

# Stochastic Local Search Variants

Computer Science cpsc322, Lecture 16  
*(Textbook Chpt 4.8)*

February, 9, 2009



# Lecture Overview

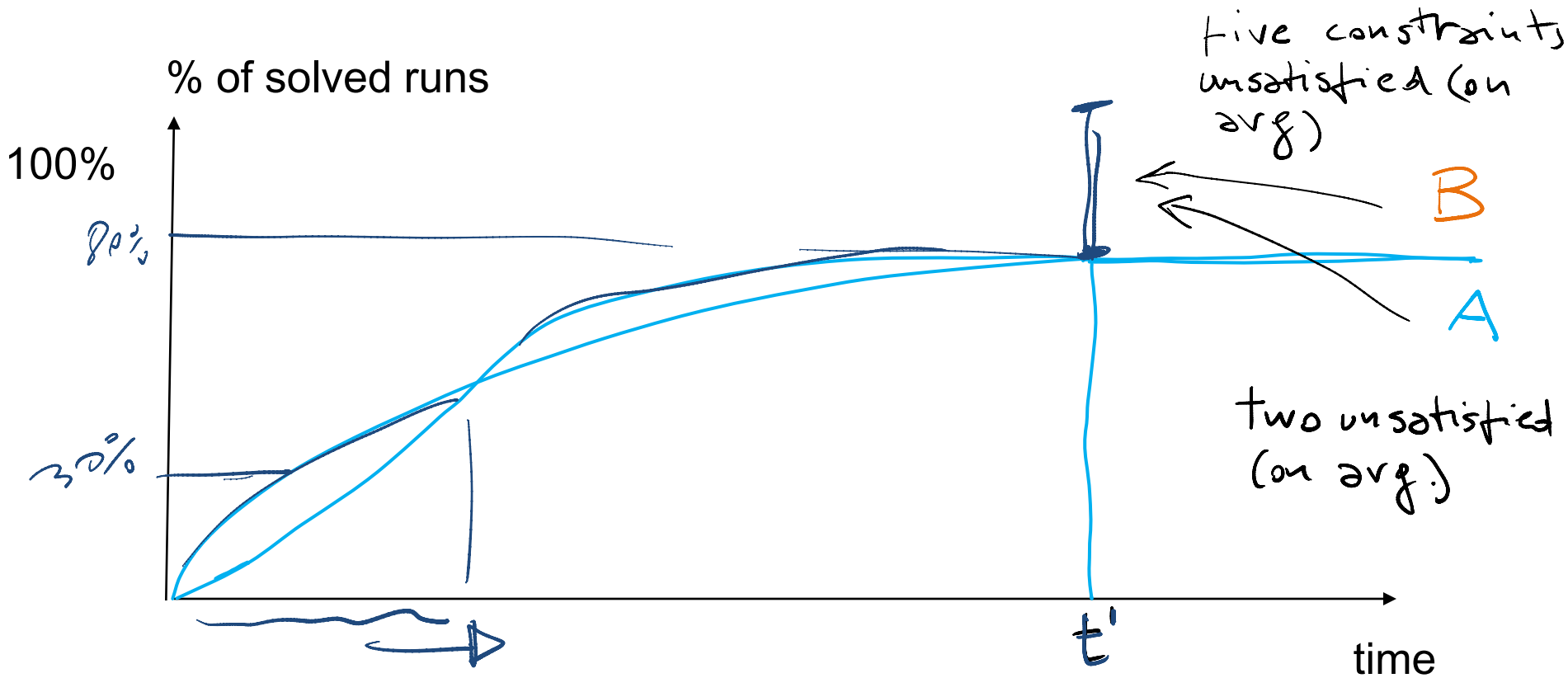
- **Recap SLS**
- SLS variants

# Stochastic Local Search

min max

- **Key Idea:** combine greedily improving moves with randomization
- As well as improving steps we can allow a “small probability” of:
  - Random steps: move to a random neighbor. ← 9% ↵
  - Random restart: reassign random values to all variables. ← 1% ↵
- Always keep best solution found so far ←
- Stop when
  - Solution is found (in vanilla CSP ..... satisfying all C .....)
  - Run out of time (return best solution so far)

# Runtime Distributions



Which one would you use if you could wait  $t = t'$  ?

you should look at the quality of the answers on the unsolved problems. In pure CSP # of unsatisfied constraints so use A

# Lecture Overview

- Recap SLS
- **SLS variants**
  - Tabu lists
  - Simulated Annealing
  - Beam search
  - Genetic Algorithms

# Tabu lists

- To avoid search to
  - Immediately going back to previously visited candidate
  - To prevent cycling
- Maintain a **tabu list** of the  $k$  last nodes visited.
  - Don't visit a poss. world that is already on the **tabu list**.
- Cost of this method depends on.....

# Simulated Annealing

- **Key idea:** Change the degree of randomness....
- **Annealing:** a metallurgical process where metals are hardened by being slowly cooled.
  - Analogy: start with a high ``temperature": a high tendency to take random steps
  - Over time, cool down: more likely to follow the scoring function
- Temperature reduces over time, according to an annealing schedule

# Simulated Annealing: algorithm

Here's how it works (for maximizing):

- You are in node  $n$ . Pick a variable at random and a new value at random. You generate  $n'$
- If it is an improvement i.e.,  $h(n') > h(n)$ , adopt it.
- If it isn't an improvement, adopt it probabilistically depending on the difference and a temperature parameter,  $T$ .  
 $h(n) > h(n') \Rightarrow h(n') - h(n) < 0$
- we move to  $n'$  with probability  $e^{(h(n') - h(n))/T}$

see next slide

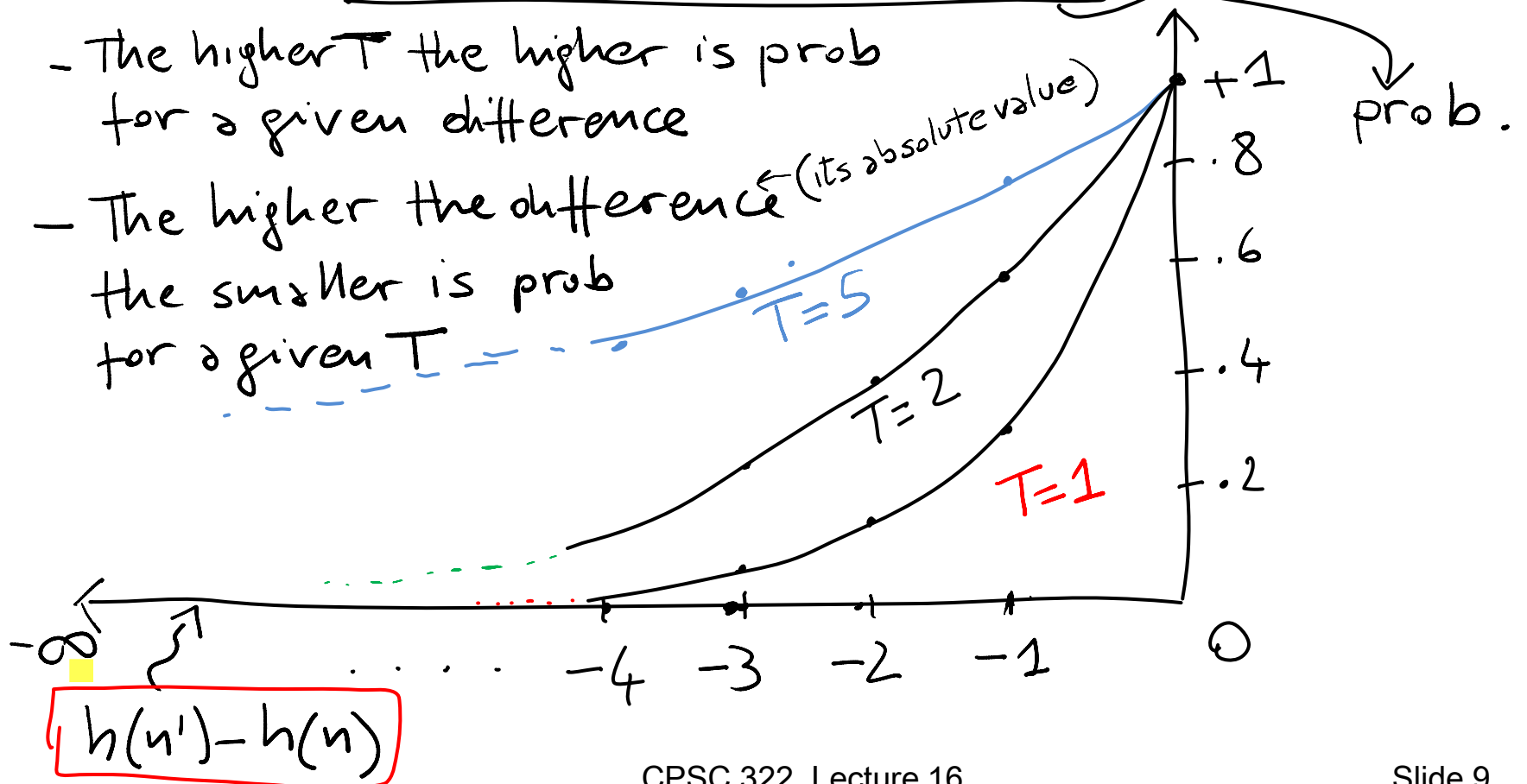


- If it isn't an improvement, adopt it probabilistically depending on the difference and a temperature parameter,  $T$ .

$$h(n) > h(n') \Rightarrow h(n') - h(n) < 0$$

- we move to  $n'$  with probability  $e^{(h(n')-h(n))/T}$


- The higher  $T$  the higher is prob for a given difference
- The higher the difference the smaller is prob for a given  $T$



# Properties of simulated annealing search

One can prove: If  $T$  decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1

Widely used in VLSI layout, airline scheduling, etc.



# Lecture Overview

- Recap SLS
- SLS variants
  - Simulated Annealing
  - **Population Based**
    - ✓ Beam search
    - ✓ Genetic Algorithms

# Population Based SLS

Often we have more memory than the one required for current node (+ best so far + tabu list)

**Key Idea:** maintain a population of  $k$  individuals

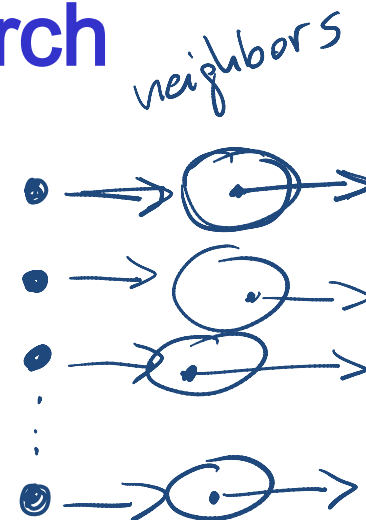
- At every stage, update your population.
- Whenever one individual is a solution, report it.

## Simplest strategy: Parallel Search

- All searches are independent
- Like  $k$  restarts

*but more memory :)*  
*No reason to use it*

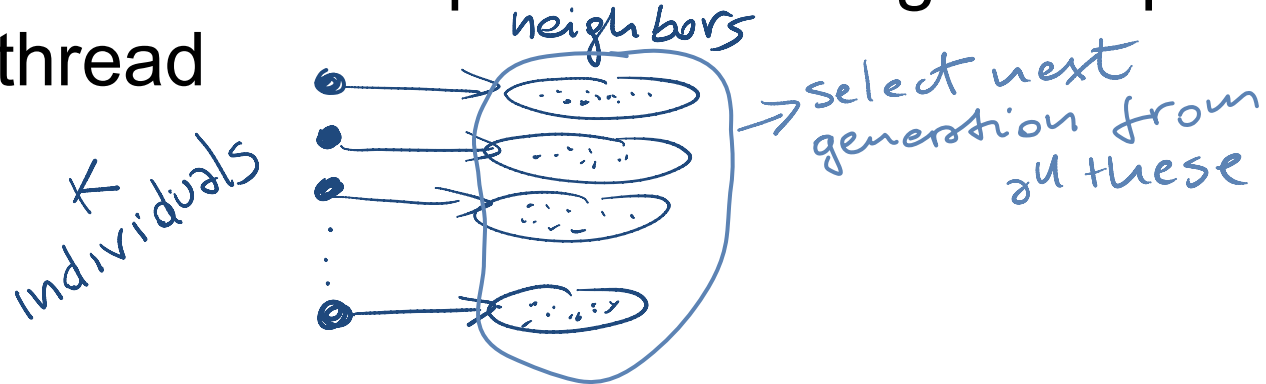
*k poss worlds*



# Population Based SLS: Beam Search

## Non Stochastic

- Like parallel search, with  $k$  individuals, but you choose the  $k$  best out of **all of the neighbors**.
- Useful information is passed among the  $k$  parallel search threads



- **Troublesome case:** If one individual generates several good neighbors and the other  $k-1$  all generate bad successors....

the next generation will comprise very similar individuals

# Population Based SLS: Stochastic Beam Search

- **Non Stochastic** Beam Search may suffer from lack of diversity among the  $k$  individual (just a more expensive hill climbing)
- **Stochastic** version alleviates this problem:
  - Selects the  $k$  individuals at random
  - But probability of selection proportional to their value

$m$  neighbors  $\{n_1 \dots n_m\}$

$h$ : scoring function

Probability of selecting ( $n_a$ ) =

according to scoring function

$$\frac{h(n_a)}{\sum_{i=1}^m h(n_i)}$$

# Stochastic Beam Search: Advantages

- It maintains diversity in the population.
- **Biological metaphor** (asexual reproduction):
  - ✓ each individual generates “mutated” copies of itself (its neighbors)
  - ✓ The scoring function value reflects the fitness of the individual
  - ✓ the higher the fitness the more likely the individual will survive (i.e., the neighbor will be in the next generation)

# Lecture Overview

- Recap SLS
- SLS variants
  - Simulated Annealing
  - Population Based
    - ✓ Beam search
    - ✓ **Genetic Algorithms**

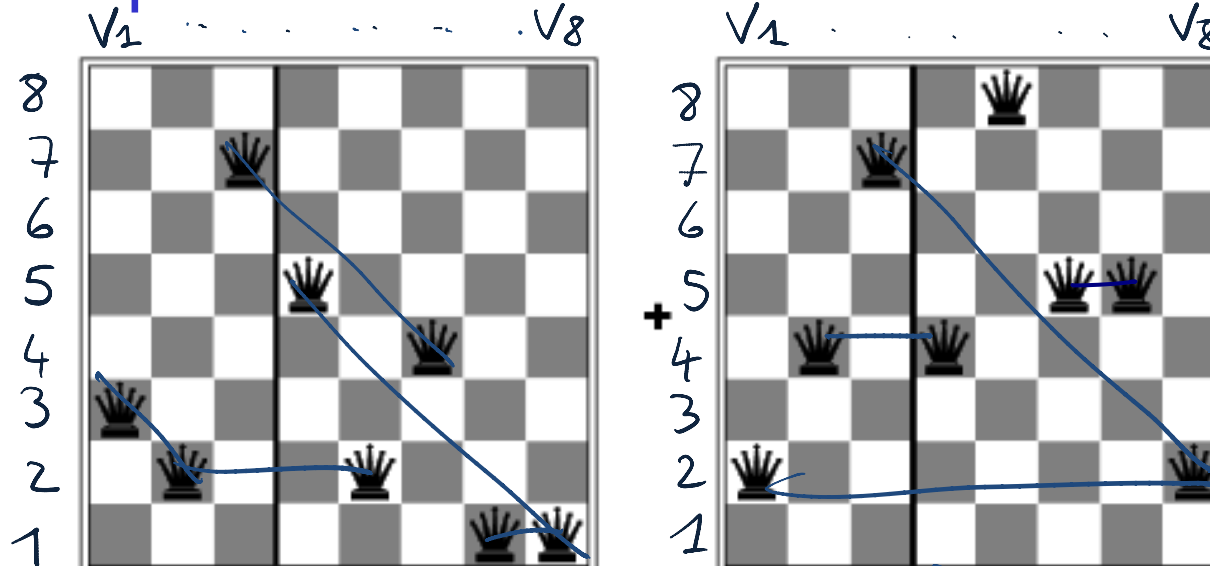


# Population Based SLS: Genetic Algorithms

- Start with  $k$  randomly generated individuals (population)
- An individual is represented as a string over a finite alphabet (often a string of 0s and 1s)
- A successor is generated by combining two parent individuals (loosely analogous to how DNA is spliced in sexual reproduction)
- Evaluation/Scoring function (fitness function). Higher values for better individuals.
- Produce the next generation of individuals by selection, crossover, and mutation

# Genetic algorithms: Example 8-queen

## Representation and fitness function



# of queen pairs  
possibly attacking  
each other

$$\frac{8 \cdot 7}{2} = 28$$

**State:** string over finite alphabet

24748552

**Fitness function:** higher value

32752411

better states. # queen pairs not  
attacking each other

(28-4)

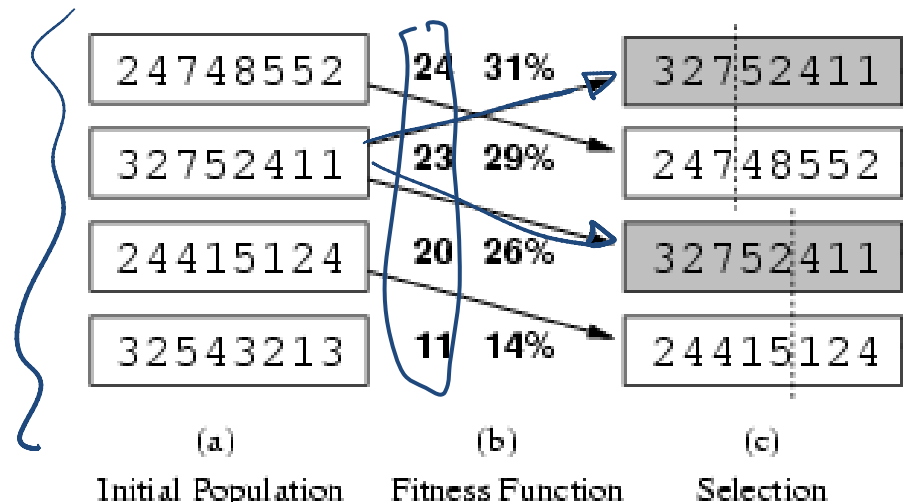
24

23

(28-5)

# Genetic algorithms: Example

**Selection:** common strategy, probability of being chosen for reproduction is directly proportional to fitness score



$$\rightarrow 24/(24+23+20+11) = 31\%$$

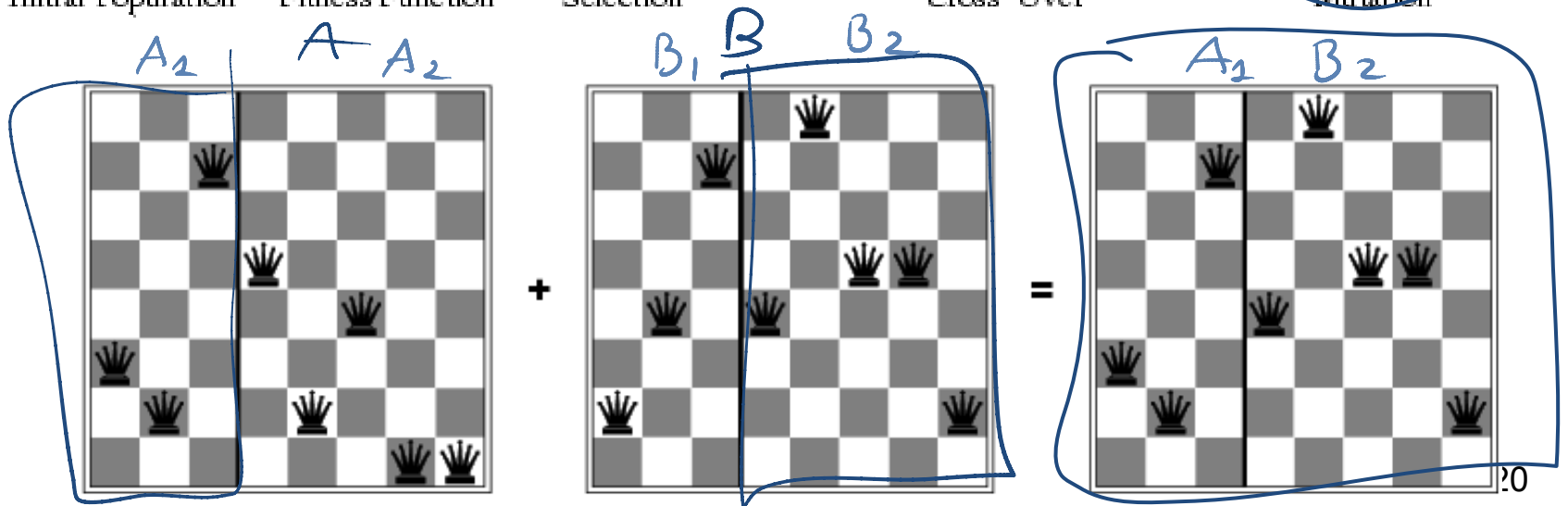
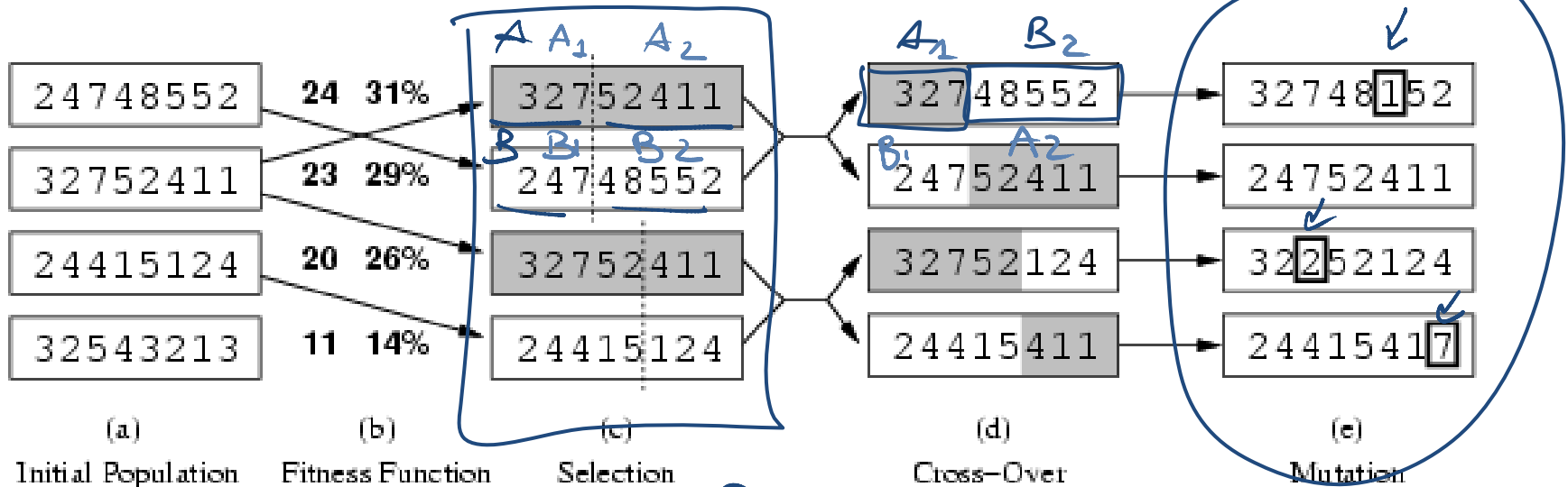
$$\rightarrow 23/(24+23+20+11) = 29\% \text{ etc}$$

.....

same as Beam Search  
slide 14

# Genetic algorithms: Example

## Reproduction: cross-over and mutation



# Genetic Algorithms: Conclusions

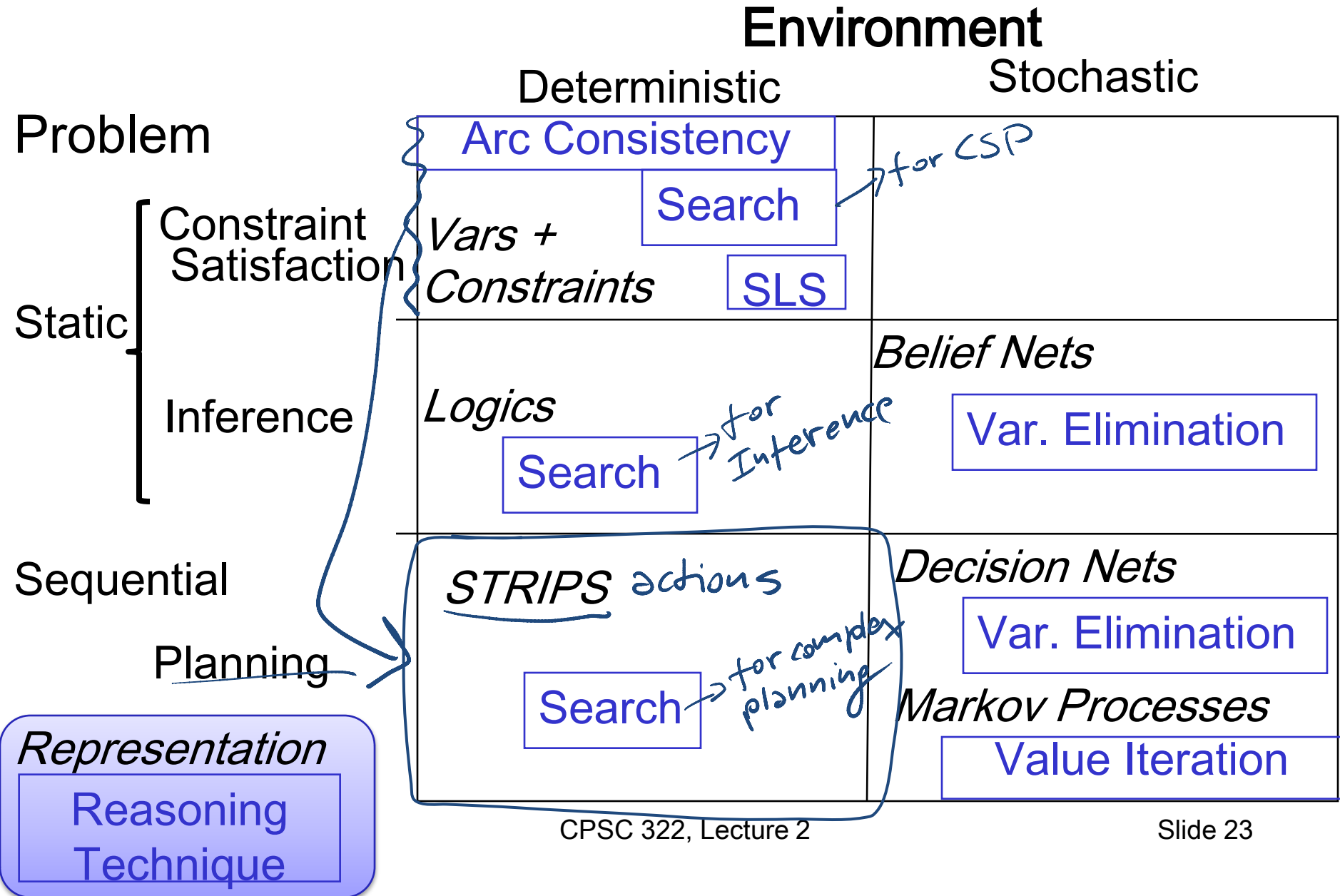
- Their performance is very sensitive to the choice of state representation and fitness function
- Extremely slow (not surprising as they are inspired by evolution!)

# Learning Goals for today's class

**You can:**

- Implement a tabu-list.
- Implement the simulated annealing algorithm
- Implement population based SLS algorithms:
  - Beam Search
  - Genetic Algorithms.
  - Explain pros and cons

# Modules we'll cover in this course: R&Rsys



# Next class

Start Planning (Chp 11)



# Feedback summary



|   |    |   |   |       |
|---|----|---|---|-------|
| • Assignments ( <i>prog. , unclear</i> )        | 7  | 1 | 7 | (0)   |
| • TAs   | 0  | 0 | 1 | (-1)  |
| • Textbook                                      | 6  | 2 | 2 | (+4)  |
| • Lectures ( <i>more interactive</i> )          | 5  | 5 | 1 | (+4)  |
| • Practice Exercises ( <i>one per lecture</i> ) | 6  | - | 1 | (+5)  |
| • Course Topics                                 | 6  | 1 | - | (+6)  |
| • Learning Goals                                | 6  | - | - | (+6)  |
| • Slides ( <i>hard to read</i> )                | 10 | 1 | 3 | (+7)  |
| • <u>Alspace</u>                                | 13 | 1 | 1 | (+12) |
| • <u>Exams...</u>                               |    |   |   |       |

lots of sample questions

# What is coming next?

How to select and organize a sequence of actions to achieve a given goal...

.....

# Systematically solving CSPs: Summary

- Build Constraint Network
- Apply Arc Consistency
  - One domain is empty → *no solution* ↙
  - • Each domain has a single value → *unique solution* ↙
  - Some domains have more than one value →  
*may or may not be a solution* ↙
- Apply Depth-First Search with Pruning ↙
- Split the problem in a number of disjoint cases ↙
  - • Apply Arc Consistency to each case ↙

# CSPs summary

Find a single variable assignment that satisfies all of our constraints (atemporal)

- Systematic Search approach (search space .....?)
  - Constraint network support  $n^2d^3$ 
    - ✓ inference e.g., Arc Consistency (can tell you if solution does not exist)
    - ✓ Decomposition
  - Heuristic Search (degree, min-remaining)
- (Stochastic) Local Search (search space .....?)
  - Huge search spaces and highly connected constraint network but solutions densely distributed
  - No guarantee to find a solution (if one exists).
  - Unable to show that no solution exists