Computational Intelligence

A Logical Approach

Problems for Chapter 2

Here are some problems to help you understand the material in Computational Intelligence: *A Logical Approach*. They are designed to help students understand the material and practice for exams.

This file is available in html, or in pdf format, either without solutions or with solutions. (The pdf can be read using the free acrobat reader or with recent versions of Ghostscript).

1 Models and Logical Consequences (ground)

Given the knowledge base:

$a \leftarrow b \wedge c.$

$a \leftarrow g.$

$b \leftarrow d.$

$b \leftarrow f.$

$c \leftarrow e.$

$d \leftarrow h.$

$e.$

$f \leftarrow e.$

where $\{a, b, c, d, e, f, g, h\}$ is the set of all atoms.

(a) Give a model of the knowledge base.

(b) Give an interpretation that is not a model of the knowledge base.

(c) Give two atoms that are logical consequences of the knowledge base.

(d) Give two atoms that are not logical consequences of the knowledge base.

Solution to part (a)

Give a model of the knowledge base.

One model is where all of the atoms are true.

Another model is where e, f, c, b, a are true and d, g, h are all false. [This is the minimal model.]

Solution to part (b)

Give an interpretation that is not a model of the knowledge base.

There are lots of possibilities. Just choose a clause to be false, say the first one, and fill if the other variables arbitrarily.

The interpretation with a false, b and c true and all the other atoms true isn't a model of the knowledge base, as the first clause is false in this interpretation.

The interpretation with a false, b and c true and all the other atoms false isn't a model of the knowledge base, as the first (and seventh) clauses are false in this interpretation.

Solution to part (c)

Give two atoms that are logical consequences of the knowledge base.

Answer: Any two of e, f, c, b, a.

Solution to part (d)

Give two atoms that are not logical consequences of the knowledge base.

Answer: Any two of d, g, h.

2 Interpretations and Models (with variables)

Suppose we had a domain with two individuals, x and y. Suppose we had two predicate symbols p and q and three constants a, b, and c. Suppose we had the knowledge base KB defined by

$$p(X) \leftarrow q(X).$$

$$q(a).$$

(a) Give one interpretation that is a model of KB.

(b) Give one interpretation that is not a model of KB.

(c) How many interpretations are there? Give a brief justification for your answer.

(d) How many of these interpretations are models of KB? Give a brief justification for your answer.

Solution to part (a)

Give one interpretation that is a model of KB.

$D = \{x, y\}$ (this is true of all the interpretations.)

$\phi(a) = x, \phi(b) = x, \phi(c) = x.$

$\pi(p)(< x >) = TRUE. \pi(p)(< y >) = TRUE.$

$\pi(q)(< x >) = TRUE. \pi(q)(< y >) = TRUE.$

Solution to part (b)

Give one interpretation that is not a model of KB.

Both clauses of KB are false in the following interpretation:

$D = \{x, y\}$

$\phi(a) = x, \phi(b) = x, \phi(c) = x.$

$\pi(p)(< x >) = TRUE. \pi(p)(< y >) = FALSE.$

$\pi(q)(< x >) = FALSE. \pi(q)(< y >) = TRUE.$

Solution to part (c)

How many interpretations are there? Give a brief justification for your answer.

There are two possible denotations for a, two for b and two for c. Thus there are $2^3 = 8$ possible ϕ's.

There are two possible values for $\pi(p)(< x >)$, and two for $\pi(p)(< x >)$. Thus there are 4 possible denotations for p.

Similarly, there are 4 possible denotations for q.

Thus there are $8 * 4 * 4 = 128$ different interpretations.

Solution to part (d)

How many of these interpretations are models of KB? Give a brief justification for your answer.

There are the same number of possible ϕ's (the KB doesn't constrain this).

For each of the ϕ's, there are two values for the $\pi(q)$. (Depending on whether q of the other individual (whichever of x and y isn't denoted by a) is true or false).

For the case where q is true of both individuals, p must also be true of both individuals. There are thus 8 interpretations where q is true of both individuals.

For the case where q is true of just one individual (it must be the individual denoted by a), p must be true of that individual and can either be true or false of the other individual. There are thus 16 interpretations where q is true of one individual.

Thus there are $8 + 16 = 24$ models of KB.

3 Proofs and Logical Consequences (ground)

Given the knowledge base *KB* containing the clauses:

$a \leftarrow b \wedge d.$
$b \leftarrow e \wedge f.$
$c \leftarrow h \wedge e.$
$d \leftarrow e.$
$d \leftarrow b \wedge g.$
$e \leftarrow h.$
$g \leftarrow c \wedge d.$
$h.$

(a) Show how the bottom-up proof procedure works for this example. Show at each stage the value of C. Give all logical consequences of *KB*.

(b) a isn't a logical consequence of *KB*. Explain what this means. Show why a isn't a logical consequence of *KB*.

(c) g is a logical consequence of *KB*. Explain what this means. Give a top-down derivation for the query $?g$.

Solution to part (a)

The following shows the elements of C at each stage:

$\{h\}$
$\{e, h\}$
$\{c, e, h\}$
$\{d, c, e, h\}$
$\{g, d, c, e, h\}$

The set of logical consequences of *KB* is $\{g, d, c, e, h\}$.

Solution to part (b)

"a isn't a logical consequence of *KB*" means there exists a model of *KB* in which a is false.

To show why a isn't a logical consequence of *KB*, we can give such a model. The minimal model will always be one such model. This model has $\{h, e, d, c, g\}$ all true and $\{a, b, f\}$ all false.

Solution to part (c)

g is a logical consequence of *KB* means that g is true in all models of *KB*.

Here is a top-down derivation for the query ?g, where we always select the leftmost atom to resolve against:

$yes \leftarrow g$

$yes \leftarrow c \wedge d$

$yes \leftarrow h \wedge e \wedge d$

$yes \leftarrow e \wedge d$

$yes \leftarrow h \wedge d$

$yes \leftarrow d$

$yes \leftarrow e$

$yes \leftarrow h$

$yes \leftarrow$

4 Unification

For each of the following pairs of atoms, either give a most general unifier, or explain why one doesn't exist.

(a) $p(X, Y, a, b, W)$
$p(E, c, F, G, F)$

(b) $p(X, Y, Y)$
$p(E, E, F)$

(c) $p(Y, a, b, Y)$
$p(c, F, G, F)$

(d) $ap(F0, c(b, c(B0, L0)), c(a, c(b, c(b, c(a, emp)))))$
$ap(c(H1, T1), L1, c(H1, R1))$

Solution to part (a)

To unify: $p(X, Y, a, b, W)$ and $p(E, c, F, G, F)$
One mgu: $\{X/E, Y/c, F/a, G/b, W/a\}$.
Another mgu: $\{X/Z, E/Z, Y/c, F/a, G/b, W/a\}$.

Solution to part (b)

To unify: $p(X, Y, Y)$ and $p(E, E, F)$
One mgu: $\{X/E, Y/E, F/E\}$.
Another mgu: $\{X/Z, Y/Z, F/Z, E/Z\}$.

Solution to part (c)

To unify: $p(Y,a,b,Y)$ and $p(c,F,G,F)$

There is no unifier. We can't have $c = Y = F = a$; we can't have any substitution that makes these terms have be identical.

Solution to part (d)

To unify: $ap(F0,c(b,c(B0,L0)),c(a,c(b,c(b,c(a,emp)))))$ and $ap(c(H1,T1),L1,c(H1,R1))$

Unique mgu: $\{F0/c(a,T1), L1/c(b,c(B0,L0)), H1/a, R1/c(b,c(b,c(a,emp)))\}$

5 Proofs (with variables)

Consider the following knowledge base:

```
ap(emp,L,L).
ap(c(H,T),L,c(H,R)) <-
   ap(T,L,R).
adj(A,B,L) <-
   ap(F,c(A,c(B,E)),L).
```

(a) Give a top down derivation (including all substitutions) for one answer to the query:

```
? adj(b,Y,c(a,c(b,c(b,c(a,emp))))).
```

(b) Are there any other answers? If so, explain where a different choice could be made in the derivation in the previous answer, and continue the derivation showing another example. If there are no other answers explain why not.

[You are meant to do this exercise as would a computer, without knowing what the symbols mean. If you want to give a meaning to this program, you could read *ap* as *append*, *c* as *cons*, *emp* as *empty*, and *adj* as *adjacent*.]

Solution to part (a)

Give a top down derivation (including all substitutions) for one answer to the query:

```
? adj(b,Y,c(a,c(b,c(b,c(a,emp))))).
```

Here is a top-down derivation:

```
yes(Y) <- adj(b,Y,c(a,c(b,c(b,c(a,emp))))).
  choose clause 3, with {A/b,B/Y,L/c(a,c(b,c(b,c(a,emp)))),F/F1,E/E1}
yes(Y) <- ap(F1,c(b,c(Y,E1)),c(a,c(b,c(b,c(a,emp)))))
  choose clause 2, under
```

```
  {F1/c(a,T2),L/c(b,c(Y,E1)),H/a,R/c(b,c(b,c(a,emp))),T/T2}
yes(Y) <- ap(T2,c(b,c(Y,E1)),c(b,c(b,c(a,emp))))
  *** choose clause 1 under {T2/emp,Y/b,E1/c(b,c(a,emp))}
yes(b) <-
```

Solution to part (b)

Are there any other answers? If so, explain where a different choice could be made in the derivation in the previous answer, and continue the derivation showing another example. If there are no other answers explain why not.

Yes, there is one more answer (if you weren't sure you should have run the program!).

```
   at *** choose clause 2 under
   {T2/c(b,T3),L/c(b,c(Y,E1)),H/b,R/c(b,c(a,emp)),T/T3}
yes(Y) <- ap(T3,c(b,c(Y,E1)),c(b,c(a,emp)))
   choose clause 1 under {T3/emp,L/c(b,c(a,emp)),Y/a,E1/emp}
yes(a) <-
```