Stochastic Simulation

- **Idea:** probabilities $\leftrightarrow$ samples
- Get probabilities from samples:

X	$count$
x_1	n_1
$\vdots$	$\vdots$
x_k	n_k
$total$	m

$\leftrightarrow$

X	$probability$
x_1	n_1/m
$\vdots$	$\vdots$
x_k	n_k/m

- If we could sample from a variables (posterior) probability, we could estimate its (posterior) probability.

Generating samples from a distribution

- For a variable X with a discrete domain or a (one-dimensional) real domain:
  - Totally order the values of the domain of X.
  - Generate the cumulative probability distribution: $f(x) = P(X \leq x)$.
  - Select a value y uniformly in the range [0, 1].
  - Select the x such that $f(x) = y$.

Forward sampling in a belief network

- Sample the variables one at a time; sample parents of X before you sample X.
- Given values for the parents of X, sample from the probability of X given its parents.

Rejection Sampling

- To estimate a posterior probability given evidence $Y_1 = v_1 \wedge \ \ldots \wedge Y_j = v_j$:
- If, for any i, a sample assigns Y_i to any value other than v_i reject that sample.
- The non-rejected samples are distributed according to the posterior probability.

Importance Sampling

- If we can compute $P(evidence|sample)$ we can weight the (partial) sample by this value.
- To get the posterior probability, we do a weighted sum over the samples; weighting each sample by its probability.
- We don't need to sample all of the variables as long as we weight each sample appropriately.
- We thus mix exact inference with sampling.

Particle Filtering

- Suppose the evidence is $e_1 \wedge e_2$
  $P(e_1 \wedge e_2|sample) = P(e_1|sample)P(e_2|e_1 \wedge sample)$
- After computing $P(e_1|sample)$, we may know the sample will have an extremely small probability.
- Idea: we use lots of samples: "particles". A particle is a sample on some of the variables.
- Based on $P(e_1|sample)$, we resample the set of particles. We select from the particles according to their weight.
- Some particles may be duplicated, some may be removed.