Stat 302 Assignment 2

Please remember to **INCLUDE A COVER SHEET** when you submit your assignment. It is due on Wednesday, 31st March at 5pm in the designated drop off boxes next to the Statistics general office in the LSK building. When answering the questions, writing down the final answer will not be sufficient to receive full marks. Please show all calculations unless otherwise specified.

Q1. (10) The number of UBC students infected by H1N1 is a Poisson random variable with rate 1/5,000 per week.

(a) What is the probability that there are more than 10 students infected in a week?

(b) What is the probability that in the next 4 weeks there will be at least 2 weeks with more than 10 infected students each?

Q2. (10) (a) Suppose X is $Binomial(n, p)$, with probability mass function $f(k; n, p)$. show that

$$\frac{f(k; n, p)}{f(k-1; n, p)} = 1 + \frac{(n+1)p - k}{k(1-p)}.$$

(b) Suppose X is $Binomial(n, p)$ and Y is $Binomial(n, 1-p)$, show that

$$P(X \geq k) = P(Y \leq n-k), k = 0, 1, 2, \ldots, n.$$

Q3. (10) Assume that the number of alpha particles emitted by a radioactive substance follows a Poisson process with rate $\lambda = 4$ per minute.

(a) What is the probability that exactly three particles will be counted during a minute?

(b) What is the probability that no more than three particles will be counted during a minute?

(c) What is the expected amount of time you need to observe the 100^{th} particle is emitted? (Hint: use exponential distribution to answer this question.)

Q4. (10) Suppose that in a certain country commercial airplanes crashes occur at a rate of 1.5 per year. Assuming the frequency of crashes per year to be a Poisson random variable, find the probability that three or more crashes will occur next year. Also, find the probability that the next two crashes will occur within three months of one another. (Hint: Let Y denote the interval length between consecutive occurrences of an event described by a Poisson distribution. If the Poisson events are occuring at a rate of λ per unit time, then we have $f_Y(y) = \lambda e^{-\lambda y},\ y > 0$.)

Q5. (10) You are allowed to take a certain test three times, and your final score will be the maximum of the test scores. Thus,

$$X = \max\{X_1, X_2, X_3\},$$

where X_1, X_2, X_3 are the three test scores and X is the final score. Assume that your test scores are (integer) values between 1 and 10 with equal probability 1/10, independently from each other. What is the probability mass function of the final score?

Q6. (10) The metro train arrives at the station, always on time, near your home every quarter hour starting at 6:00 AM. You walk into the station every morning between 7:10 and 7:30 AM, with the time in this interval being a uniform random variable.

(a) What is the probability density function of the amount of the time, in minutes, that you have to wait for the first train to arrive?

(b) What is the expected waiting time?

(c) What is the median waiting time?

(d) Which of the expected waiting time and the median waiting time is a better summary? Why?

Q7. (10) Suppose that the probability density function of a random variable X is as follows:

$$f(x) = \begin{cases} \frac{1}{8}x & \text{for } 0 < \text{x} < 4, \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of t such that $P(X \leq t) = 1/4$.

(b) Find the value of t such that $P(X \geq t) = 1/2$.

(c) After the value of X has been observed, let Y be the integer closest to X, and Y is set equal to zero in case of tie. Find the probability mass function of the random variable Y.

Q8. (10) A system consists of 3 components arranged in series. The lifetime (in days) of each component follows approximately an exponential distribution with a mean lifetime of 100 days. The lifetimes of the components are independent.

(a) What is the probability that the first component lasts between 50 and 100 days?

(b) What is the probability that 2 of the 3 components have lifetimes between 50 and 150 days?

(c) What is the cumulative distribution function of the lifetime of the entire system? What are the corresponding median and mean lifetime?

(d) If the 3 components are arranged in parallel, what is the c.d.f. of the lifetime of the entire system? What is the corresponding median lifetime?

Q9. (10) Suppose that X is a normal random variable with mean 3 and variance 4.

(a) Calculate $P(X > 4)$,

(b) Calculate $P(2 < X < 4)$,

(c) Calculate $P(|X - 3| < 2)$,

(d) How does the the probability in (c) compare with $P(|X - 2| < 2)$ (larger / equal / smaller)? Why?

(e) Find c such that $P(|X - 3| < c) = 0.90$

Q10. (10) A machine fills 25-pound bags of dry concrete mix. The actual weight of the mix that is put in the bag is a normal random variable with standard deviation $\sigma = 0.05\mu$ pound. The mean μ can be set by the machine operator.

(a) At what mean weight should the machine be set so that at most 10 per cent of the bags are underweight?

(b) Find the corresponding mean weight to be set for larger 50-pound bags.