CPSC 520
Numerical methods for time-dependent partial differential equations

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This course explores numerical methods based on finite difference and finite volume discretizations for solving time-dependent partial differential equations (PDEs). Such mathematical models arise in many diverse applications, e.g. fluid flow, image processing and computer vision, animation, mechanical systems, earth sciences and mathematical finance. We will methodically develop tools for understanding and assessing methods and software for these problems.

The course will start at a relatively basic level, and elementary knowledge of PDEs and numerical methods is a sufficient prerequisite. Talk to me if you are uncertain. Towards the end of the course we will discuss advanced techniques required for problems with nonlinearities, multi-dimensions, interfaces, and discontinuities such as shocks. Many examples will be given.

A fairly complete set of course notes will be available. I'll be absorbed in trying to transform them, hopefully with your help, into a book.

http://www.cs.ubc.ca/spider/ascher/520.html

Tentative course outline

1. Initial value PDE problems [1 week]
   (a) Well-posed initial value problems
   (b) A taste of finite differences
2. Methods and concepts for initial value ODEs [2 weeks +]
   (a) Linear multistep and Runge-Kutta methods
   (b) Stability and stiffness
3. Finite difference and finite volume methods [1 week +]
   (a) Semi-discretization

    (b) Boundary conditions
    (c) Full discretization

4. Stability for constant coefficient PDEs [1 week -]
    (a) Fourier analysis

5. Variable coefficient and nonlinear PDEs [1 week +]
    (a) Freezing coefficients and dissipativity
    (b) Schemes for hyperbolic systems in one dimension
    (c) Upwind and centered schemes
    (d) Nonlinear stability and "energy methods"

6. Hamiltonian systems and long time integration [1 week]
    (a) Hamiltonian systems
    (b) Symplectic methods

7. Dispersion and dissipation [1 week]
    (a) Dispersion
    (b) The classical wave equation
    (c) The KdV equation

8. More on handling boundary conditions [1 week -]
    (a) BC for hyperbolic problems
    (b) Infinite or large domains

9. Problems in several space variables and splitting methods [2 weeks +]
    (a) Extending the methods we already know
    (b) Solving large algebraic systems for implicit methods
    (c) Splitting methods