S_t conveys all of the information about the history that can affect the future states.
- "The future is independent of the past given the present."

Simplest Possible DBN (cont')



- How many CPTs do we need to specify?

4 $P(S_1|S_0)$ $P(S_2|S_1)$ etc.

A. *1*

C . *2*

D. *3*

[✓ B. *4*]

- *Stationary process assumption:* the mechanism that regulates how state variables change overtime is **stationary**, that is it can be described by a single transition model

- $P(S_t|S_{t-1})$ is the same for all t

Stationary Markov Chain (SMC)

A stationary Markov Chain : for all t >0

- $\underline{P(S_{t+1}| S_0,\ldots,S_t) = P(S_{t+1}|S_t)}$ and Markov assumption
- $\underline{P(S_{t+1}|S_t)}$ is the same stationary

We only need to specify $P(S_0)$ and $P(S_{t+1}|S_t)$

- Simple Model, easy to specify ←
- Often the natural model ←
- The network can extend indefinitely ←
- **Variations of SMC are at the core of most Natural Language Processing (NLP) applications!** also used in the PageRank algo (used by Google to rank web pages)

Stationary Markov Chain (SMC)

A stationary Markov Chain : for all t >0

- $P(S_{t+1}| S_0,\ldots,S_t) = P(S_{t+1}|S_t)$ and Markov assumption
- $P(S_{t+1}|S_t)$ is the same stationary

So we only need to specify?

iclicker

[✓ A. $P(S_{t+1}|S_t)$ and $P(S_0)$]

B. $P(S_0)$

C . $P(S_{t+1}|S_t)$

D. $P(S_t|S_{t+1})$

Stationary Markov-Chain: Example

Domain of variable S_i is {t , q, p, a, h, e}

Probability of initial state $P(S_0)$

t	.6
q	.4
p	0
a	0
h	0
e	

Stochastic Transition Matrix $P(S_{t+1}|S_t)$

Which of these two is a possible STM?

S_{t+1}

S_t

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

S_{t+1}

S_t

	t	q	p	a	h	e
t	1	0	0	0	0	0
q	0	1	0	0	0	0
p	.3	0	1	0	0	0
a	0	0	0	1	0	0
h	0	0	0	0	0	1
e	0	0	0	.2	0	1

$\Sigma > 1$

[✓ **A.** Left one only]

B. *Right one only*

C . *Both*

D. *None*

Stationary Markov-Chain: Example

six possible values

Domain of variable S_i is {t , q, p, a, h, e}

We only need to specify…

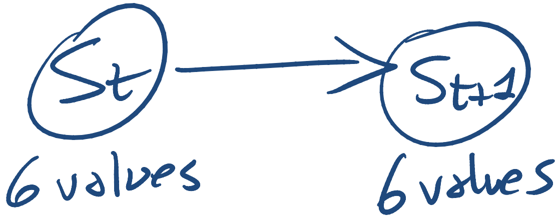
$P(S_0)$

t	.6
q	.4
p	0
a	0
h	0
e	0

Probability of initial state

Stochastic Transition Matrix

$P(S_{t+1}|S_t)$



S_{t+1}

S_t

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

. . .

$\leftarrow P(S_{t+1}|S_t = q)$

$\leftarrow P(S_{t+1}|S_t = p)$

. . .

. . .

. .

Markov-Chain: Inference

Probability of a sequence of states $S_0 \dots S_T$

$$P(S_0,\dots,S_T) = P(S_0)\,P(S_1|S_0)\,P(S_2|S_1)\,\dots\dots$$

$P(u,e,e) \rightarrow$

$P(S_0)$

t	.6
q	.4
p	0
a	0
h	0
e	0

$P(S_{t+1}|S_t)$

	t	q	p	a	h	e
t	0	.3	0	.3	.4	0
q	.4	0	.6	0	0	0
p	0	0	1	0	0	0
a	0	0	.4	.6	0	0
h	0	0	0	0	0	1
e	1	0	0	0	0	0

Example:

$$P(t, q, p) =$$

$$P(t) * P(q|t) * P(p|q)$$

$$.6 * .3 * .6 = .108$$

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Key problems in NLP

Noun Verb

"Book me a room near UBC"

w_1 w_2 w_3 w_4 w_5 w_6

$$\boxed{P(w_1,..,w_n)?}$$

Assign a probability to a sentence (a sequence of words)

- Part-of-speech tagging
- Word-sense disambiguation,
- Probabilistic Parsing

Summarization, Machine Translation……..

Predict the next word

$$P(w_n | w_1 \ldots\ldots w_{n-1}) =$$

$$= P(w_1 \ldots w_n) / P(w_1 \ldots w_{n-1})$$

- Speech recognition
- Hand-writing recognition
- Augmentative communication for the disabled

$$\boxed{P(w_1,..,w_n)?}$$

Impossible to estimate ☹

$\boxed{P(w_1,..,w_n)?}$ **Impossible to estimate!**

Assuming 10^5 words and average sentence contains 10 words …….

$(10^5)^{10} = 10^{50}$

would contain ↗ probabilities

Google language repository (22 Sept. 2006) ↗ collected from the whole web

contained "only": 95,119,665,584 sentences

$\sim 10^{11}$

Most sentences will not appear or appear only once ☹

What can we do?

Make a strong simplifying assumption!

Sentences are generated by a Markov Chain

w_1 at the beginning of a sentence

$$P(w_1,..,w_n) = P(w_1|<S>)\prod_{k=2}^{n} P(w_k \mid w_{k-1})$$

$$= P(w_1|<S>)\,P(w_2|w_1)\,P(w_3|w_2)\,..P(w_k|w_{k-1})$$

P(The big red dog barks)=

**P(The|<S>) *** P(big | the) * P(red | big) * ...

* P(dog | red) * P(barks | dog)

These probs can be assessed in practice!

Estimates for Bigrams

$P(w_i \mid w_{i-1})$

Silly language repositories with **only two sentences**:

"<S> *The big red dog barks against the big pink dog*"

"<S> *The big pink dog is much smaller*"

Count How many times in your documents you have "big red" and "big"

count

$$P(red \mid big) = \frac{P(big, red)}{P(big)} = \frac{\dfrac{C(big, red)}{\cancel{N_{pairs}}}}{\dfrac{C(big)}{\cancel{N_{words}}}} = \frac{C(big, red)}{C(big)} = \frac{1}{3}$$

$P(w_i \mid w_{i-1})$

$10^5 * 10^5$ matrix

$\cancel{10}^5$

$$\boxed{P(w_i \mid w_{i-1}, w_{i-2})}$$

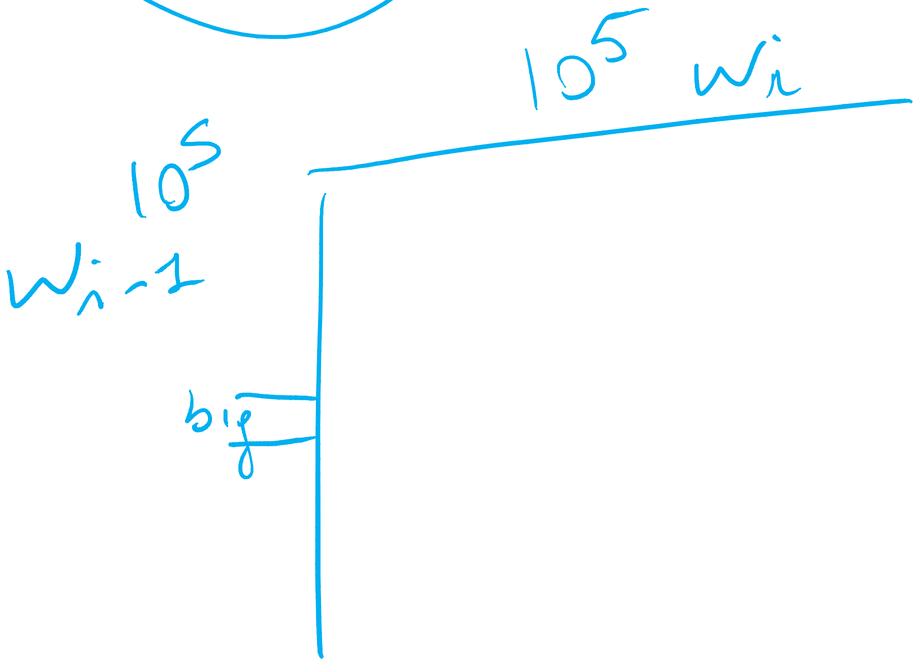
some models use two preceeding words

Bigrams in practice…

If you have 10^5 words in your dictionary $P(w_i \mid w_{i-1})$

will contain this many numbers…. ??

- A.2 *10^5
- [✓ B. 10^{10}]
- C. 5 *10^5
- D.2 *10^{10}



$10^5 * 10^5$

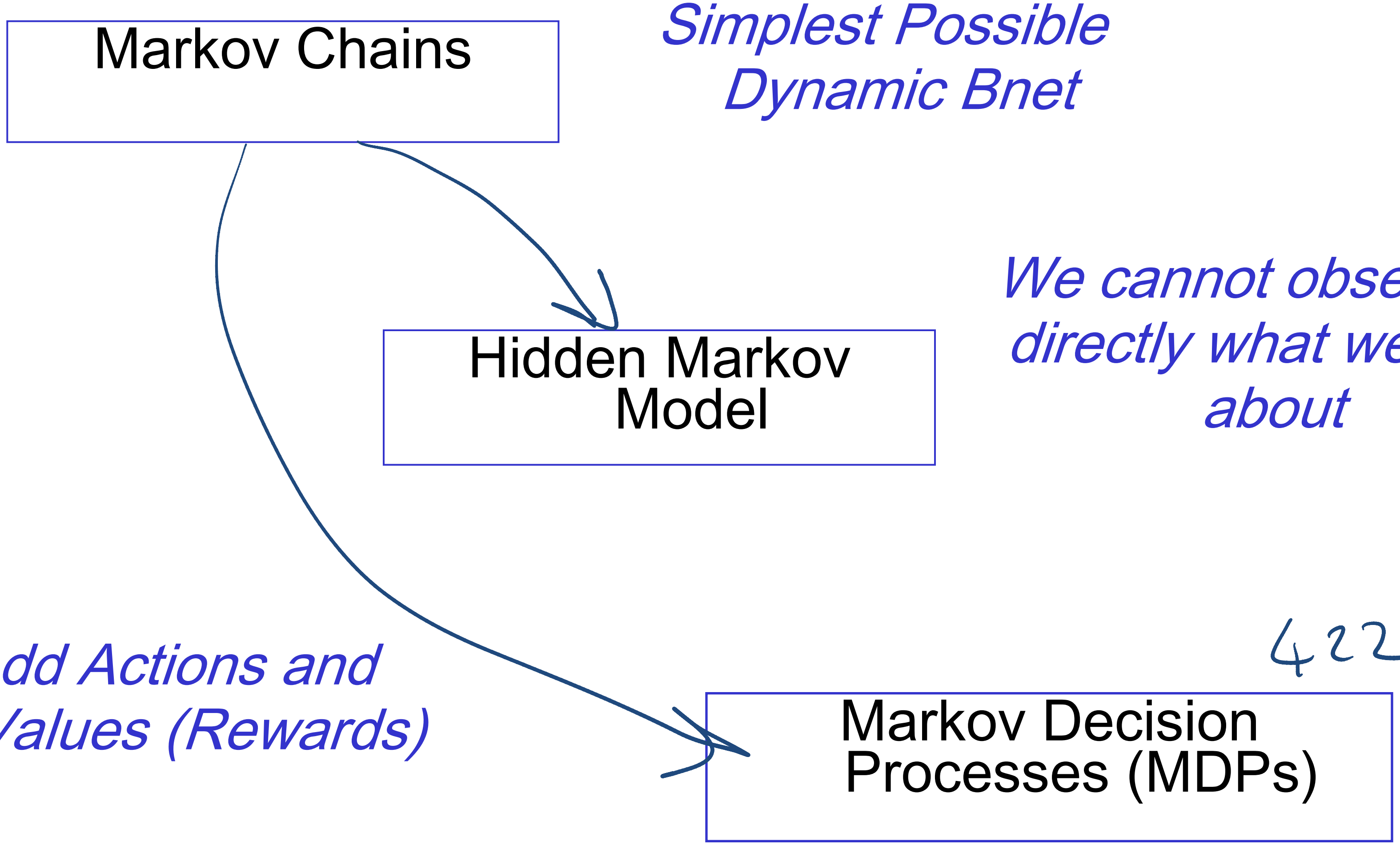
Learning Goals for today's class

You can:

- Specify a Markov Chain ✓ and compute the ✓ probability of a sequence of states

- Justify and apply Markov Chains to compute the probability of a Natural Language sentence (NOT to compute the conditional probabilities - slide 18)

Markov Models



Markov Chains

Simplest Possible Dynamic Bnet

Hidden Markov Model

We cannot observe directly what we care about

Add Actions and Values (Rewards)

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Markov Decision Processes (MDPs)

Next Class

- **Finish Probability and Time:** Hidden Markov Models (HMM) *(TextBook 6.5.2)*
- **Start Decision networks** *(TextBook chpt 9)*

Course Elements

- **Assignment 4 is available on Connect** Due on Dec the 2nd .

Office Hours today

2-3 => 3-4