Logic: Domain Modeling /Proofs + Top-Down Proofs

Computer Science cpsc322, Lecture 22

(Textbook Chpt 5.2)

March, 2, 2009

Lecture Overview

- Recap
- Using Logic to Model a Domain (Electrical System)
- Reasoning/Proofs (in the Electrical Domain)
- Top-Down Proof Procedure

Soundness & completeness of proof procedures

- A proof procedure X is sound …

$$KB \vdash_X G \Rightarrow KB \models G$$

- A proof procedure X is complete….

$$KB \models G \Rightarrow KB \vdash_X G$$

- BottomUp for PDCL is

sound & complete $\leftarrow$

- We proved this in general even for domains represented by thousands of propositions and corresponding KB with millions of definite clauses !

Can you think of a proof procedure for PDCL….

A: $C_A = \{\text{all clauses with empty bodies}\}$

$G \subseteq C_A$ $\qquad$ $C_A \subseteq C_{BU}$

B: $C_B = \{\text{all atoms of } KB\}$

$G \subseteq C_B$ $\qquad$ $C_{BU} \subseteq C_B$

KB

$a \leftarrow e \wedge g.$

$b \leftarrow f \wedge g.$

$d \leftarrow e.$

$f \leftarrow c \wedge e.$

$e.$

$d.$

C_B

C_A

- That is sound but not complete?

A $\quad$ $KB \vdash_A G \Rightarrow G \subseteq C_A \Rightarrow G \subseteq C_{BU} \Rightarrow KB \vdash_{BU} G$

$KB \models G$

- That is complete but not sound?

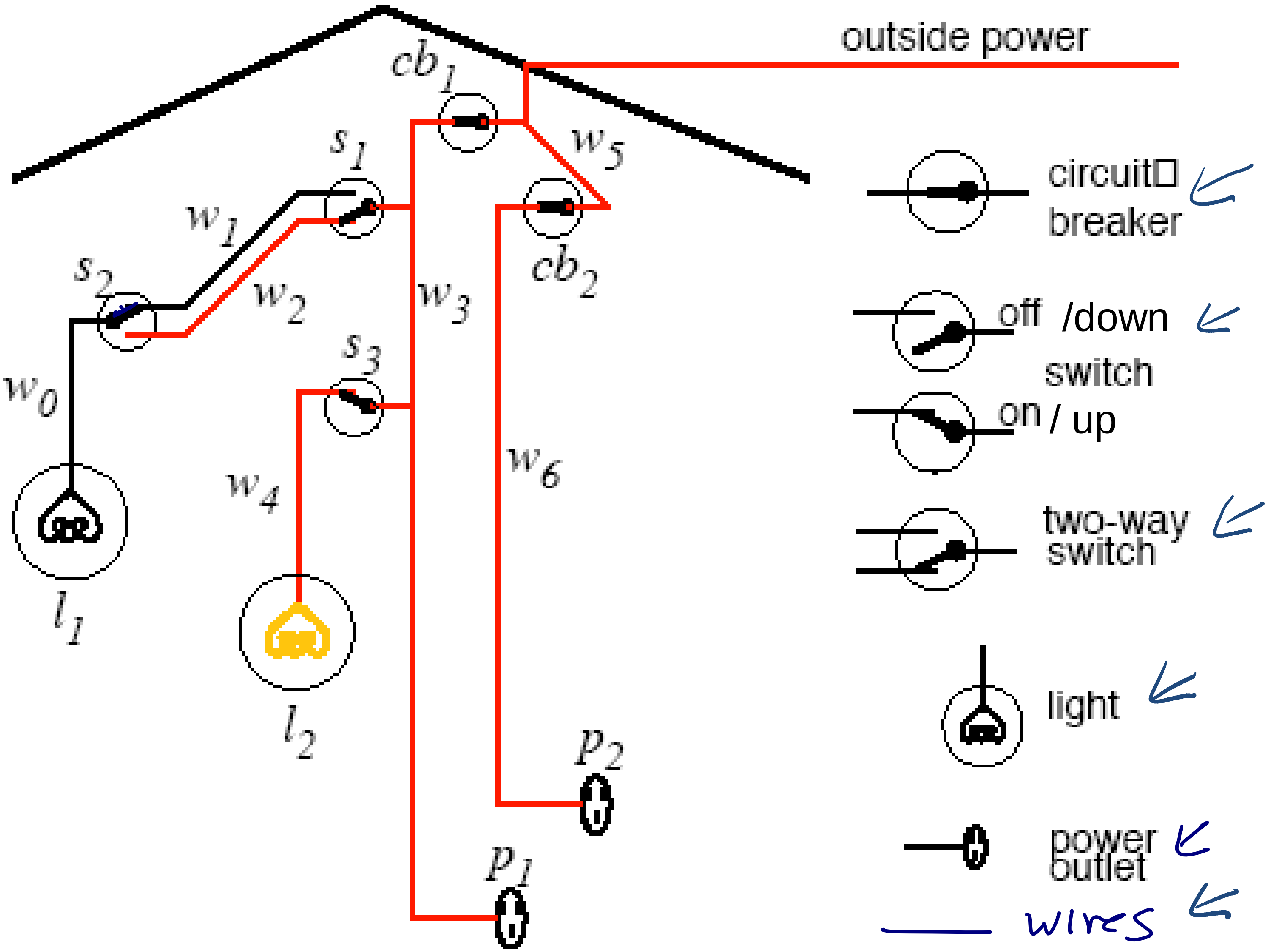
B $\quad$ $KB \models G \Rightarrow KB \vdash_{BU} G \Rightarrow G \subseteq C_{BU} \Rightarrow$

$G \subseteq C_B \Rightarrow KB \vdash_B G$

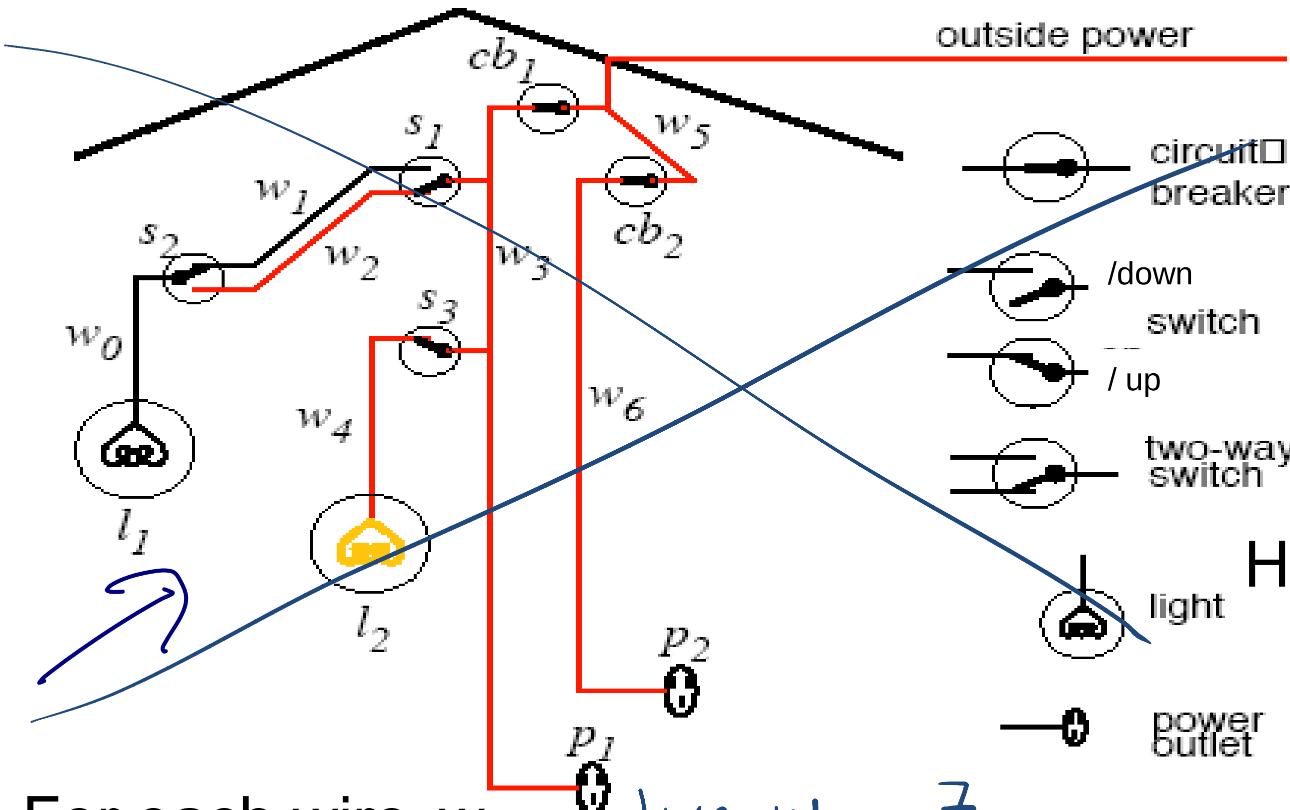
Lecture Overview

- Recap
- Using PDCL Logic to Model a Domain (Electrical System)
- Reasoning/Proofs (in the Electrical Domain)
- Top-Down Proof Procedure

Electrical Environment



Let's define relevant propositions



2^{19}

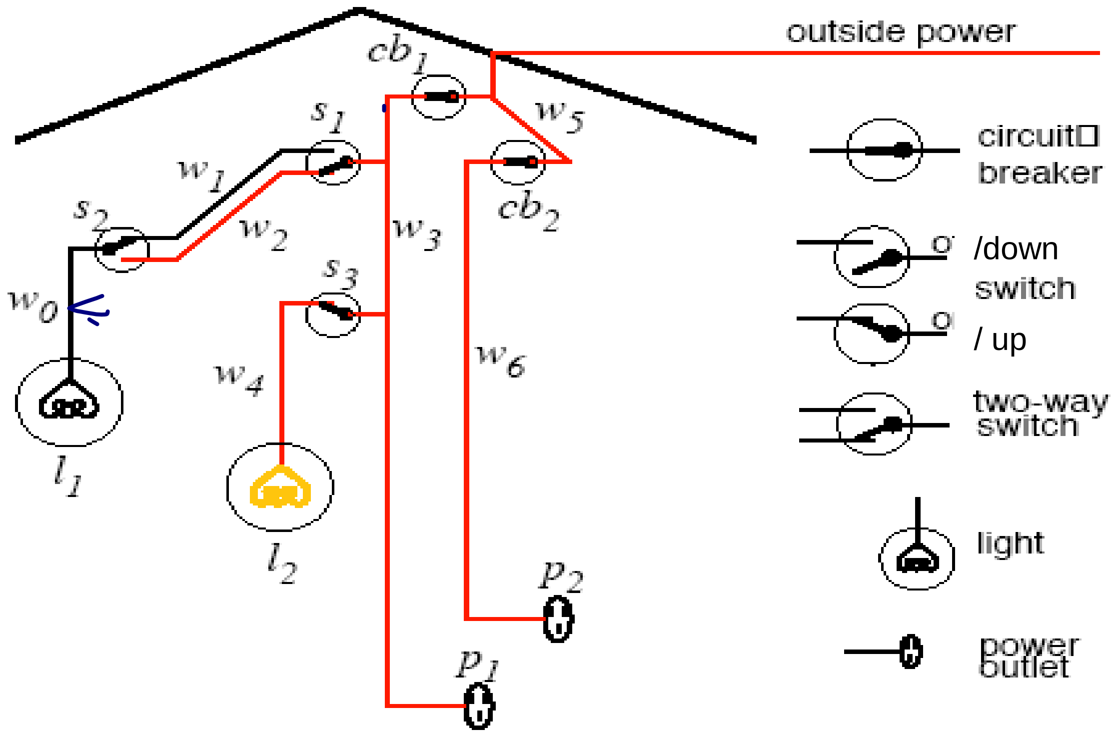
How many interpretations?

- For each wire *w* — live_w_i — 7
- For each circuit breaker *cb* — ok_cb_i — 2
- For each switch *s* — up_s_i, down_s_i — 3x2
- For each light *l* — live_l_i — 2
- For each outlet *p* — live_p_i — 2

19

- 7
- 2
- 3 x 2
- 2
- 2

Let's now tell system knowledge about how the domain works



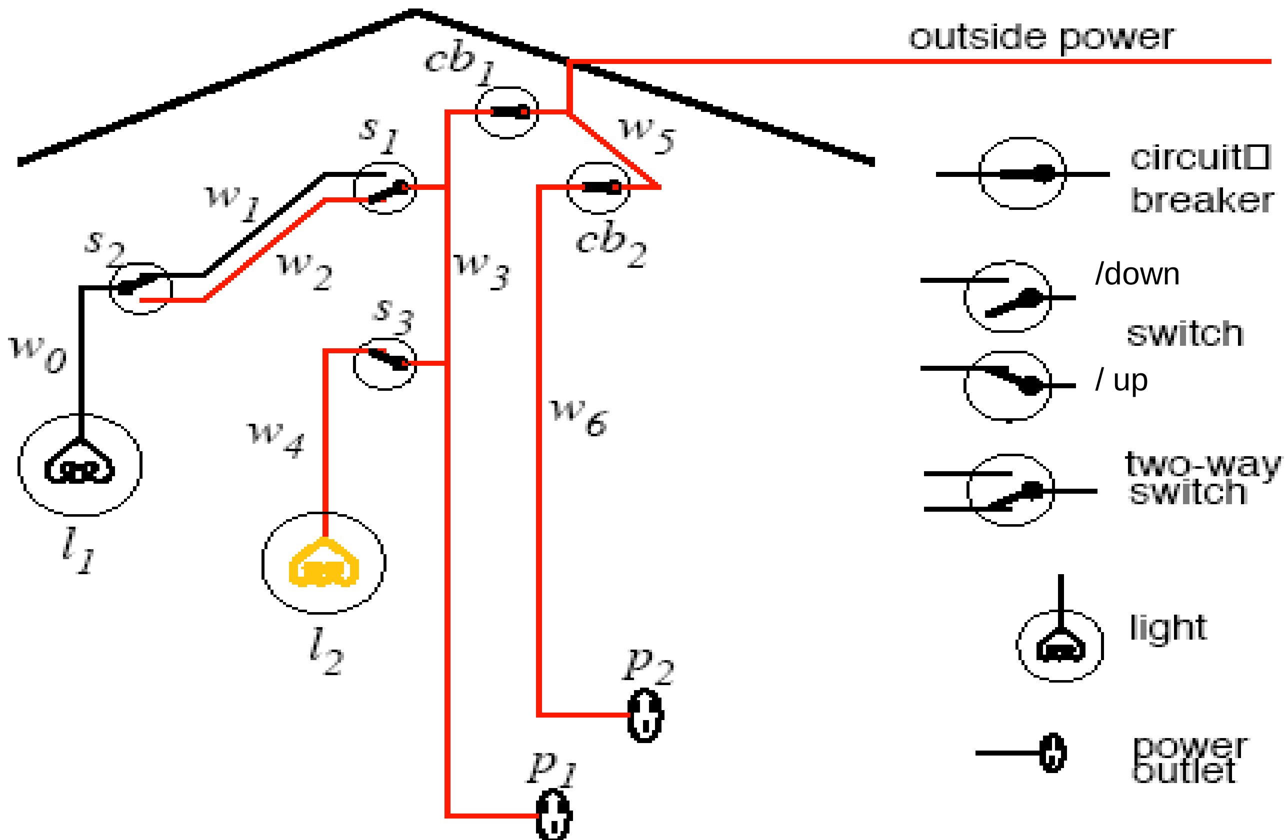
$live\_l_1 \leftarrow live\_w_0$

$live\_w_0 \leftarrow up\_s_2 \wedge live\_w_1$

$live\_w_0 \leftarrow down\_s_2 \wedge live\_w_2$

$live\_w_1 \leftarrow up\_s_1 \wedge live\_w_3$

More on how the domain works….



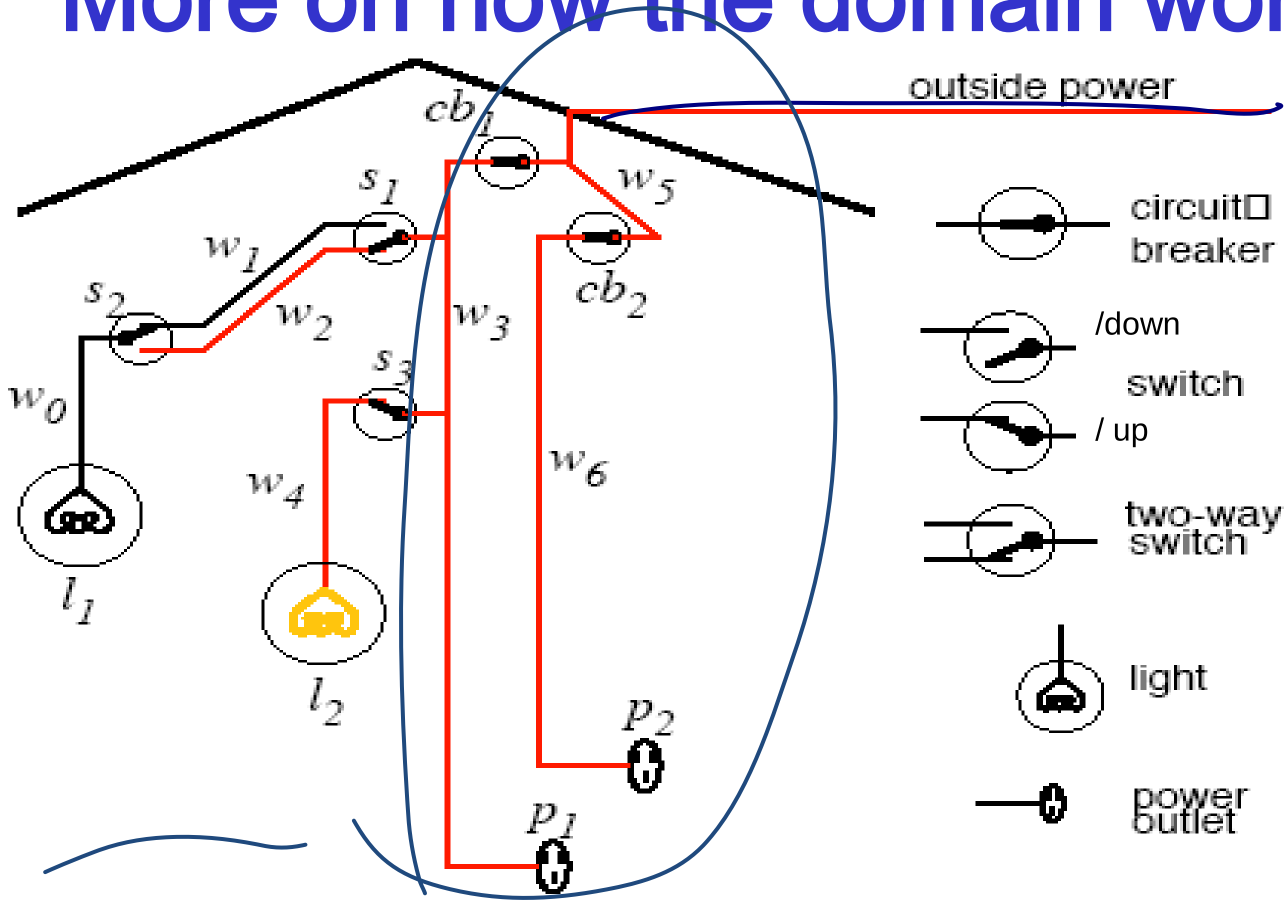
$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$

$live\_l_2 \leftarrow live\_w_4.$

$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$

$live\_p_1 \leftarrow live\_w_3.$

More on how the domain works….



$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$

$live\_p_2 \leftarrow live\_w_6.$

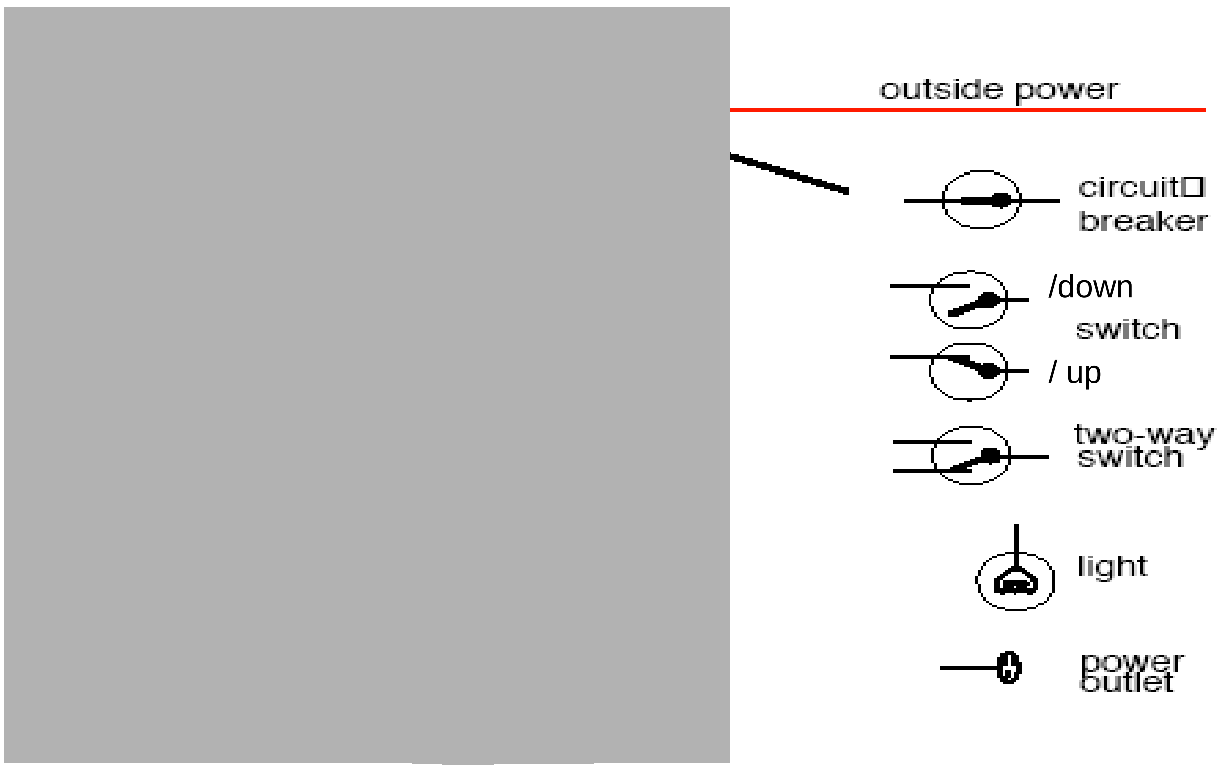
$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$

$live\_w_5 \leftarrow live\_outside.$

What else we may know about this domain?

- That some simple propositions are true

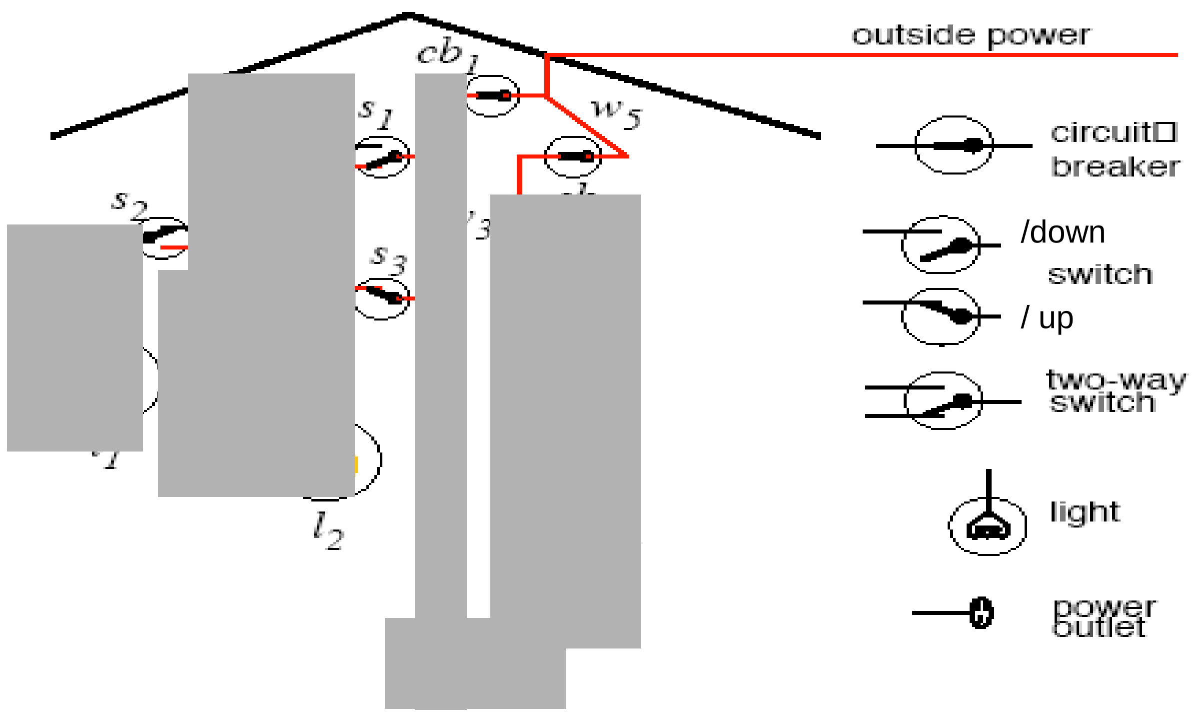
    live_outside.



What else we may know about this domain?

- That some additional simple propositions are true

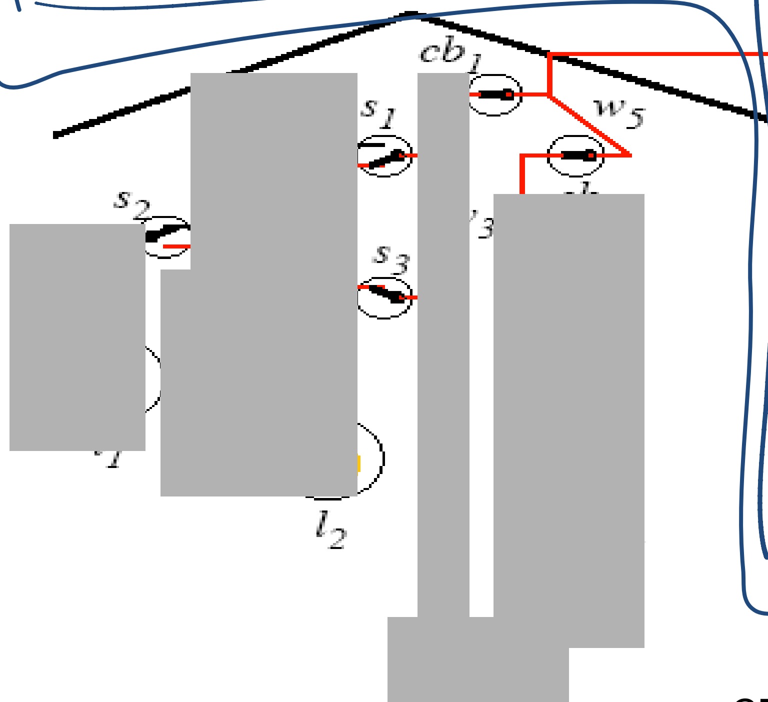
$down\_s_1$. up_s_2. up_s_3. ok_cb_1. ok_cb_2. $live\_outside$.



All our knowledge…..

KB

$down\_s_1.$
$up\_s_2.$
$up\_s_3.$
$ok\_cb_1.$
$ok\_cb_2.$
$live\_outside$



$live\_l_1 \leftarrow live\_w_0$
$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2.$
$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2.$
$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1.$
$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1.$
$live\_l_2 \leftarrow live\_w_4.$
$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3.$
$live\_p_1 \leftarrow live\_w_3.$
$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1.$
$live\_p_2 \leftarrow live\_w_6.$
$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2.$
$live\_w_5 \leftarrow live\_outside.$

Lecture Overview

- Recap
- Using Logic to Model a Domain (Electrical System)
- Reasoning/Proofs (in the Electrical Domain)
- Top-Down Proof Procedure

What Semantics is telling us

- Our KB (all we know about this domain) is going to be true only in a subset of all possible ___2^{19}___ interpretations

- What is logically entailed by our KB are all the propositions that are true in all those interpretations

- This is what we should be able to derive given a sound and complete proof procedure

If we apply the bottom-up (BU) proof procedure

$down\_s_1$.
up_s_2.
up_s_3.
ok_cb_1.
ok_cb_2.
$live\_outside$

$live\_l_1 \leftarrow live\_w_0$
$live\_w_0 \leftarrow live\_w_1 \wedge up\_s_2$.
$live\_w_0 \leftarrow live\_w_2 \wedge down\_s_2$.
$live\_w_1 \leftarrow live\_w_3 \wedge up\_s_1$.
$live\_w_2 \leftarrow live\_w_3 \wedge down\_s_1$.
$live\_l_2 \leftarrow live\_w_4$.
$live\_w_4 \leftarrow live\_w_3 \wedge up\_s_3$.
$live\_p_1 \leftarrow live\_w_3$..
$live\_w_3 \leftarrow live\_w_5 \wedge ok\_cb_1$.
$live\_p_2 \leftarrow live\_w_6$.
$live\_w_6 \leftarrow live\_w_5 \wedge ok\_cb_2$.
$live\_w_5 \leftarrow live\_outside$.

BU generates C

all the atoms added to C are in green

$live\_l_2 \subseteq C$

$KB \vdash_{BU} live\_l_2$

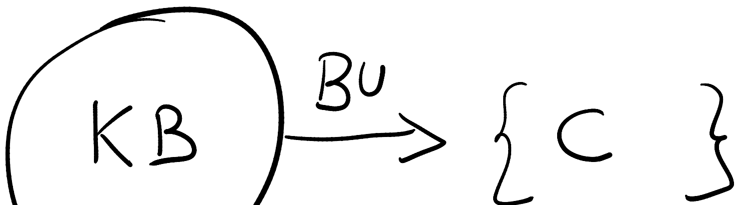
Lecture Overview

- Recap
- Using Logic to Model a Domain (Electrical System)
- Reasoning/Proofs (in the Electrical Domain)
- Top-Down Proof Procedure

Top-down Ground Proof Procedure

Key Idea: search backward from a query *G* to determine if it can be derived from *KB*.

Bottom Up

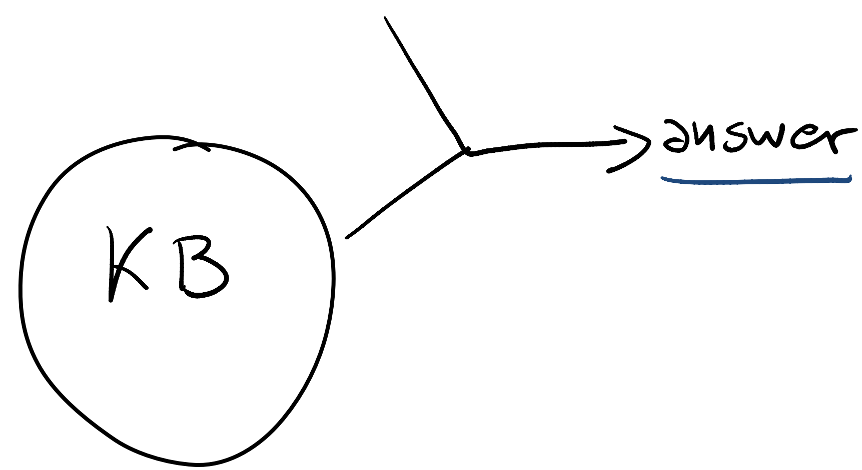


G is proved if
$G \subseteq C$

look at query only
at the end

Top Down

query G



Top-down Proof Procedure: Basic elements

Notation: An answer clause is of the form:

$yes \leftarrow a_1 \wedge a_2 \wedge \ldots \wedge a_m$

Express query as an answer clause (e.g., query $a_1 \wedge a_2 \wedge \ldots \wedge a_m$)

$yes \leftarrow a_1 \wedge \ldots \wedge a_m$

Rule of inference (called SLD Resolution)

Given an answer clause of the form:

$yes \leftarrow a_1 \wedge a_2 \wedge \ldots \wedge a_m$

and the clause:

$a_i \leftarrow b_1 \wedge b_2 \wedge \ldots \wedge b_p$

You can generate the answer clause

$yes \leftarrow a_1 \wedge \ldots \wedge a_{i-1} \wedge b_1 \wedge b_2 \wedge \ldots \wedge b_p \wedge a_{i+1} \wedge \ldots \wedge a_m$

Rule of inference: Examples

Rule of inference (called SLD Resolution)
Given an answer clause of the form:

$$yes \leftarrow a_1 \wedge a_2 \wedge \ldots \wedge a_m$$

and the clause:

$$a_i \leftarrow b_1 \wedge b_2 \wedge \ldots \wedge b_p$$

You can generate the answer clause

$$yes \leftarrow a_1 \wedge \ldots \wedge a_{i-1} \wedge b_1 \wedge b_2 \wedge \ldots \wedge b_p \wedge a_{i+1} \wedge \ldots \wedge a_m$$

KB clause

$yes \leftarrow b \wedge c.$ $\qquad$ $b \leftarrow k \wedge f.$ $\qquad \Rightarrow yes \leftarrow k \wedge f \wedge c$

$Yes \leftarrow e \wedge f.$ $\qquad$ $e.$ $\qquad \Rightarrow yes \leftarrow f$

(successful) Derivations

- An answer is an answer clause with $m = 0$. That is, it is the answer clause $\boxed{yes \leftarrow .}$

- A (successful) derivation of query "$?q_1 \wedge \ldots \wedge q_k$" from KB is a sequence of answer clauses such that $\gamma_0, \gamma_1, \ldots, \gamma_n$ such that
  - γ_0 is the answer clause $\boxed{yes \leftarrow q_1 \wedge \ldots \wedge q_k}$ G
  - γ_i is obtained by resolving γ_{i-1} with a clause in KB, and
  - γ_n is an answer. yes ←

- An unsuccessful derivation…..

yes ← a ∧ b

Example: derivations

KB

$a \leftarrow e \wedge f.$	$a \leftarrow b \wedge c.$	$b \leftarrow k \wedge f.$
$c \leftarrow e.$	$d \leftarrow k.$	$e.$
$f \leftarrow j \wedge e.$	$f \leftarrow c.$	$j \leftarrow c.$

Query: a `(two ways)`

$yes \leftarrow a.$

$yes \Leftarrow e \wedge f$

$yes \Leftarrow f$

$yes \Leftarrow c$

$yes \Leftarrow e$

$yes \Leftarrow$

$yes \leftarrow a.$

$yes \Leftarrow b \wedge c$

$yes \Leftarrow b \wedge e$

$yes \Leftarrow b$

$yes \Leftarrow k \wedge f$

⋮

Query: b `(k, f different order)`

$yes \leftarrow b.$

Course Big Picture

Problem		Environment: Deterministic	Environment: Stochastic
Static	Constraint Satisfaction	*Vars + Constraints* Arc Consistency Search → for CSP SLS	
Static	Inference	*Logics* CSP Search → for Inference	*Belief Nets* Var. Elimination
Sequential	Planning	*STRIPS* → CSP Search → for complex planning	*Decision Nets* Var. Elimination *Markov Processes* Value Iteration

Representation

Reasoning Technique

Standard Search vs. Specific R&R systems

Constraint Satisfaction (Problems):

- State: assignments of values to a subset of the variables
- Successor function: assign values to a "free" variable
- Goal test: set of constraints
- Solution: possible world that satisfies the constraints
- Heuristic function: *none (all solutions at the same distance from start)*

Planning :

- State possible world
- Successor function states resulting from valid actions
- Goal test assignment to subset of vars
- Solution sequence of actions
- Heuristic function empty-delete-list (solve simplified problem)

Logical Inference

- State answer clause
- Successor function states resulting from substituting one atom with all the clauses of which it is the head
- Goal test empty answer clause
- Solution start state
- Heuristic function number of atoms in given state

Learning Goals for today's class

You can:

- Model a relatively simple domain with propositional definite clause logic (PDCL)
- Define/read/write/trace/debug the TopDown proof procedure

Midterm: this Wed March 4

Tue3 office hours
<2 - 4>

SAME ROOM – 1.5 hours

~10 short questions (~6pts each) + 2 problems (~20pts each)

- Study: textbook and **inked** slides
- Work on **all** practice exercises
- Work-on/Study the posted **learning goals**, **review questions** (I may even reuse some verbatim ☺), **two problems** from previous offering (solutions also posted ☺)