

# Semantics: General Idea

A **semantics** specifies the meaning of sentences in the language.

An **interpretation** specifies:

- what objects (individuals) are in the world
- the correspondence between symbols in the computer and objects & relations in world
  - constants denote individuals
  - predicate symbols denote relations

# Formal Semantics

An **interpretation** is a triple  $I = \langle D, \phi, \pi \rangle$ , where

- $D$ , the **domain**, is a nonempty set. Elements of  $D$  are **individuals**.
- $\phi$  is a mapping that assigns to each constant an element of  $D$ . Constant  $c$  **denotes** individual  $\phi(c)$ .
- $\pi$  is a mapping that assigns to each  $n$ -ary predicate symbol a relation: a function from  $D^n$  into  $\{TRUE, FALSE\}$ .

# Example Interpretation

**Constants:** *phone*, *pencil*, *telephone*.

**Predicate Symbol:** *noisy* (unary), *left\_of* (binary).

➤  $D = \{ \langle \text{scissors} \rangle, \langle \text{phone} \rangle, \langle \text{pencil} \rangle \}$ .

➤  $\phi(\text{phone}) = \langle \text{phone} \rangle$ ,  $\phi(\text{pencil}) = \langle \text{pencil} \rangle$ ,  $\phi(\text{telephone}) = \langle \text{phone} \rangle$ .

➤  $\pi(\text{noisy})$ :

$\langle \text{scissors} \rangle$	<i>FALSE</i>	$\langle \text{phone} \rangle$	<i>TRUE</i>	$\langle \text{pencil} \rangle$	<i>FALSE</i>
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$\pi(\text{left\_of})$ :

$\langle \text{scissors}, \text{scissors} \rangle$	<i>FALSE</i>	$\langle \text{scissors}, \text{phone} \rangle$	<i>TRUE</i>	$\langle \text{scissors}, \text{pencil} \rangle$	<i>TRUE</i>
$\langle \text{phone}, \text{scissors} \rangle$	<i>FALSE</i>	$\langle \text{phone}, \text{phone} \rangle$	<i>FALSE</i>	$\langle \text{phone}, \text{pencil} \rangle$	<i>TRUE</i>
$\langle \text{pencil}, \text{scissors} \rangle$	<i>FALSE</i>	$\langle \text{pencil}, \text{phone} \rangle$	<i>FALSE</i>	$\langle \text{pencil}, \text{pencil} \rangle$	<i>FALSE</i>

## Important points to note

- The domain  $D$  can contain real objects. (e.g., a person, a room, a course).  $D$  can't necessarily be stored in a computer.
- $\pi(p)$  specifies whether the relation denoted by the  $n$ -ary predicate symbol  $p$  is true or false for each  $n$ -tuple of individuals.
- If predicate symbol  $p$  has no arguments, then  $\pi(p)$  is either *TRUE* OR *FALSE*.

# Truth in an interpretation

Each ground term denotes an individual in an interpretation.

A constant  $c$  denotes in  $I$  the individual  $\phi(c)$ .

Ground (variable-free) atom  $p(t_1, \dots, t_n)$  is

- true in interpretation  $I$  if  $\pi(p)(t'_1, \dots, t'_n) = \text{TRUE}$ , where  $t_i$  denotes  $t'_i$  in interpretation  $I$  and
- false in interpretation  $I$  if  $\pi(p)(t'_1, \dots, t'_n) = \text{FALSE}$ .

Ground clause  $h \leftarrow b_1 \wedge \dots \wedge b_m$  is false in interpretation  $I$  if  $h$  is false in  $I$  and each  $b_i$  is true in  $I$ , and is true in interpretation  $I$  otherwise.



# Example Truths

In the interpretation given before:

$noisy(phone)$	true
$noisy(telephone)$	true
$noisy(pencil)$	false
$left\_of(phone, pencil)$	true
$left\_of(phone, telephone)$	false
$noisy(pencil) \leftarrow left\_of(phone, telephone)$	true
$noisy(pencil) \leftarrow left\_of(phone, pencil)$	false
$noisy(phone) \leftarrow noisy(telephone) \wedge noisy(pencil)$	true



# Models and logical consequences

- A knowledge base,  $KB$ , is true in interpretation  $I$  if and only if every clause in  $KB$  is true in  $I$ .
- A **model** of a set of clauses is an interpretation in which all the clauses are true.
- If  $KB$  is a set of clauses and  $g$  is a conjunction of atoms,  $g$  is a **logical consequence** of  $KB$ , written  **$KB \models g$** , if  $g$  is true in every model of  $KB$ .
- That is,  $KB \models g$  if there is no interpretation in which  $KB$  is true and  $g$  is false.



# Simple Example

$$KB = \begin{cases} p \leftarrow q. \\ q. \\ r \leftarrow s. \end{cases}$$

	$\pi(p)$	$\pi(q)$	$\pi(r)$	$\pi(s)$	
$I_1$	TRUE	TRUE	TRUE	TRUE	is a model of $KB$
$I_2$	FALSE	FALSE	FALSE	FALSE	not a model of $KB$
$I_3$	TRUE	TRUE	FALSE	FALSE	is a model of $KB$
$I_4$	TRUE	TRUE	TRUE	FALSE	is a model of $KB$
$I_5$	TRUE	TRUE	FALSE	TRUE	not a model of $KB$

$KB \models p, KB \models q, KB \not\models r, KB \not\models s$





# User's view of Semantics

- ① Choose a task domain: **intended interpretation.**
- ② Associate constants with individuals you want to name.
- ③ For each relation you want to represent, associate a predicate symbol in the language.
- ④ Tell the system clauses that are true in the intended interpretation: **axiomatizing the domain.**
- ⑤ Ask questions about the intended interpretation.
- ⑥ If  $KB \models g$ , then  $g$  must be true in the intended interpretation.



# Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If  $KB \models g$  then  $g$  must be true in the intended interpretation.
- If  $KB \not\models g$  then there is a model of  $KB$  in which  $g$  is false. This could be the intended interpretation.