

# Logic: TD as search, Datalog (variables)

Computer Science cpsc322, Lecture 23

*(Textbook Chpt 5.2 &  
some basic concepts from Chpt 12)*



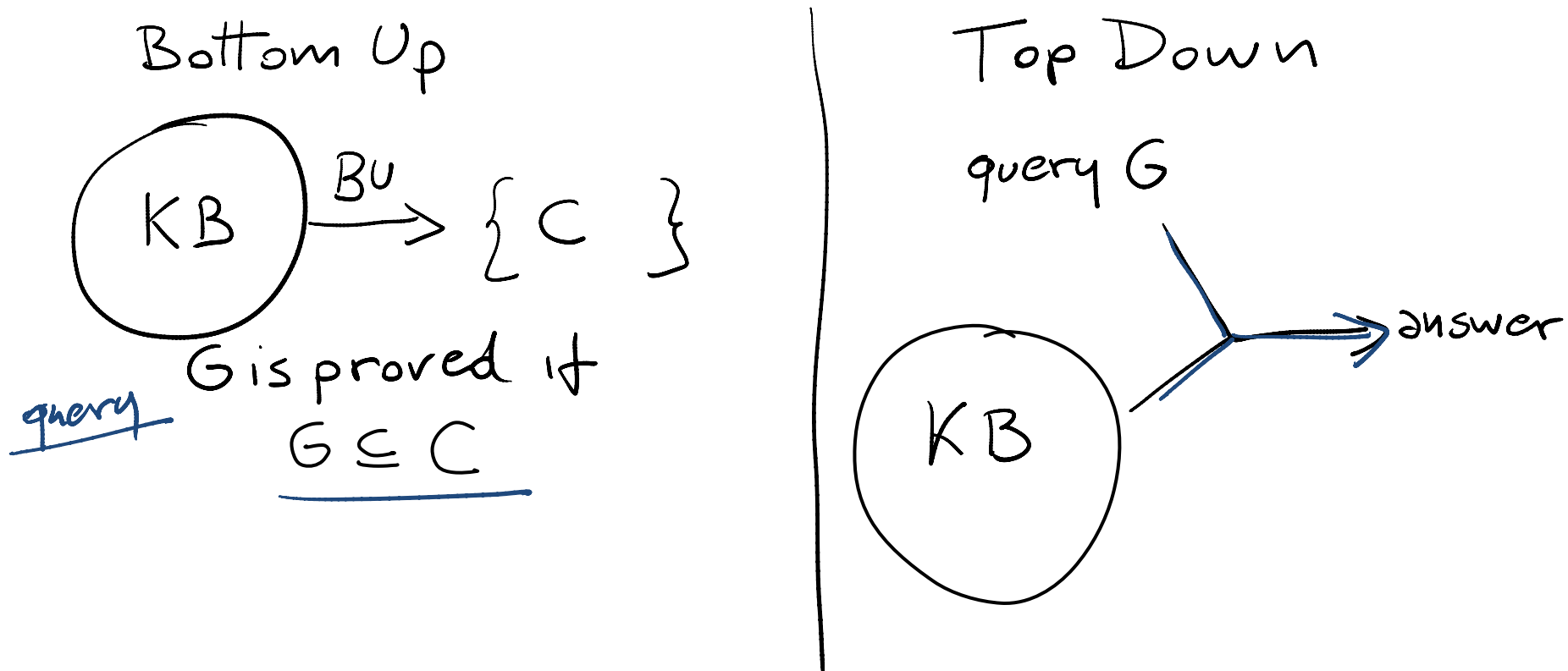
March, 6, 2009

# Lecture Overview

- Recap Top Down
- TopDown Proofs as search
- Datalog

# Top-down Ground Proof Procedure

**Key Idea:** search backward from a query  $G$  to determine if it can be derived from  $KB$ .



# Top-down Proof Procedure: Basic elements

**Notation:** An **answer clause** is of the form:

$$\text{yes} \leftarrow a_1 \wedge a_2 \wedge \dots \wedge a_m$$

Express query as an **answer clause** (e.g., query  $a_1 \wedge a_2 \wedge \dots \wedge a_m$ )

$$\text{yes} \leftarrow a_1 \wedge \dots \wedge a_m$$

**Rule of inference** (called SLD Resolution)

Given an **answer clause** of the form:

$$\text{yes} \leftarrow a_1 \wedge a_2 \wedge \dots \wedge a_m \leftarrow$$

and the clause: from  $K \cup B$

$$a_i \leftarrow b_1 \wedge b_2 \wedge \dots \wedge b_p$$

$$a_i \leftarrow \square$$

You can generate the answer clause

$$\text{yes} \leftarrow a_1 \wedge \dots \wedge a_{i-1} \wedge b_1 \wedge b_2 \wedge \dots \wedge b_p \wedge a_{i+1} \wedge \dots \wedge a_m$$

- **Successful Derivation:** When by applying the inference rule you obtain the answer clause yes  $\leftarrow$  . *empty body*

$a \leftarrow e \wedge f.$

$c \leftarrow e.$

$f \leftarrow j \wedge e.$

$a \leftarrow b \wedge c.$

$d \leftarrow k.$

$f \leftarrow c.$

$b \leftarrow k \wedge f.$

$e.$

$j \leftarrow c.$

KB

Query: a (two ways)

$yes \leftarrow a.$

$yes \leftarrow e \wedge f$

$yes \leftarrow f$

$yes \leftarrow c$

$yes \leftarrow e \rightarrow yes$

$yes \leftarrow a.$

$yes \leftarrow b \wedge c$

~~$yes \leftarrow e \wedge f \wedge c$~~

$\vdots$  fails

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# Systematic Search in different R&R systems

## Constraint Satisfaction (Problems):

- **State**: assignments of values to a subset of the variables
- **Successor function**: assign values to a “free” variable
- **Goal test**: set of constraints
- **Solution**: possible world that satisfies the constraints
- **Heuristic function**: *none (all solutions at the same distance from start)*

## Planning (forward) :

- **State** possible world
- **Successor function** states resulting from valid actions
- **Goal test** assignment to subset of vars
- **Solution** sequence of actions
- **Heuristic function** empty-delete-list (solve simplified problem)

## Logical Inference (top Down)

- **State** answer clause
- **Successor function** states resulting from substituting one atom with all the clauses of which it is the head
- **Goal test** empty answer clause ←
- **Solution** start state
- **Heuristic function** .....

start state: query as answer clause

number of atoms in body of state/answer clause

# Search Graph

Prove:  $? \leftarrow a \wedge d.$

KB

$a \leftarrow b \wedge c.$

$a \leftarrow h.$

$b \leftarrow k.$

$d \leftarrow p.$

$f \leftarrow p.$

$g \leftarrow f.$

$h \leftarrow m.$

$a \leftarrow g.$

$b \leftarrow j.$

$d \leftarrow m.$

$f \leftarrow m.$

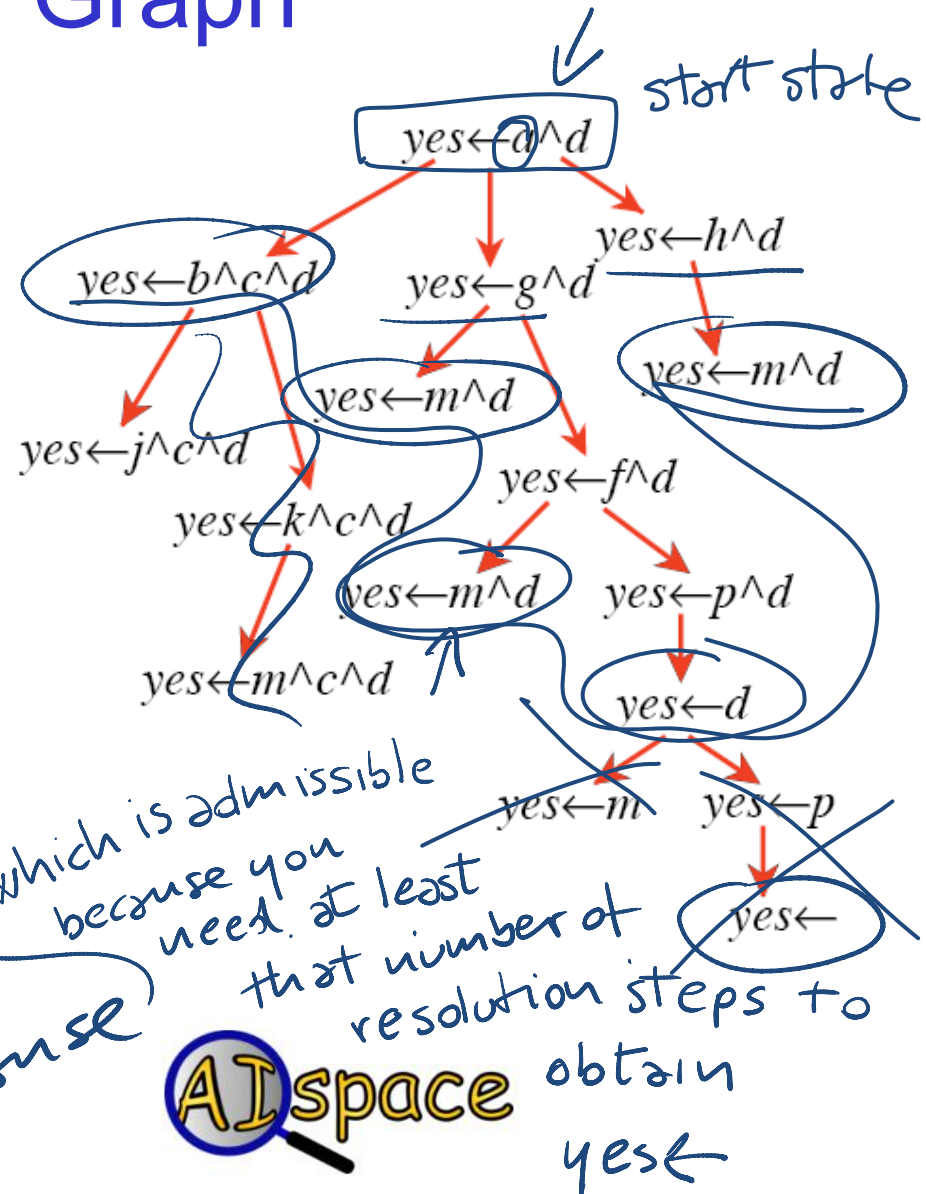
$g \leftarrow m.$

$k \leftarrow m.$

$p.$

Heuristics?

# atoms in the answer clause





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- **Datalog**

# Representation and Reasoning in Complex domains

- In complex domains expressing knowledge with **propositions** can be quite limiting

*up\_s<sub>2</sub>*  
*up\_s<sub>3</sub>*  
*ok\_cb<sub>1</sub>*  
*ok\_cb<sub>2</sub>*  
*live\_w<sub>1</sub>*  
*connected\_w<sub>1</sub>\_w<sub>2</sub>*

*The system  
can reason  
about*

- It is often **natural** to consider individuals and their **properties**

*aka relations/predicates*

*up(s<sub>2</sub>)*  
*up(s<sub>3</sub>)*  
*ok(cb<sub>1</sub>)*  
*ok(cb<sub>2</sub>)*  
*live(w<sub>1</sub>)*  
*connected(w<sub>1</sub>, w<sub>2</sub>)*

There is no notion that

*are about the same property 'up'*

*are about the same individual w<sub>1</sub>*

# What do we gain....

By breaking propositions into relations applied to individuals?

- Express knowledge that holds for set of individuals  
(by introducing *variables* )  
*variables*

$$\underline{live(W)} \leftarrow \underline{connected\_to(W, W1)} \wedge \underline{live(W1)} \wedge \underline{wire(W)} \wedge \underline{wire(W1)}.$$

- We can ask generic queries (i.e., containing *variables* )  
*variables*

$$? \underline{connected\_to(W, w_1)}$$

# Datalog vs PDCL (better with colors)

## First Order Logic

$$\forall X \exists Y p(X, Y) \Leftrightarrow \neg q(Y)$$

$$p(a_1, a_2) \\ \neg q(a_5)$$

## Propositional Logic

$$\neg (p \vee q) \rightarrow (r \wedge s \wedge t), \\ p, r$$

## Datalog

$$p(X) \leftarrow q(X) \wedge r(X, Y)$$

$$r(X, Y) \leftarrow s(Y)$$

$$s(a_1), q(a_2)$$

## PDCL

$$p \leftarrow s \wedge t$$

$$r \leftarrow s \wedge q \wedge p$$

$$r \\ p$$

# Datalog: a relational rule language

It expands the syntax of PDCL....

A **variable** is a symbol starting with an upper case letter

X Y

A **constant** is a symbol starting with lower-case letter or a sequence of digits.

w<sub>1</sub> 2/24

A **term** is either a variable or a constant.

A **predicate symbol** is a symbol starting with lower-case letter.

in part-of live

# Datalog Syntax (cont')

An **atom** is a symbol of the form  $p$  or  $p(t_1 \dots t_n)$  where  $p$  is a predicate symbol and  $t_i$  are terms

*summary*  
↑  
includes propositions  
 $in(alam, X)$

A **definite clause** is either an atom (a fact) or of the form:

$$h \leftarrow b_1 \wedge \dots \wedge b_m$$

where  $h$  and the  $b_i$  are atomic symbol (Read this as " $h$  if  $b$ .")

$$in(X, Y) \leftarrow in(X, Z) \wedge part-of(Z, Y)$$

A **knowledge base** is a set of definite clauses

# Datalog: Top Down Proof

Extension of TD for PDCL.

How do you deal with variables?

**Example:**

in(alan, r123).

part\_of(r123, cs\_building).

in(X, Y) <- part\_of(Z, Y) & in(X, Z).

first resolution  
step

**Query:** in(alan, cs\_building).

yes <- in(alan, cs\_building).

KB

output of resolution  
part-of(Z, cs-bldg) & in(alan, Z)

AI space

# Datalog: queries with variables

`in(alan, r123).`  $\rightarrow$   $yes(r123) \leftarrow$

`part_of(r123, cs_building).`

`in(X, Y) <- in(X, Z). & part_of(Z, Y)`

$\rightarrow yes(X1) \leftarrow in(alan, Z) \& part\_of(Z, X1)$

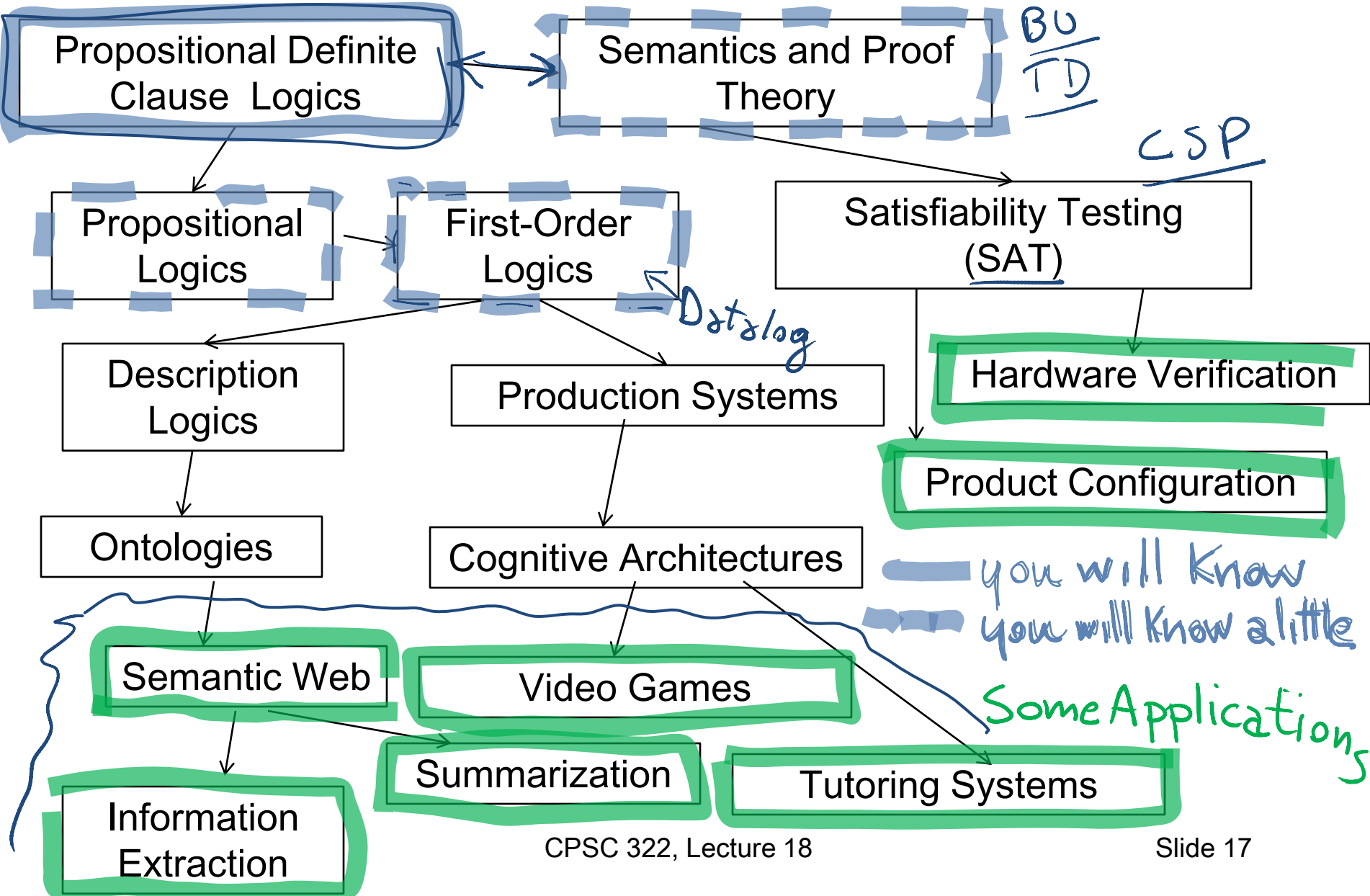
Query: `in(alan, X1).`

`Yes(X1) <- in(alan, X1).`

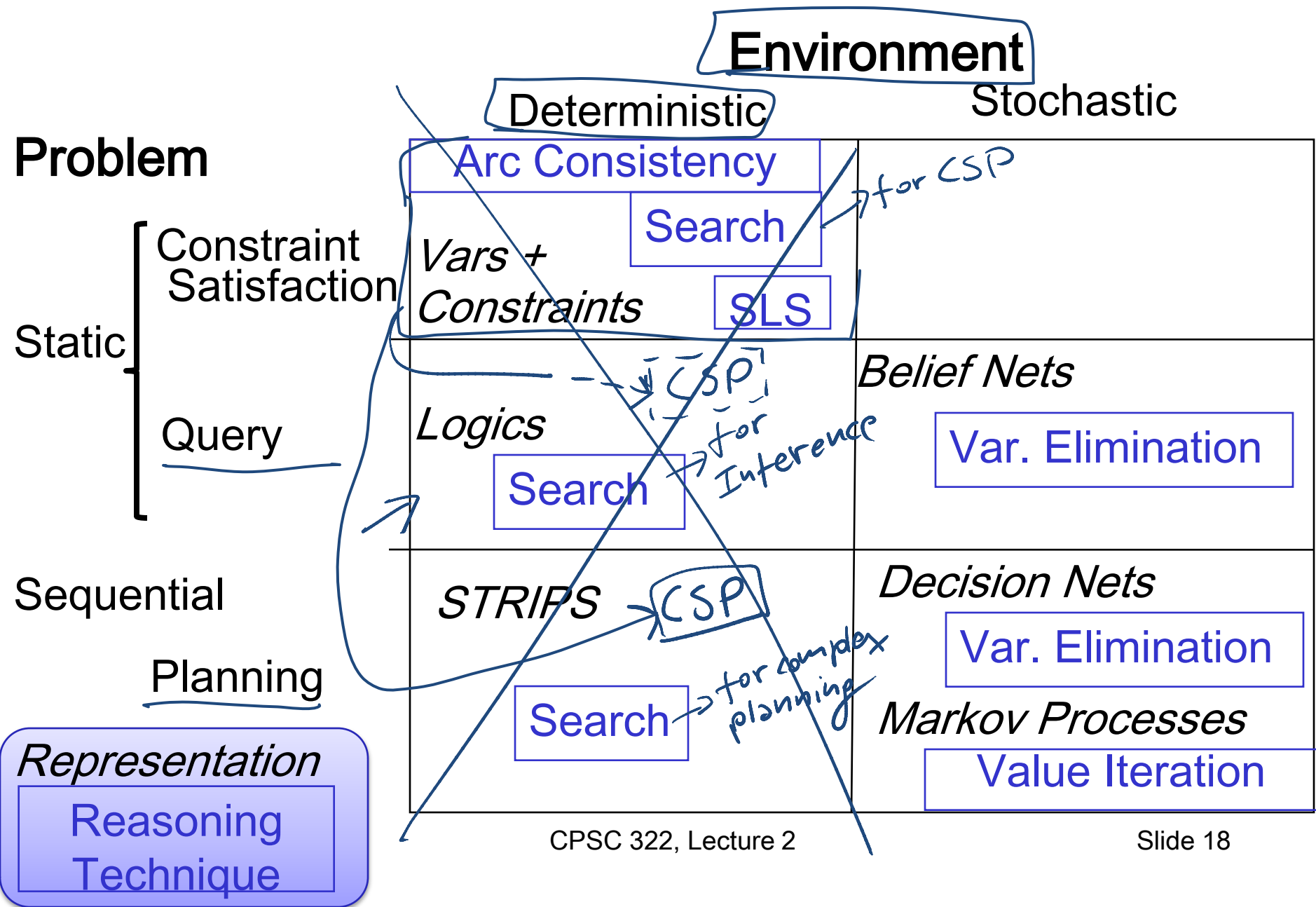




# Logics in AI: Similar slide to the one for planning



# Big Picture: R&R systems



# Learning Goals for today's class

You can:

- Define/read/write/trace/debug the **TopDown** proof procedure (as a **search** problem)
- Represent simple domains in **Datalog**
- Apply **TopDown** proof procedure in **Datalog**

# Midterm review

*(without outlier who did 0%)*

**Average 72%** 😊

**Best 93%, Worst 23%**

**Only three <50%**

## How to learn more from midterm

- Carefully examine your mistakes (and our feedback)
- If you still do not see the correct answer/solution go back to your notes, the slides and the textbook
- If you are still confused come to office hours with specific questions