CSPs: Search and Arc Consistency

Computer Science cpsc322, Lecture 12

(Textbook Chpt 4.3-4.5)

Oct, 2, 2013

Lecture Overview

- **Recap CSPs**
- Generate-and-Test
- Search
- Consistency
- Arc Consistency

Constraint Satisfaction Problems: definitions

Definition (Constraint Satisfaction Problem)

A constraint satisfaction problem consists of # possible worlds

$3 * 5 * 5$

- a set of variables

  A, B, C

- a domain for each variable

  dom C = {1, 2, 3}

  dom A = {1, 2, 3, 4, 5} = dom(B)

- a set of constraints

  → B = 5 C > B A = B no sol

  → " C < B "

Definition (model / solution)

A model of a CSP is an assignment of values to variables that satisfies all of the constraints.

→ B = 5 A = 5 C = 1

→ " " C = 2

→ " " C = 3

Modules we'll cover in this course: R&Rsys

Environment

Problem	Deterministic	Stochastic
Static: Constraint Satisfaction	Arc Consistency; *Vars + Constraints*; Search → for CSP	
Static: Query	*Logics*; Search → for Inference	*Belief Nets*; Var. Elimination
Sequential: Planning	*STRIPS* actions; Search → for complex planning	*Decision Nets*; Var. Elimination; *Markov Processes*; Value Iteration

Representation

Reasoning Technique

Standard Search vs. Specific R&R systems

Constraint Satisfaction (Problems):

- State & start state
- Successor function
- Goal test
- Solution
- Heuristic function

Planning :

- State
- Successor function
- Goal test
- Solution
- Heuristic function

Answering Queries

- State
- Successor function
- Goal test
- Solution
- Heuristic function

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Generate-and-Test Algorithm

- **Algorithm:**
  - Generate possible worlds one at a time
  - Test them to see if they violate any constraints

→ domA = {1, 2, 3, 4, 5}
→ domB = {1, 2, 3, 4, 5}
→ domC = {1, 2, 3}

```
For a in domA
   For b in domB
        For c in domC
        if (a b c) satisfies all constraints
        return (a b c)
return fail
```

- This procedure is able to solve any CSP
- However, the running time is proportional to the number of possible worlds
  - always exponential in the number of variables
  - far too long for many CSPs ☹

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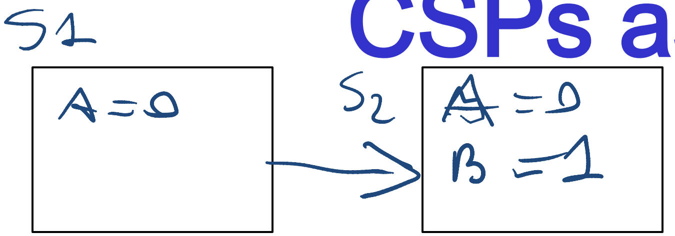
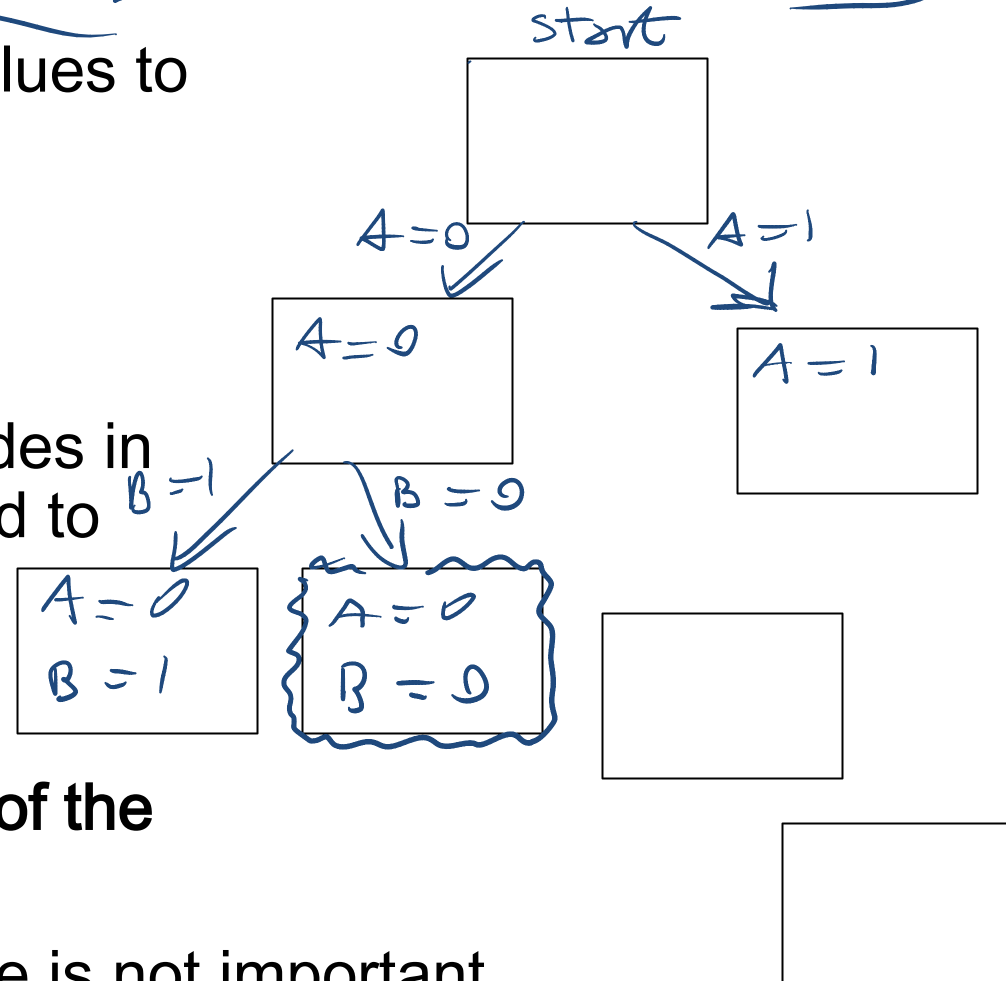
CSPs as search problems



A, B dom A = dom B = {0,1}

→ A=B



- **states:** assignments of values to a subset of the variables
- **start state:** the empty assignment (no variables assigned values)
- **neighbours** of a state: nodes in which values are assigned to one additional variable
- **goal state:** a state which **assigns a value to each variable**, and **satisfies all of the constraints**

Note: the path to a goal node is not important

CSPs as Search Problems

What search strategy will work well for a CSP?

- If there are n variables every solution is at depth……. n

Is there a role for a heuristic function?

A. Yes B. No C. It depends

The search space is always?

A. Finite with cycles
B. Infinite without cycles
C. Finite without cycles
D. Infinite with cycles

So which search strategy is better?

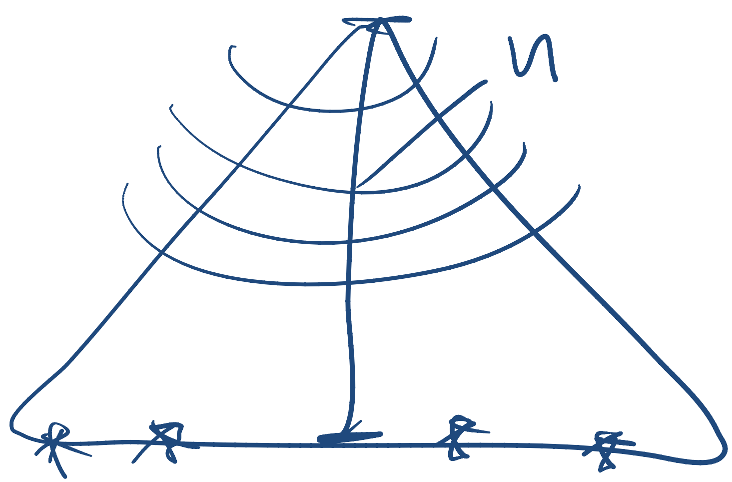
A. BFS
B. IDS
C. A*
D. DFS

CSPs as Search Problems

What search strategy will work well for a CSP?

- If there are n variables every solution is at depth…n…….
- Is there a role for a heuristic function?

- the tree is always …finite…… and has no…cycles……, so which one is better ~~BFS~~ or ~~IDS~~ or DFS?

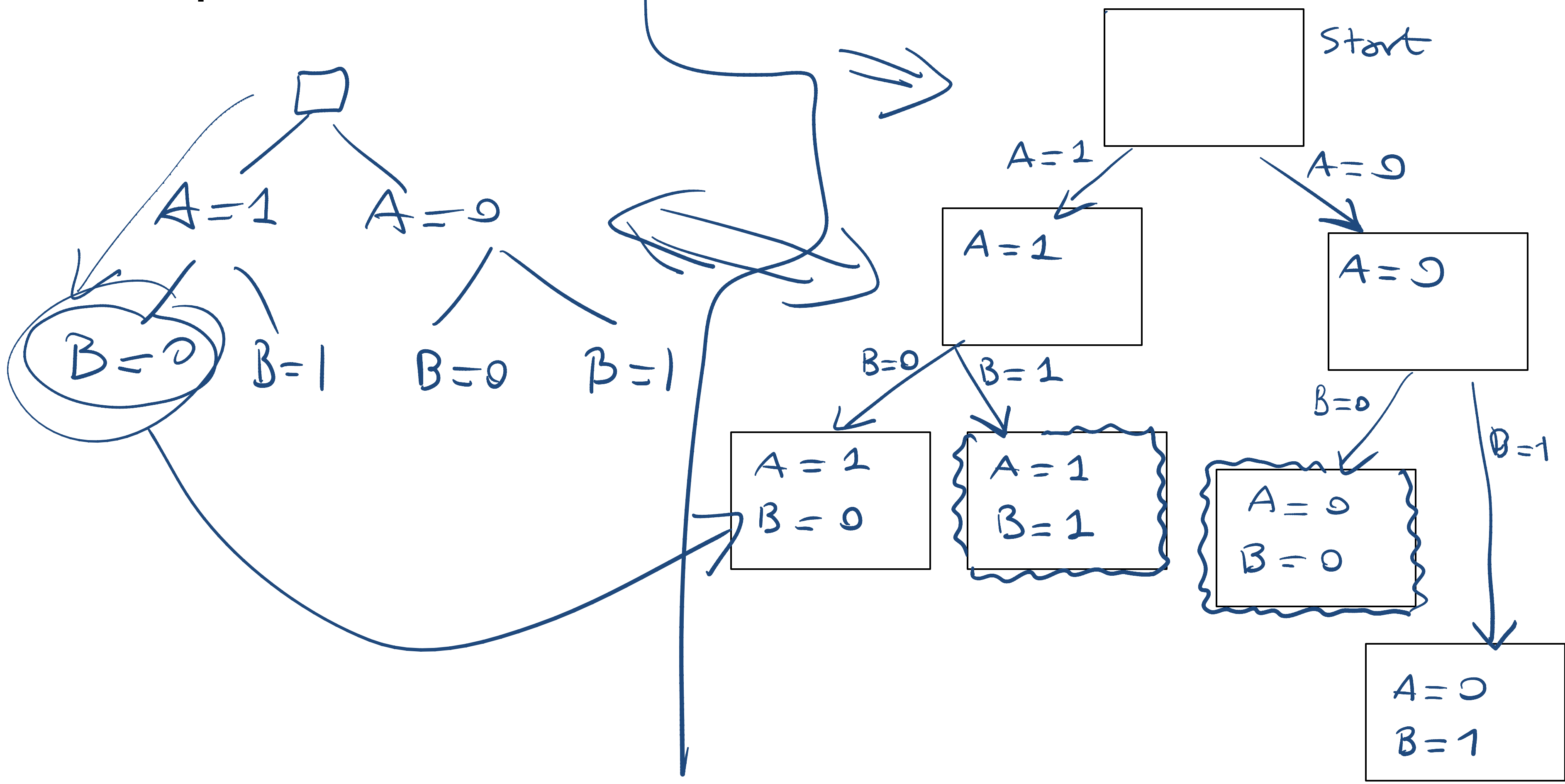


CSPs as search problems

$A, B \quad \mathrm{dom}\, A = \mathrm{dom}\, B = \{0, 1\}$

const $A = B$

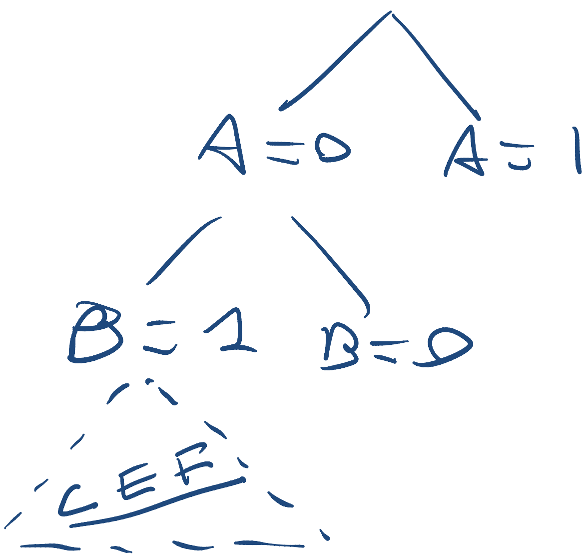
Simplified notation



CSPs as Search Problems

How can we avoid exploring some sub-trees i.e., prune the DFS Search tree?

- once we consider a path whose end node violates one or more constraints, we know that a solution cannot exist below that point
- thus we should **remove that path** rather than continuing to search



A B C E F {1, 0}

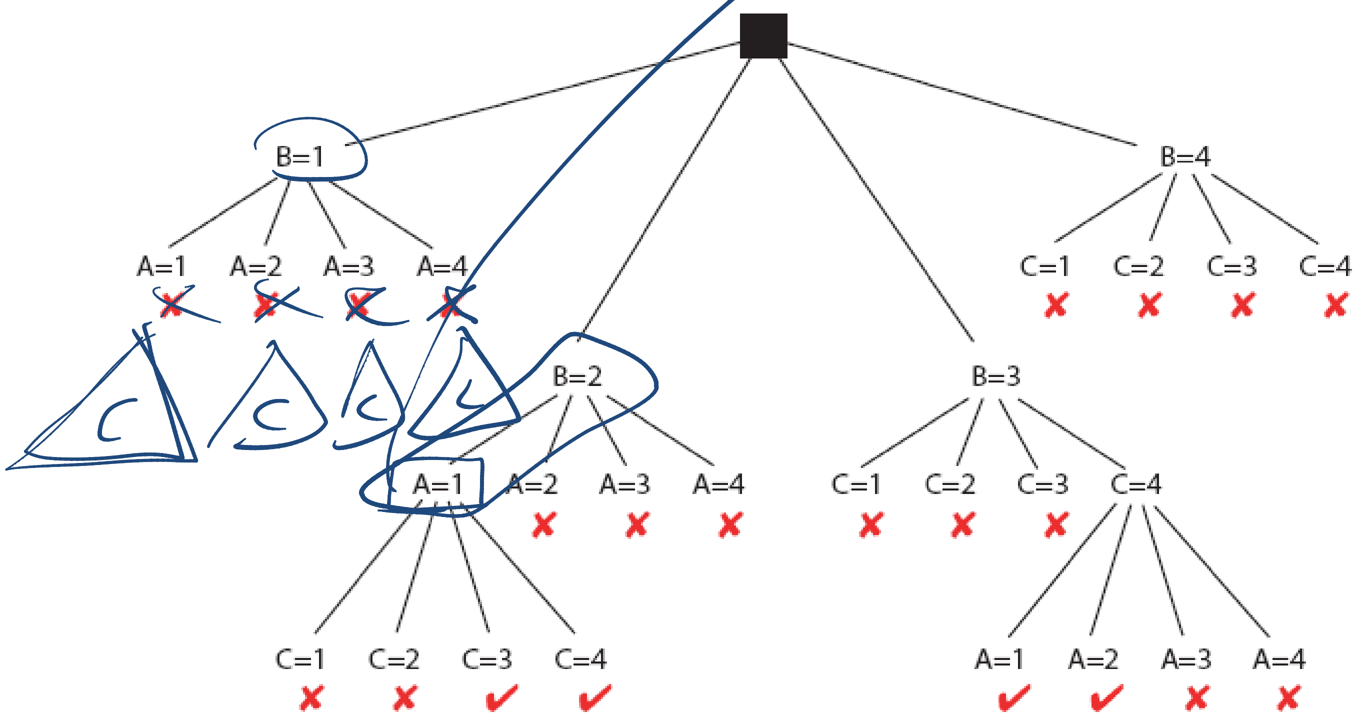
constraints (A = B) (F > A) (C = E)

Solving CSPs by DFS: Example

Problem:

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints: A < B, B < C

B = 2
A = 1



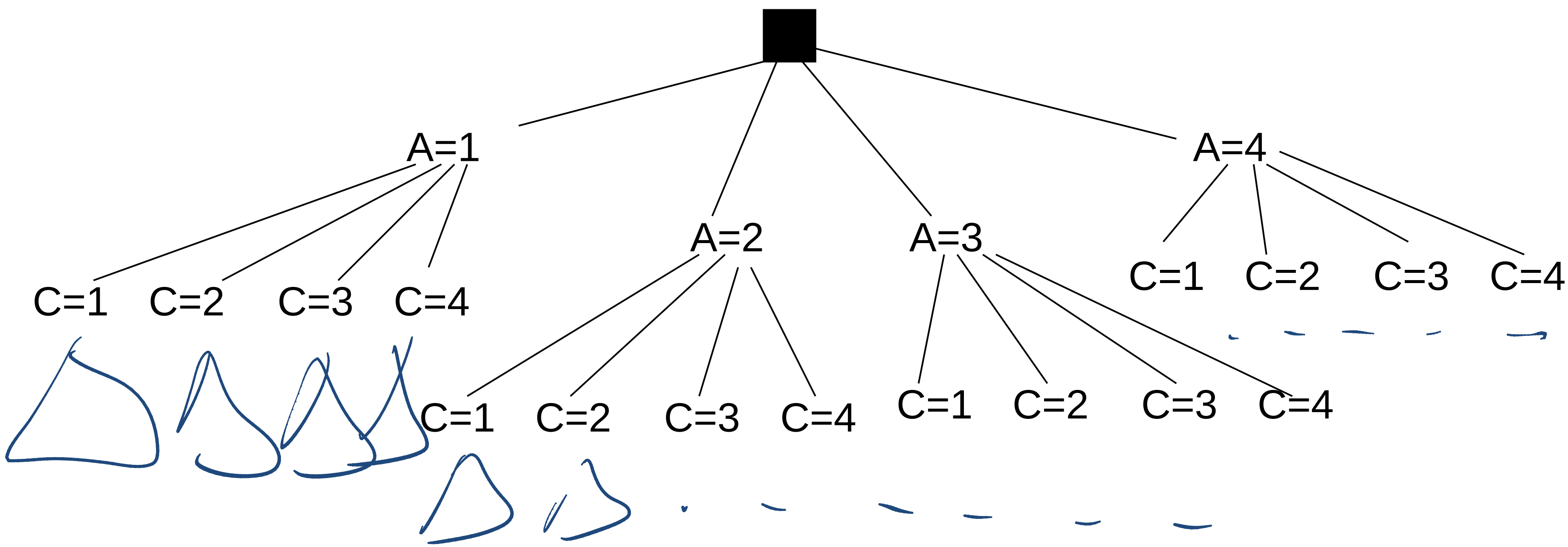
Solving CSPs by DFS: Example Efficiency

Problem:

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints: A < B, B < C

Note: the algorithm's efficiency depends on the order in which variables are expanded

Degree "Heuristics"



Standard Search vs. Specific R&R systems

Constraint Satisfaction (Problems):

- State: assignments of values to a subset of the variables
- Successor function: assign values to a "free" variable
- Goal test: set of constraints
- Solution: possible world that satisfies the constraints
- Heuristic function: *none (all solutions at the same distance from start)*

Planning :

- State
- Successor function
- Goal test
- Solution
- Heuristic function

Inference

- State
- Successor function
- Goal test
- Solution
- Heuristic function

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Can we do better than Search?

Key ideas:

- **prune the domains** as much as possible **before "searching"** for a solution.

Simple when using constraints involving single variables (technically enforcing **domain consistency**)

> **Definition:** A variable is domain consistent if no value of its domain is ruled impossible by any unary constraints.

- Example: if we have the constraint $B \neq 3$ $\mathbf{D}_B = \{1, 2, \cancel{3}, 4\}$ is not domain consistent.

to make it consistent

How do we deal with constraints involving multiple variables?

Definition (**constraint network**)

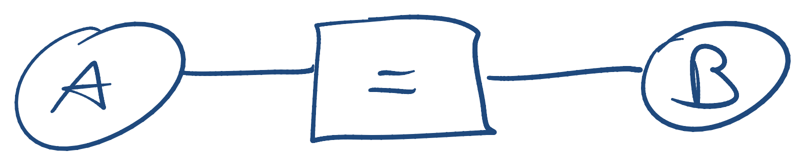
A constraint network is defined by a graph, with

- one **node** for every **variable**
- one **node** for every **constraint**

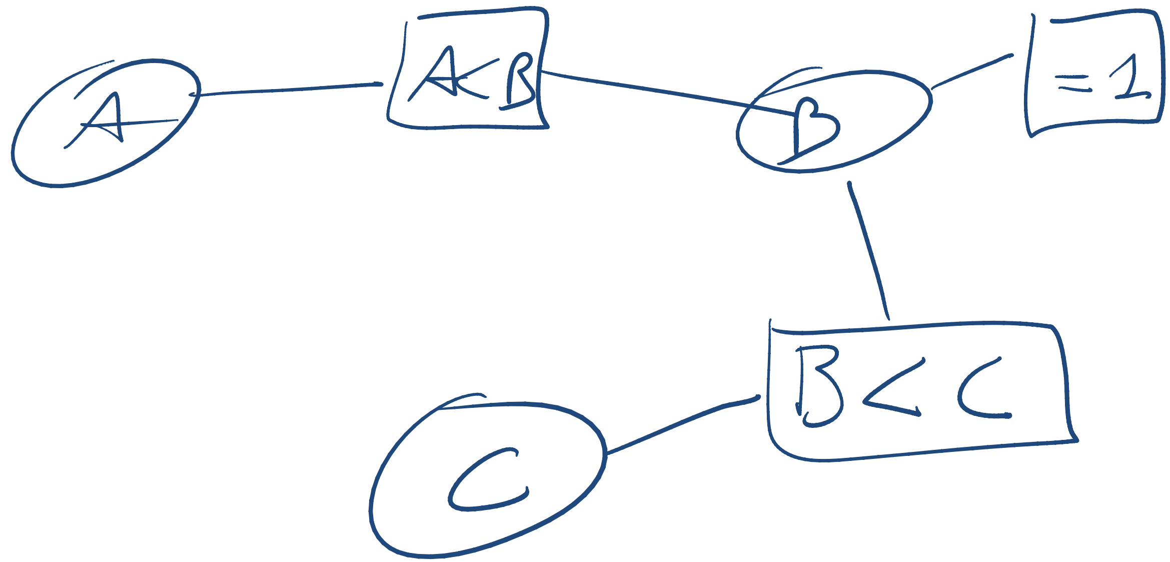
and undirected edges running between variable nodes and constraint nodes whenever a given variable is involved in a given constraint.

$A \quad B \quad \{0, 1\}$

$A = B$



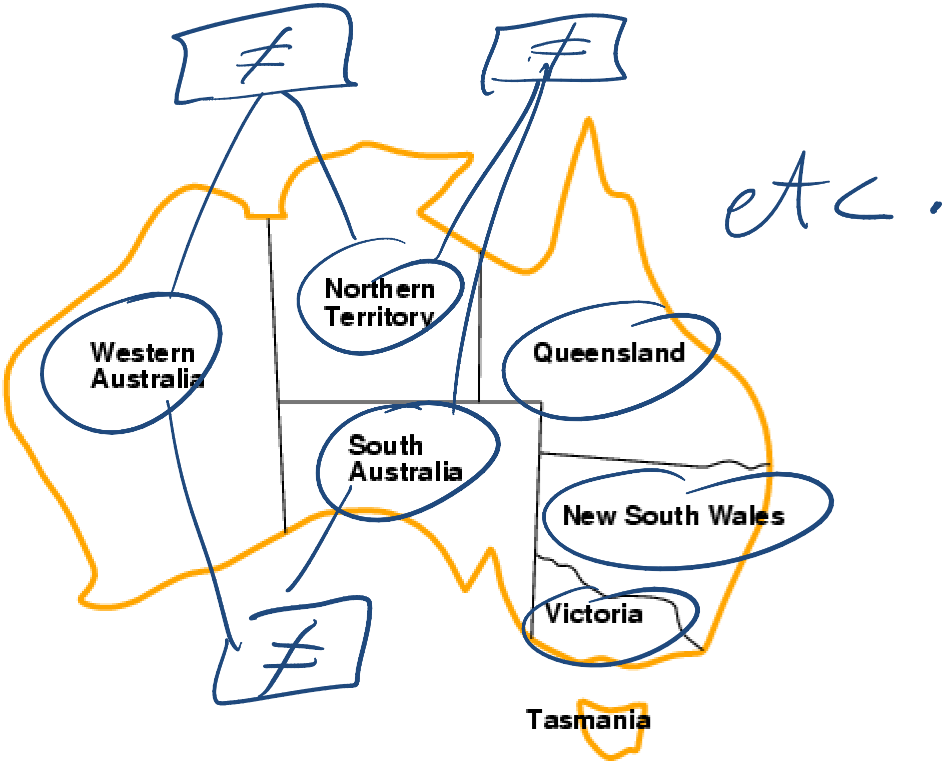
Example Constraint Network



Recall Example:

- Variables: A,B,C
- Domains: {1, 2, 3, 4}
- Constraints: A < B, B < C , B = 1

Example: Constraint Network for Map-Coloring



Variables *WA, NT, Q, NSW, V, SA, T*

Domains D_i = {red,green,blue}

Constraints: adjacent regions must have different colors

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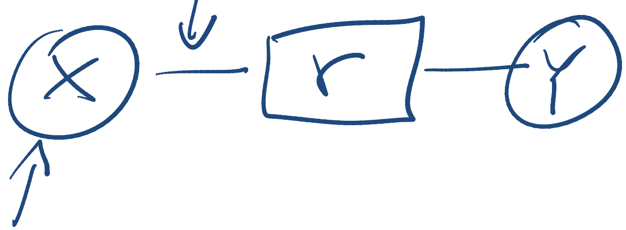
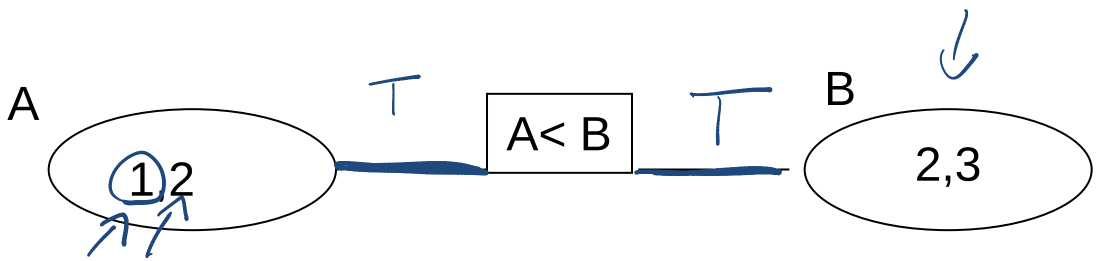
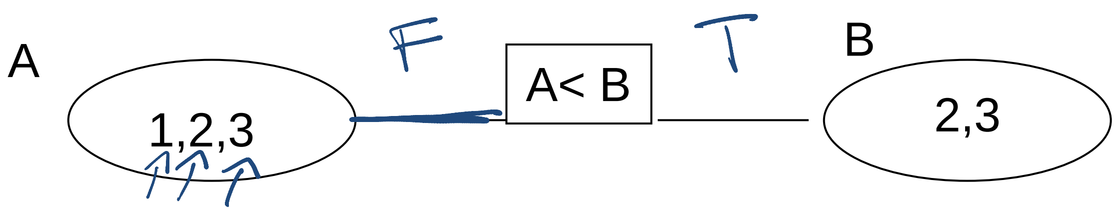
Arc Consistency

Definition (arc consistency)

An arc $\langle X, r(X,Y)\rangle$ is arc consistent if for each value x in $dom(X)$ there is some value y in $dom(Y)$ such that $r(x,y)$ is satisfied.







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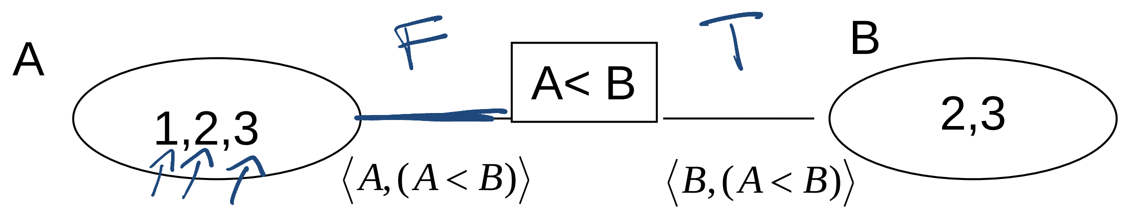
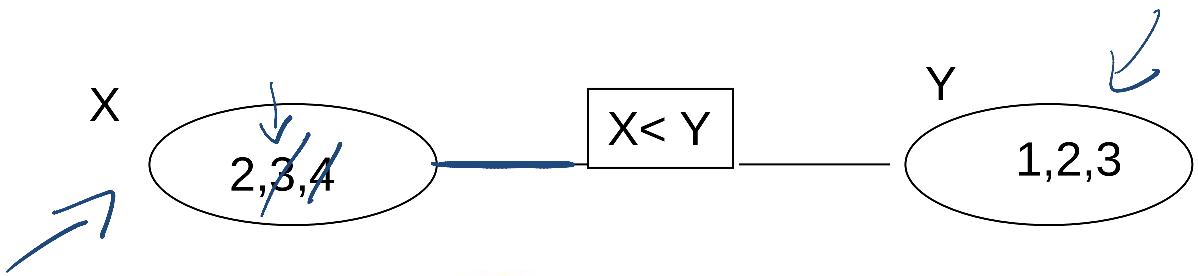


A. Both arcs are consistent
B. Left consistent, right inconsistent
C. Right ~~i~~nconsistent, left *in* consistent
D. Both arcs are inconsistent

How can we enforce Arc Consistency?

- If an arc $\langle X, r(X,Y)\rangle$ is not arc consistent, all values x in $dom(X)$ for which there is no corresponding value in $dom(Y)$ may be deleted from $dom(X)$ to make the arc $\langle X, r(X,Y)\rangle$ consistent.
  - This removal can never rule out any models/solutions



- **A network is arc consistent** if all its arcs are arc consistent.

Learning Goals for today's class

You can:

- Implement the Generate-and-Test Algorithm. Explain its disadvantages.
- Solve a CSP by search (specify neighbors, states, start state, goal state). Compare strategies for CSP search. Implement pruning for DFS search in a CSP.
- Build a constraint network for a set of constraints.
- Verify whether a network is arc consistent.
- Make an arc arc-consistent.

Next class

How to make a constraint network arc consistent?

Arc Consistency Algorithm

There are Practice Exercises for CSP