Reasoning Under Uncertainty: Belief Networks

Computer Science cpsc322, Lecture 27

(Textbook Chpt 6.3)

Nov, 13, 2013

Big Picture: R&R systems

Environment

Problem		Deterministic	Stochastic
Static	Constraint Satisfaction	*Vars + Constraints* Arc Consistency Search SLS	
Static	Query	*Logics* Search	*Belief Nets* Var. Elimination
Sequential	Planning	*STRIPS* Search	*Decision Nets* Var. Elimination *Markov Processes* Value Iteration

Representation

Reasoning Technique

→ for CSP

CSP

→ for Inference

CSP

→ for complex planning

Key points Recap

- We model the **environment** as a set of …. random vars

  $X_1 \dots X_n$ JPD $P(X_1 \dots X_n)$

- Why the joint is not an adequate representation ?

"Representation, reasoning and learning" are "exponential" in ….. ~~#~~ vars

Solution: Exploit marginal&**conditional** independence

$\boxed{P(X|Y)} = P(X)$ $\boxed{P(X|YZ)} = P(X|Z)$

But how does independence allow us to simplify the joint?

CHAIN RULE !

Lecture Overview

- Belief Networks
  - Build sample BN
  - Intro Inference, Compactness, Semantics
  - More Examples

Belief Nets: Burglary Example

There might be a **burglar** in my house

B

The **anti-burglar alarm** in my house may go off

A

I have an agreement with two of my neighbors, **John** and **Mary**, that they **call** me if they hear the alarm go off when I am at work

M J

Minor earthquakes may occur and sometimes the set off the alarm.

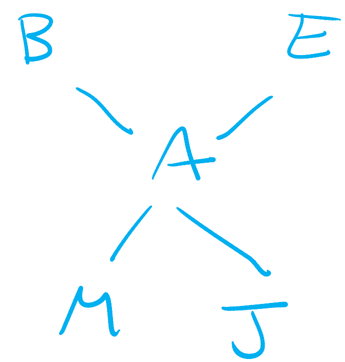
E

Variables: B A M J E $n = 5$

Joint has $2^5 - 1$ entries/probs $2^n - 1$

Belief Nets: Simplify the joint

- Typically order vars to reflect causal knowledge (i.e., causes *before effects)*
  - A burglar (B) can set the alarm (A) off
  - An earthquake (E) can set the alarm (A) off
  - The alarm can cause Mary to call (M)
  - The alarm can cause John to call (J)



$P(B, E, A, M, J)$

marginal indep.

conditional indep.

- Apply Chain Rule

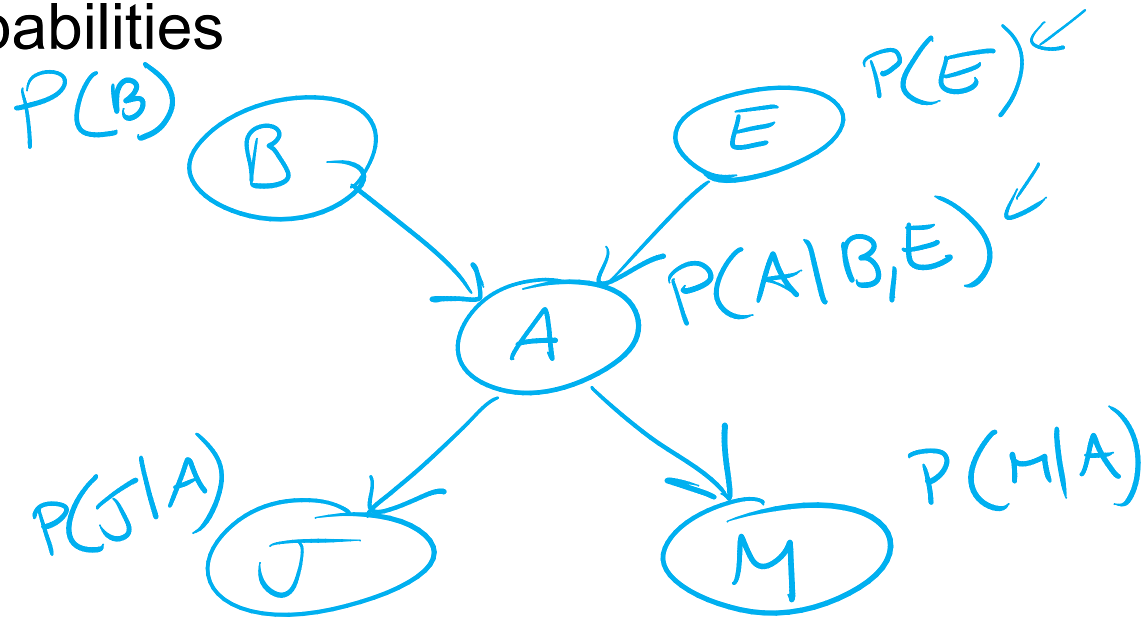
$P(B)\ P(E \mid \cancel{B})\ P(A \mid B, E)\ P(M \mid A E \cancel{B})\ P(J \mid \cancel{M} A \cancel{E} \cancel{B})$

- Simplify according to marginal&conditional independence

Belief Nets: Structure + Probs

$$P(B) * P(E) * P(A|\boxed{B,E}) * P(M|A) * P(J|A)$$

- Express remaining dependencies as a network
  - Each var is a node
  - For each var, the conditioning vars are its parents
  - Associate to each node corresponding conditional probabilities



- Directed Acyclic Graph (DAG)

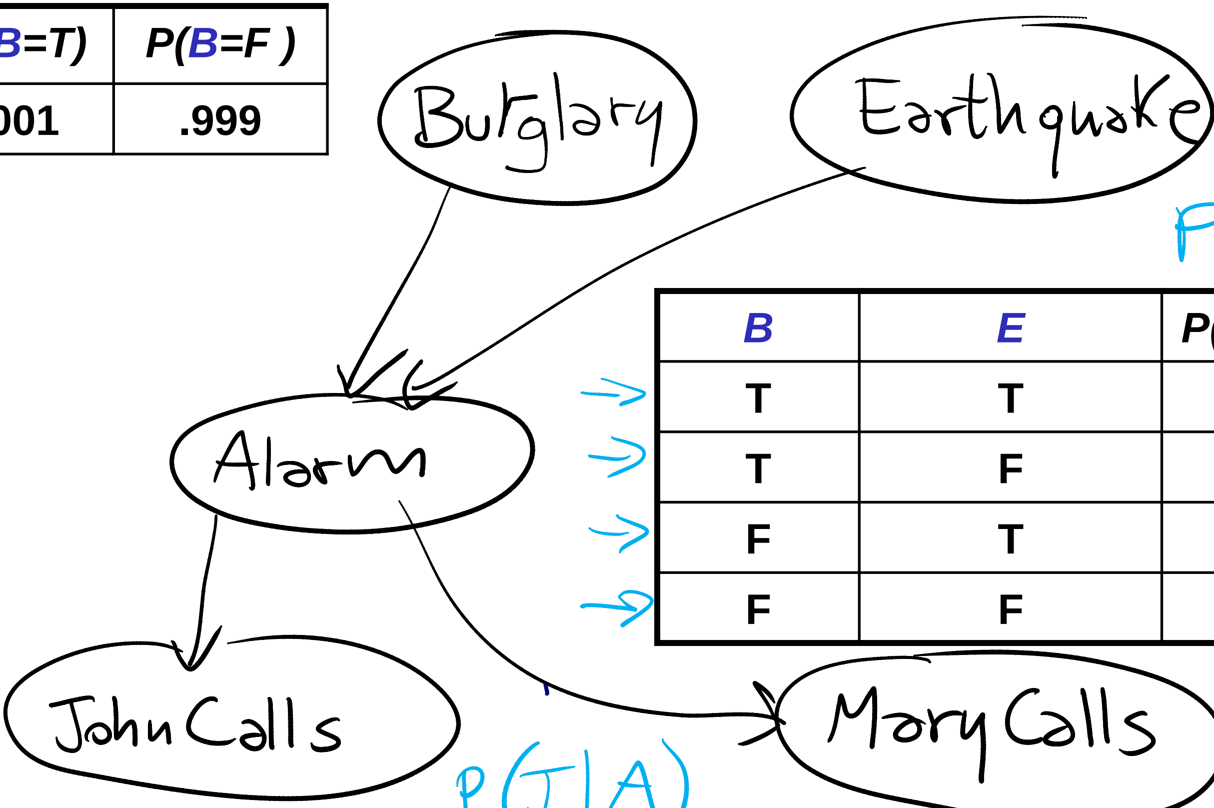
Burglary: complete BN

P(B)

P(B=T)	P(B=F)
.001	.999

P(E)

P(E=T)	P(E=F)
.002	.998



P(A | B,E)

B	E	P(A=T | B,E)	P(A=F | B,E)
T	T	.95	.05
T	F	.94	.06
F	T	.29	.71
F	F	.001	.999

P(J | A)

A	P(J=T | A)	P(J=F | A)
T	.90	.10
F	.05	.95

P(M | A)

A	P(M=T | A)	P(M=F | A)
T	.70	.30
F	.01	.99

call for any other reasons

Lecture Overview

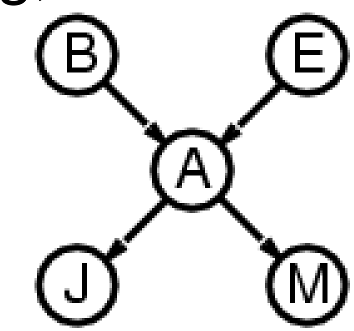
- **Belief Networks**
  - Build sample BN
  - Intro Inference, Compactness, Semantics
  - More Examples

Burglary Example: Bnets inference

Our BN can answer any probabilistic query that can be answered by processing the joint!

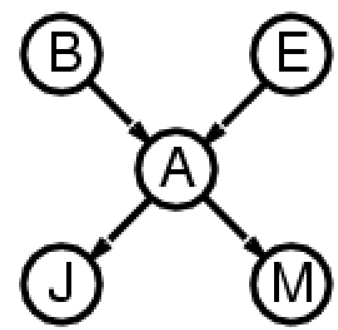
(Ex1) I'm at work,

- neighbor John calls to say my alarm is ringing,
- neighbor Mary doesn't call.
- No news of any earthquakes.
- Is there a burglar?



(Ex2) I'm at work, Try this

- Receive message that neighbor John called ,
- News of minor earthquakes.
- Is there a burglar?



Set digital places to monitor to 5

Burglary Example: Bnets inference

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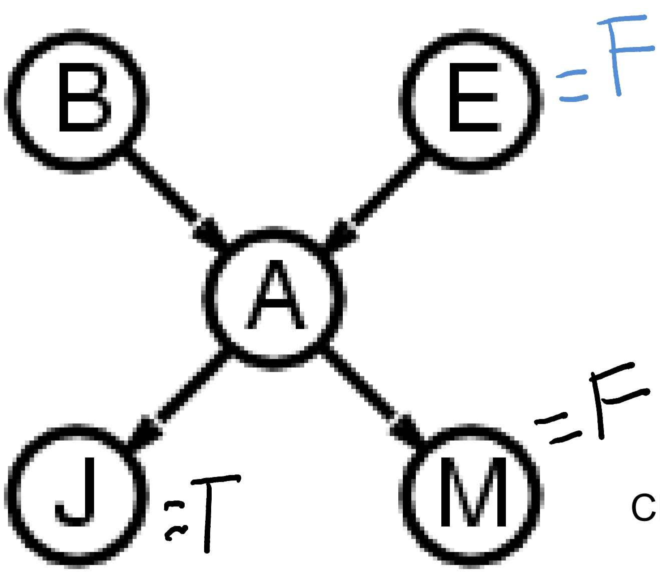
(Ex1) I'm at work,

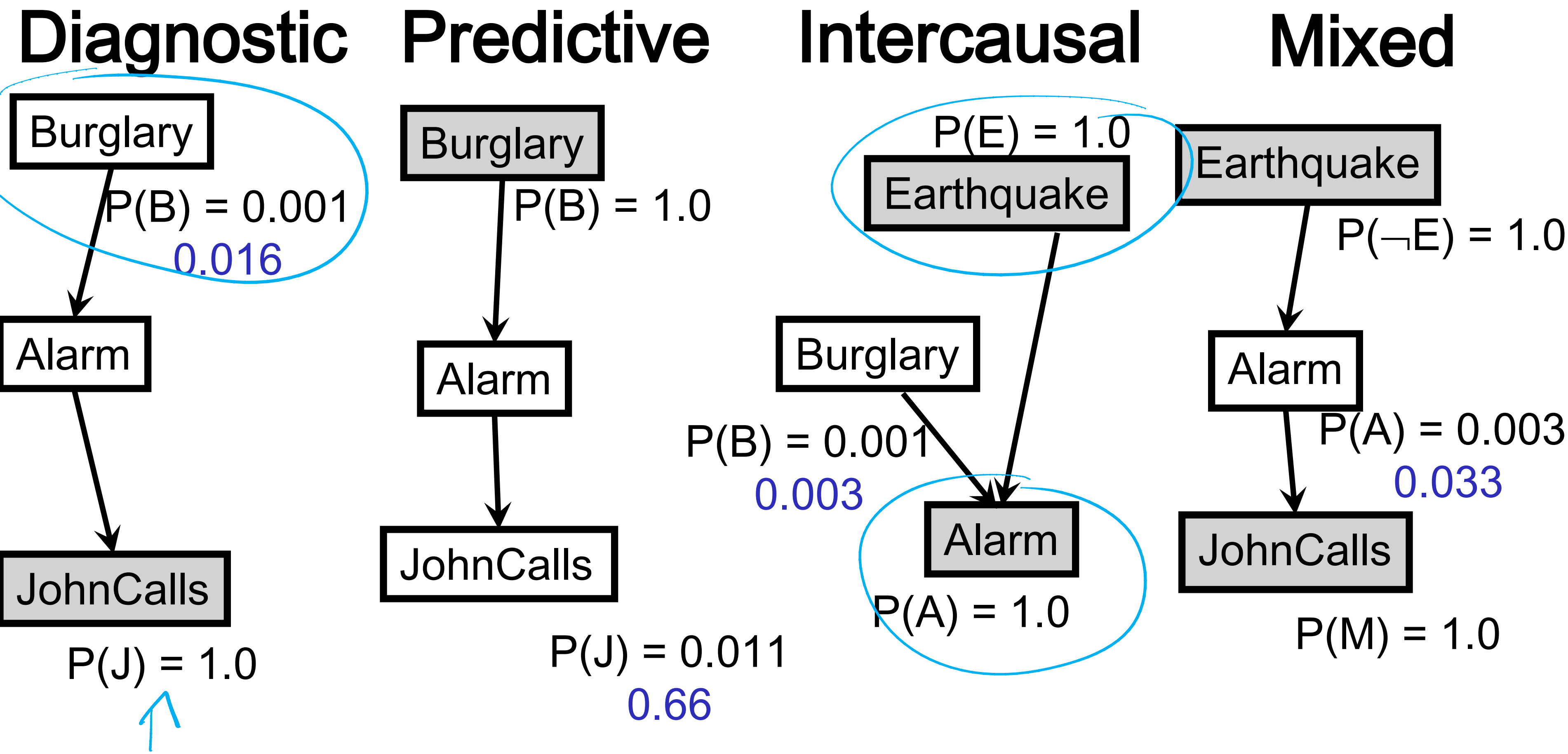
- neighbor John calls to say my alarm is ringing,
- neighbor Mary doesn't call.
- **No news of any earthquakes.**
- Is there a burglar?



The probability of Burglar will:

A. Go down
B. Remain the same
C. Go up

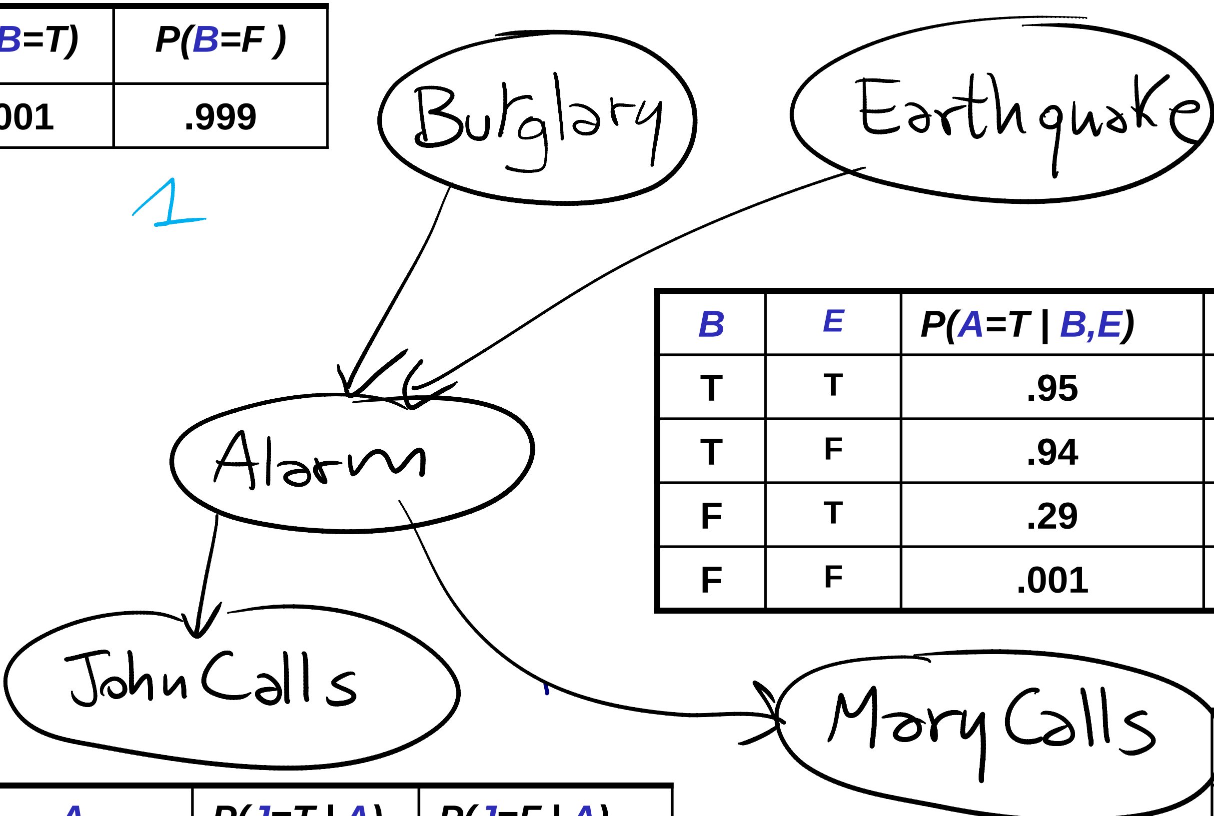
Bayesian Networks – Inference Types



BNnets: Compactness

P(B=T)	P(B=F)
.001	.999

1



P(E=T)	P(E=F)
.002	.998

1

B	E	P(A=T | B,E)	P(A=F | B,E)
T	T	.95	.05
T	F	.94	.06
F	T	.29	.71
F	F	.001	.999

4

A	P(J=T | A)	P(J=F | A)
T	.90	.10
F	.05	.95

2

A	P(M=T | A)	P(M=F | A)
T	.70	.30
F	.01	.99

2

BNet

$2+2+4+1+1=10$

$|JPD| = 2^5 - 1$

Conditional Probability Table

BNets: Compactness

In General:

A $\boxed{\text{CPT}}$ for boolean X_i with k boolean parents has 2^k rows for the combinations of parent values

Each row requires **one number** p_i for X_i = *true* (the number for X_i = *false* is just $1-p_i$)

for each node

If **each variable** has no more than ***k* parents**, the complete network requires $O(n2^k)$ numbers

For $k << n$, this is a substantial improvement,

- the numbers required grow linearly with n, vs. $O(2^n)$ for the full joint distribution

BNets: Construction General Semantics

The full joint distribution can be defined as the product of conditional distributions:

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid X_1, \ldots, X_{i-1}) \text{ (chain rule)}$$

Simplify according to marginal&conditional independence

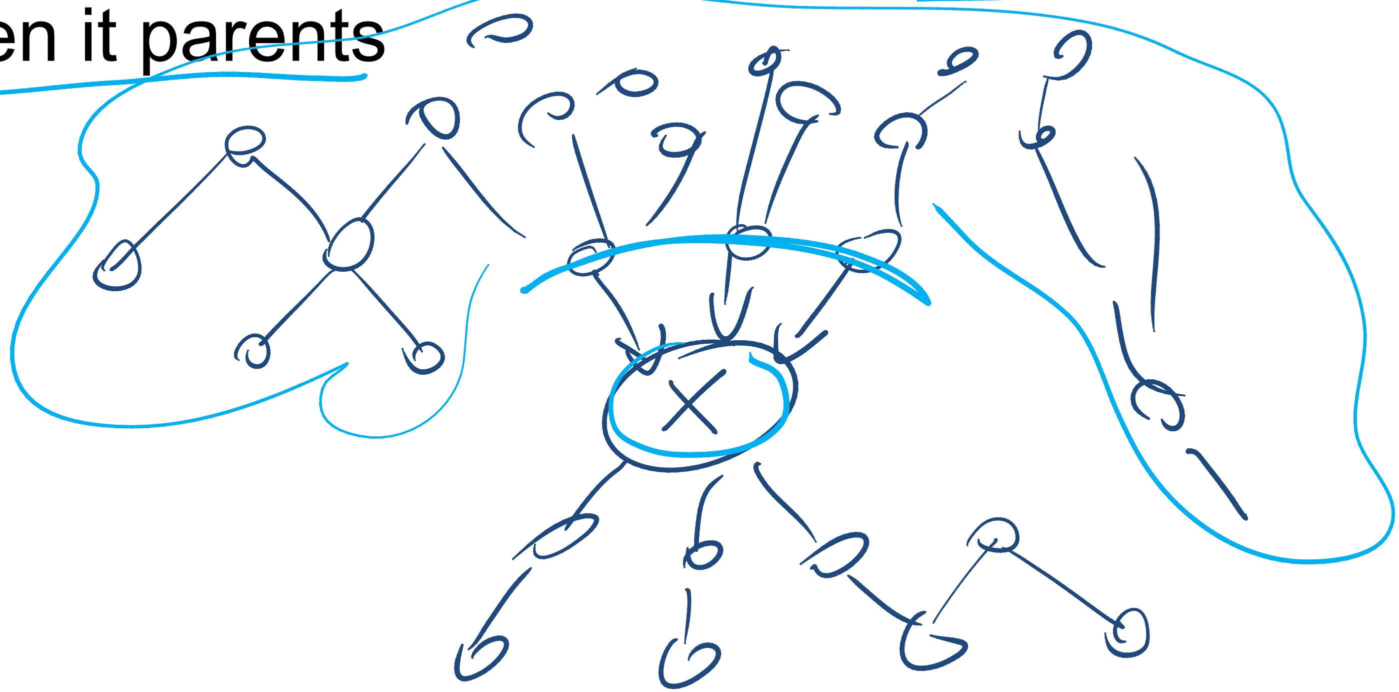
- Express remaining dependencies as a network
  - Each var is a node
  - For each var, the conditioning vars are its parents
  - Associate to each node corresponding conditional probabilities

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid Parents(X_i))$$

BNets: Construction General Semantics (cont')

$$P(X_1, \ldots, X_n) = \prod_{i=1}^{n} P(X_i \mid Parents(X_i))$$

- Every node is independent from its non-descendants given it parents



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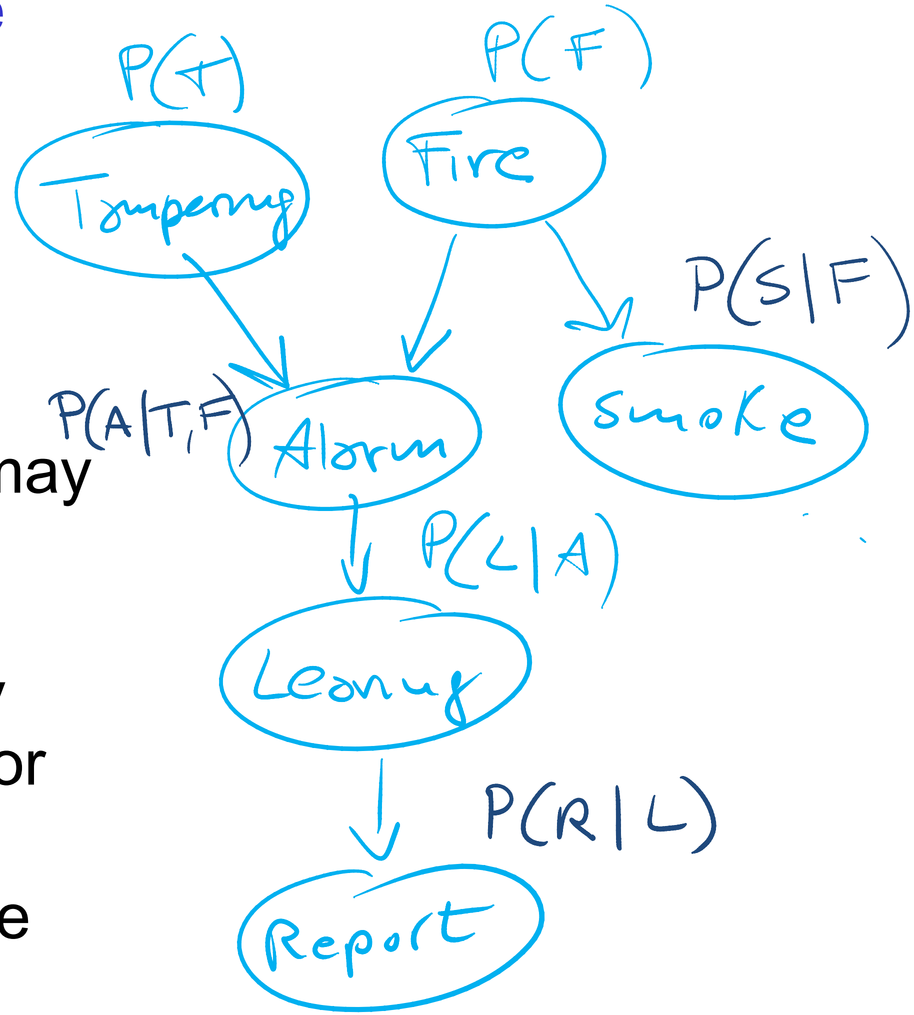
Other Examples: Fire Diagnosis (textbook Ex. 6.10)

Suppose you want to diagnose whether there is a fire in a building

- you receive a noisy report about whether everyone is leaving the building.
- if everyone is leaving, this may have been caused by a fire alarm.
- if there is a fire alarm, it may have been caused by a fire or by tampering
- if there is a fire, there may be smoke raising from the bldg.



Other Examples (cont')

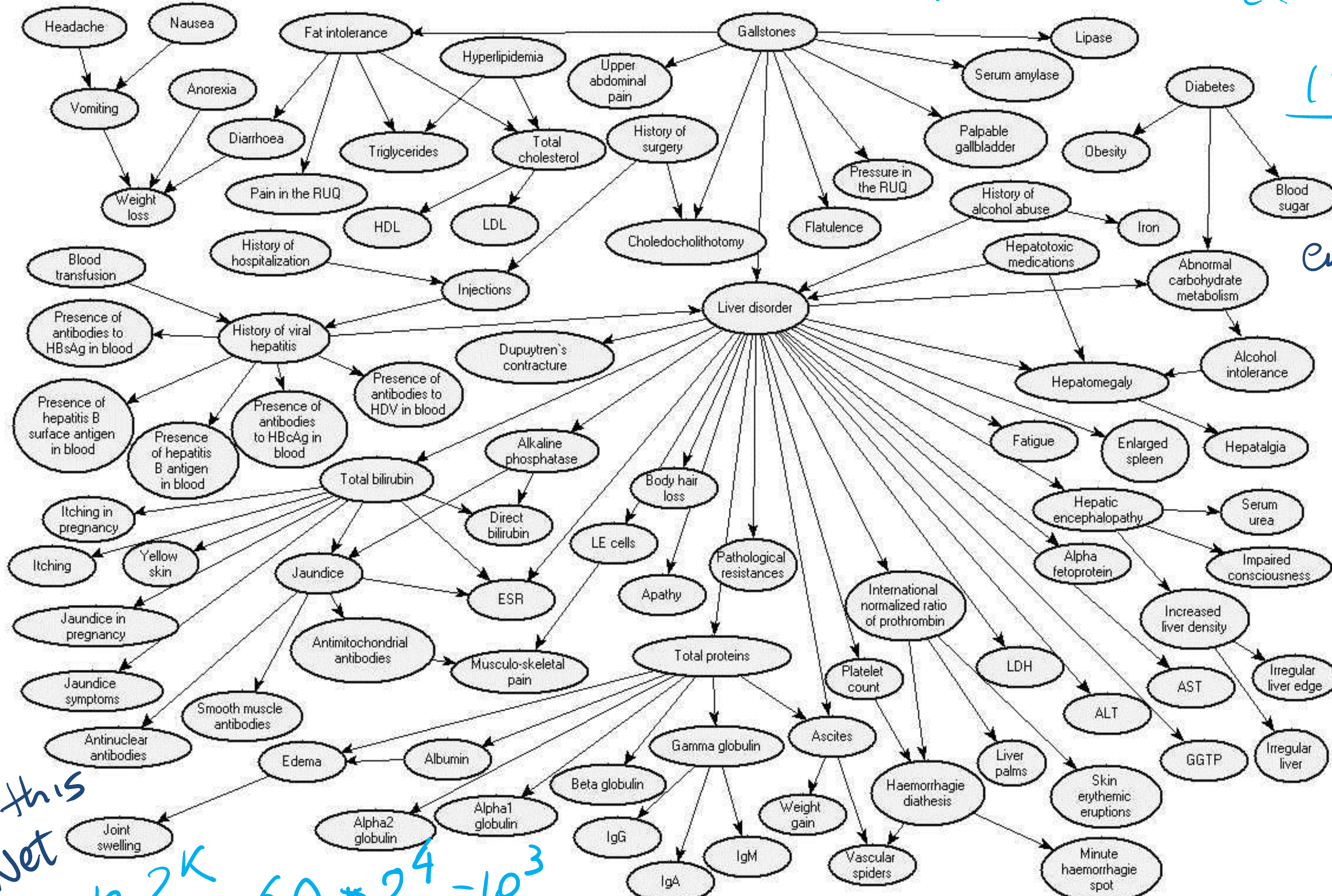
- Make sure you explore and understand the **Fire Diagnosis** example (we'll expand on it to study Decision Networks)
- **Electrical Circuit** example (textbook ex 6.11)
- **Patient's wheezing and coughing** example (ex. 6.14)
- Several other examples on

Realistic BNet: Liver Diagnosis

Source: Onisko et al., 1999

~60 nodes

$n = \sim 60$ $\sim 2^{60}$ JPD $\approx (2^{10})^6 \approx 10^{18}$ entries

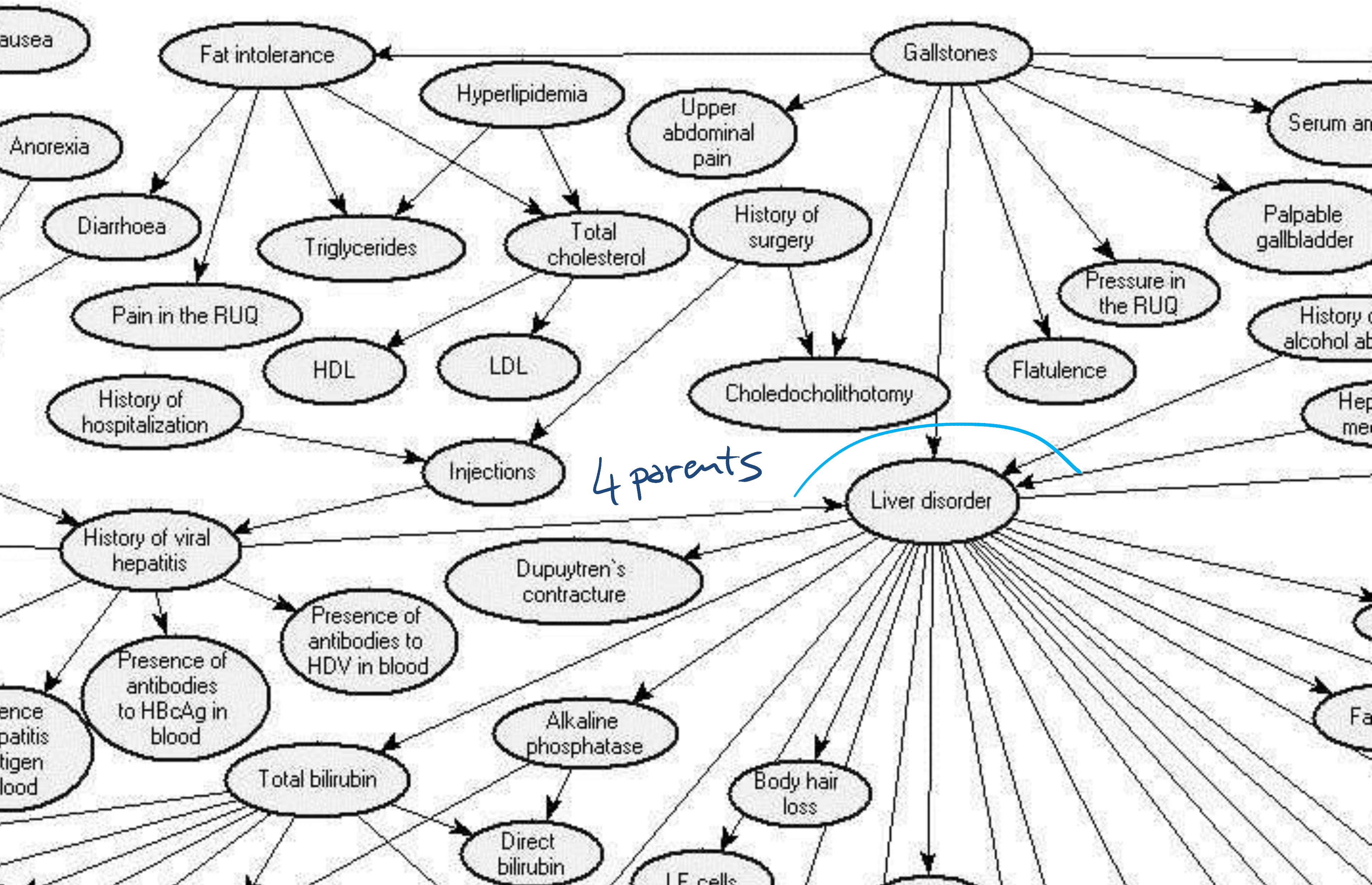


for this BNet ~2K

$60 * 2^4 = 10^3$

Realistic BNet: Liver Diagnosis

Source: Onisko et al., 1999



Realistic BNet: Liver Diagnosis

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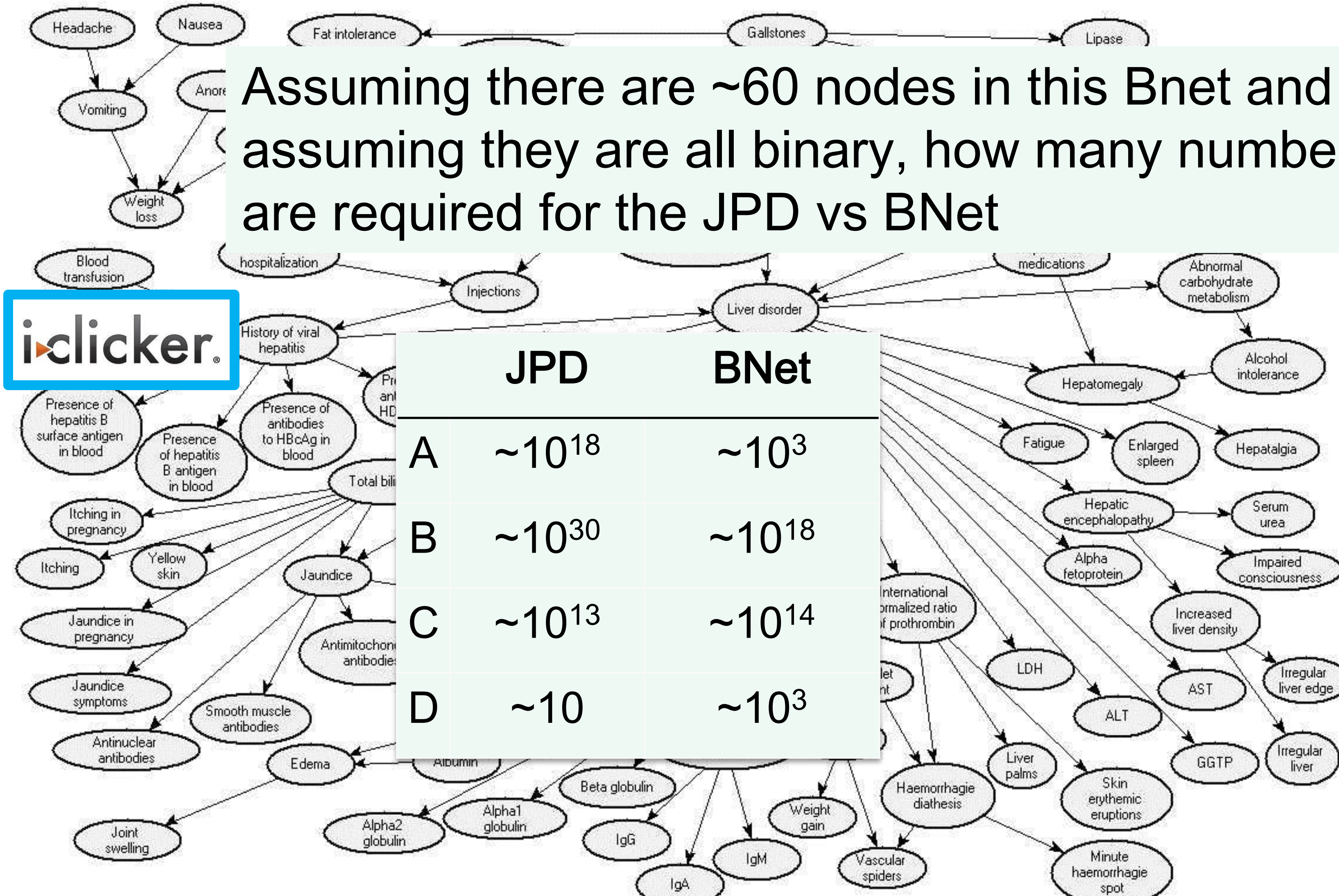
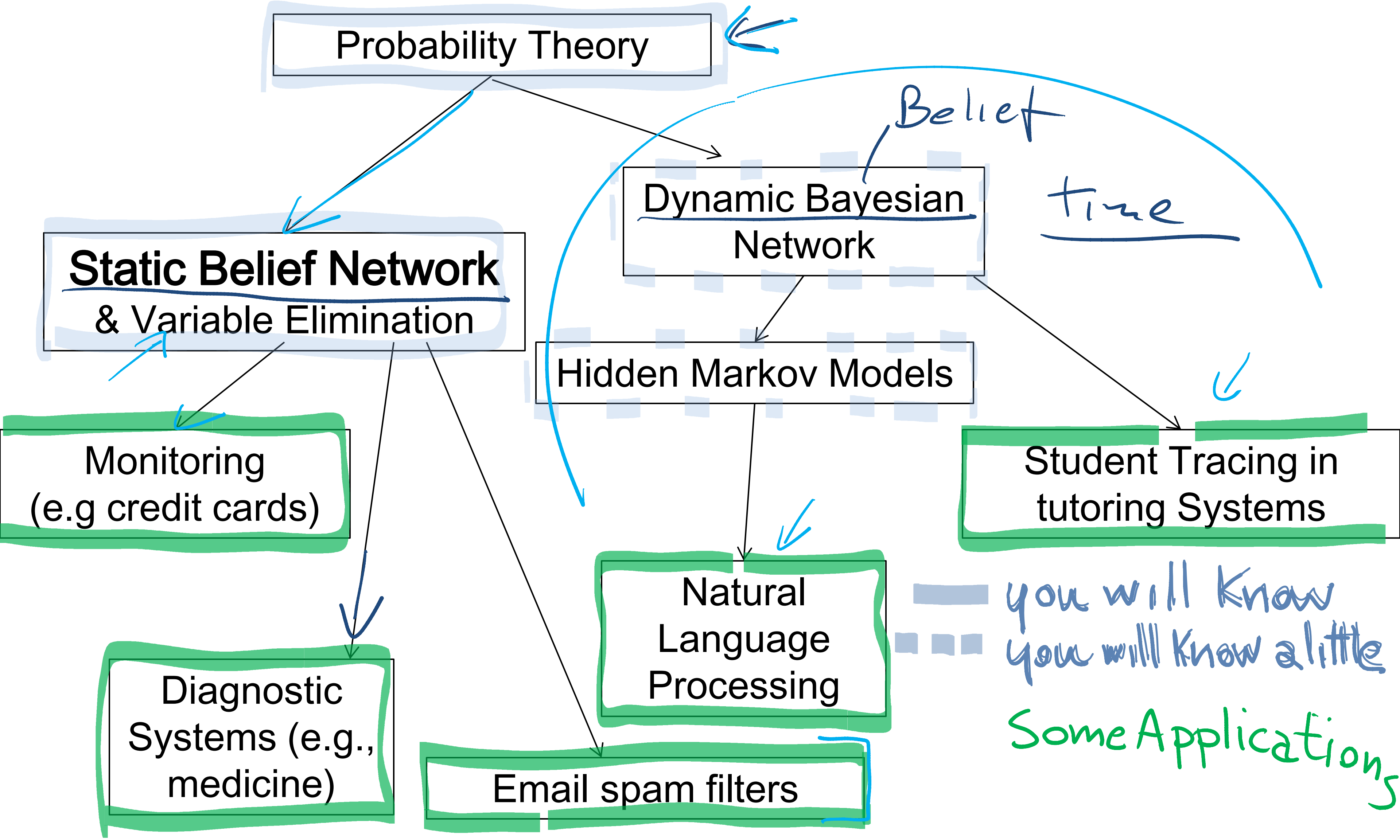


Assuming there are ~60 nodes in this Bnet and assuming they are all binary, how many numbers are required for the JPD vs BNet

	JPD	BNet
A	$\sim 10^{18}$	$\sim 10^{3}$
B	$\sim 10^{30}$	$\sim 10^{18}$
C	$\sim 10^{13}$	$\sim 10^{14}$
D	~ 10	$\sim 10^{3}$

Answering Query under Uncertainty



Learning Goals for today's class

You can:

Build a Belief Network for a simple domain

Classify the types of inference

Diagnostic, Predictive, Intercausal, Mixed

Compute the representational saving in terms on number of probabilities required

Next Class (Wednesday!)

Bayesian Networks Representation

- **Additional Dependencies** encoded by BNets
- More **compact representations** for CPT
- Very simple but extremely useful Bnet (**Bayes Classifier**)

Belief network summary

- A belief network is a directed acyclic graph (DAG) that effectively expresses independence assertions among random variables.
- The parents of a node X are those variables on which X directly depends.
- Consideration of causal dependencies among variables typically help in constructing a Bnet